# A COMPARATIVE STUDY OF SITE-DEPENDENT EARTHQUAKE RESPONSE ANALYSES -- TIME-HISTORY VERSUS SMOOTH RESPONSE SPECTRUM SRSS

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SUMMARY

This paper presents the results of a comparative study of the maximum structural responses to a set of 12 site-dependent ground acceleration time-histories. The study quantifies statistically and probabilistically the amount of conservatism associated with the use of maximum response based on the square-root-of-the-sum-of-the-squares (SRSS) combination in conjunction with smooth single earthquake response spectra for multidegree-of-freedom structures as compared to the maximum response obtained from a time-history analysis.

## INTRODUCTION

There are two fundamental types of structural dynamic analyses, time-history and response spectrum, which are used today to design large and important structures for the effects of earthquake ground motion. The time-history analysis technique involves direct integration to develop structural response as a function of time from which the "true" maximum response can be obtained. The response spectrum technique uses a pre-determined spectrum of maximum modal responses (in this case single earthquake smooth mean and mean-plus-one-sigma spectra) from which the maximum response can be estimated by a combination of each participating mode.

Use of the response spectrum technique involves uncertainty in both the shape of the spectrum and the method used to combine maximum modal responses. This paper investigates the conservatism inherent in a new method (outlined in the following sections) used to quantify the shape of response spectra for design.

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## BACKGROUND

Recently, a probabilistic site-dependent response spectrum criteria was developed (Ref. 1, 2, 3) which utilized the recorded earthquake data at a given site. The assumption was made that earthquakes are unique events and the recorded ground motions in the form of acceleration time-histories or response spectra cannot be scaled up or down to establish meaningful seismic design criteria. Smooth mean and mean-plus-one-sigma response spectra for each component of a single earthquake record are constructed using a non-linear regression to fit a five-parameter mean value model. These five shape parameters plus a variability parameter are then studied statistically for several earthquake records at a given site. It was shown (Ref. 1 and 2) that these response spectra parameters are correlated from one earthquake to the next and that a comprehensive site-dependent response spectrum criteria assumes these parameters to be jointly distributed dependent random variables. A comparative study of these probabilistic criteria with conventional response spectrum criteria was presented in Ref. 3.

This paper continues with a comparative study of the maximum structural responses computed using time-history techniques and smooth mean and mean-plus-one-sigma response spectra SRSS techniques. Included in the study are 12 main event earthquakes which were recorded at the El Centro Station during the 34-year period from 1934 to 1968 (see Table 1). It should be noted that the examples presented in this paper have natural frequencies spaced far enough apart to eliminate any problems associated with combining closely spaced modes.

## THE RESPONSE SPECTRUM MODEL

A typical single earthquake acceleration response spectrum for 5 percent critical damping is shown in Fig. 1, along with the smooth mean and mean-plus-one-sigma spectra superimposed.

The smooth mean value model has five shape parameters and is written:

$$\overline{SA} = C1 \cdot e^{-(C2 \cdot T)} + C3 \cdot T^{C4} \cdot e^{-(C5 \cdot T)} \qquad (1)$$

where $\overline{SA}$ is the mean spectral acceleration at period T, and C1 through C5 are the parameters determined from statistical analyses of the response spectrum data for a given earthquake record. The first term, C1, corresponds to the PGA, while C2 is determined by a simple statistical analysis of the spectral ordinate data in the period range of 5 to 10 seconds. Thus, C1 and C2 define the short and long period ends of the spectrum. Parameters C3, C4, and C5 define the spectrum shape in the region of dynamic amplification and are determined by employing a multiple-nonlinear regression analysis in which the model is fitted to the response spectrum data to minimize the squared error.

A sixth parameter, the root-mean-square (RMS), quantifies the variability of the response spectrum data about the mean value model for a given earthquake and is used to determine the mean-plus-one-sigma model. Development of the mean-plus-one-sigma model is discussed in Ref. 3.

## THE COMPARISON

Two multiple degree-of-freedom text book structures from Ref. 4 and 5 are used as examples in this comparison. The dynamic properties of the structures are given in Table 2. Both structures are subjected to 12 horizontal ground acceleration time-histories which were recorded at the El Centro Station. A level of five percent critical modal damping is assumed and the peak base shears and drifts are determined for each of 12 base excitations. Similarly, the maximum SRSS modal responses for base shear and drift for single component earthquakes are computed using smooth mean and mean-plus-one-sigma response spectra for the same 12 earthquakes. The peak values of base shear and drift for the two example structures are listed in Tables 3 and 4, with the ratios of maximum time-history response to SRSS modal response listed in Tables 5 and 6. Average ratios and standard deviations are shown at the bottom of each of the columns of ratios.

## OBSERVATIONS AND CONCLUSIONS

The following observations and conclusions are made from the results of this comparison:

- Different responses (e.g., base shear and drift) for a given structure differ in the amount of conservatism inherent in the use of the smooth response spectrum SRSS technique.

- Different structures differ in the amounts of conservatism their responses realize using the smooth response spectrum SRSS technique.

- The average ratios of maximum response are dependent upon the structural dynamic properties and differ from one structure to the next, although these averages are consistent with the definitions of the mean and mean-plus-one-sigma smooth spectra.

- In all cases the ratios of maximum response are widely scattered for the 12 site-dependent earthquakes considered, as indicated by the standard deviation of the ratios.

From the results of this study it is seen that the computed reliability of a structure subjected to site-dependent earthquake loads is dominated by the uncertainty inherent in the response spectrum loading criteria. Assuming the ratio of maximum response for a given site and structure are lognormally distributed the probability that the "true" time-history response will exceed a smooth response spectrum SRSS response for the same earthquake motion may be computed. Examples of such probability calculations for a mean and a mean-plus-one-sigma smooth response spectrum criteria are shown in Table 7. It is therefore possible to choose a level of conservatism in the probabilistic site-dependent response spectrum criteria which is consistent with the overall cost-benefit considerations of the structural integrity of a building.

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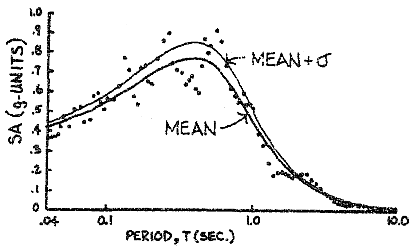



TYPICAL RECORDED AND SMOOTH RESPONSE SPECTRA

Figure 1

## Table 1

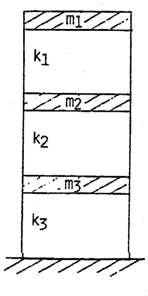
Earthquakes Recorded at Station No. 117,
El Centro Imperial Valley Irrigation District
(32.80 N 115.55 W)

| No. | Date | Location | Eipcentral Coordinates N. Lat.* | W. Long.* | Magnitude (RM) | Epicentral Distance (km) |
|---|---|---|---|---|---|---|
| 1 | 30 Dec 1934 | South of Calexico | 32.25 | 115.5 | 6.5 | 61.1 |
| 2 | 5 June 1938 | Imperial Valley | 32.9 | 115.2 | 5.0 | 33.0 |
| 3 | 6 June 1938 | Imperial Valley | 32.25 | 115.2 | 4.0 | 70.7 |
| 4 | 18 May 1940 | Southeast of Imperial Valley | 32.7 | 115.5 | 6.7 | 8.5 |
| 5 | 21 Oct 1942 | Near Borrego Valley | 33.0 | 116.0 | 6.5 | 46.1 |
| 6 | 23 Jan 1951 | Near Calipatria | 33.0 | 115.7 | 5.6 | 26.8 |
| 7 | 13 June 1953 | Imperial Valley | 32.9 | 115.7 | 5.5 | 23.0 |
| 8 | 12 Nov 1954 | Baja, California | 31.5 | 116 | 6.3 | 150.4 |
| 9 | 16 Dec 1955 | Near Brawley | 33.0 | 115.5 | 5.4 | 22.9 |
| 10 | 9 Feb 1956 | Baja, California | 31.8 | 115.9 | 6.8 | 121.5 |
| 11 | 7 Aug 1966 | Gulf of California | 31.8 | 114.5 | 6.3 | 148.3 |
| 12 | 8 Apr 1968 | South of Ocotillo Wells | 33.2 | 116.1 | 6.4 | 69.4 |

* Degrees

## Table 2

Structural Properties




| | Structure 1 | Structure 2 |
|---|---|---|
| Masses | | |
| $m_1$ | 1.0 k-sec$^2$/in | 4.0 k-sec$^2$/in |
| $m_2$ | 1.5 | 8.0 |
| $m_3$ | 2.0 | 8.0 |
| Stiffnesses | | |
| $k_1$ | 60 K/in | 500 K/in |
| $k_2$ | 120 | 1000 |
| $k_3$ | 180 | 1500 |
| Periods | | |
| $T_1$ | 1.37 sec. | 1.00 sec. |
| $T_2$ | 0.640 | 0.459 |
| $T_3$ | 0.431 | 0.315 |

Table 3

Maximum Responses for Structure 1

| Year | Base Shear (kips) | | | Drift (inches) | | |
|---|---|---|---|---|---|---|
| | T/H | SA | SA+σ | T/H | SA | SA+σ |
| 1934 | 200 | 164 | 172 | 3.31 | 2.75 | 2.85 |
| 1938 | 7 | 25 | 25 | 0.120 | 0.440 | 0.440 |
| 1938 | 3 | 8 | 8 | 0.037 | 0.144 | 0.145 |
| 1940 | 349 | 466 | 493 | 5.16 | 8.04 | 8.46 |
| 1942 | 99 | 71 | 72 | 1.82 | 1.25 | 1.25 |
| 1951 | 55 | 42 | 68 | 0.933 | 0.669 | 0.989 |
| 1953 | 85 | 65 | 75 | 1.35 | 1.15 | 1.32 |
| 1954 | 46 | 50 | 92 | 0.683 | 0.875 | 1.60 |
| 1955 | 44 | 54 | 55 | 0.788 | 0.950 | 0.952 |
| 1956 | 154 | 118 | 130 | 2.45 | 2.10 | 2.30 |
| 1966 | 27 | 37 | 79 | 0.456 | 0.651 | 1.39 |
| 1968 | 358 | 274 | 302 | 6.49 | 4.96 | 5.46 |

Table 4

Maximum Responses for Structure 2

| Year | Base Shear (kips) | | | Drift (inches) | | |
|---|---|---|---|---|---|---|
| | T/H | SA | SA+σ | T/H | SA | SA+σ |
| 1934 | 1358 | 1058 | 1142 | 3.02 | 2.17 | 2.33 |
| 1938 | 44 | 134 | 134 | 0.082 | 0.281 | 0.281 |
| 1938 | 20 | 42 | 47 | 0.034 | 0.086 | 0.088 |
| 1940 | 3870 | 3060 | 3285 | 7.06 | 6.38 | 6.86 |
| 1942 | 327 | 372 | 381 | 0.719 | 0.756 | 0.767 |
| 1951 | 261 | 309 | 566 | 0.574 | 0.638 | 1.16 |
| 1953 | 344 | 402 | 471 | 0.674 | 0.845 | 0.988 |
| 1954 | 393 | 345 | 690 | 0.880 | 0.726 | 1.46 |
| 1955 | 279 | 308 | 315 | 0.533 | 0.629 | 0.636 |
| 1956 | 753 | 738 | 819 | 1.60 | 1.55 | 1.73 |
| 1966 | 207 | 218 | 501 | 0.422 | 0.458 | 1.05 |
| 1968 | 1332 | 1358 | 1494 | 2.41 | 2.87 | 3.15 |

## Table 5

Ratio of Time-History Maximum Response to Mean Spectrum SRSS Response

| | Base Shear | | Drift | |
|---|---|---|---|---|
| Year | Struct. 1 | Struct. 2 | Struct. 1 | Struct. 2 |
| 1934 | 1.22 | 1.28 | 1.20 | 1.39 |
| 1938 | 0.26 | 0.33 | 0.27 | 0.29 |
| 1938 | 0.37 | 0.47 | 0.26 | 0.39 |
| 1940 | 0.75 | 1.26 | 0.64 | 1.11 |
| 1942 | 1.38 | 0.88 | 1.46 | 0.95 |
| 1951 | 1.31 | 0.84 | 1.39 | 0.90 |
| 1953 | 1.30 | 0.85 | 1.17 | 0.80 |
| 1954 | 0.92 | 1.14 | 0.78 | 1.21 |
| 1955 | 0.81 | 0.91 | 0.83 | 0.85 |
| 1956 | 1.30 | 1.02 | 1.17 | 1.03 |
| 1966 | 0.74 | 0.95 | 0.70 | 0.92 |
| 1968 | 1.31 | 0.98 | 1.31 | 0.84 |
| Average | 0.97 | 0.91 | 0.93 | 0.89 |
| Std. Dev. | 0.39 | 0.28 | 0.41 | 0.31 |

## Table 6

Ratio of Time-History Maximum Response to Mean Plus One Sigma Spectrum SRSS Response

| | Base Shear | | Drift | |
|---|---|---|---|---|
| Year | Struct. 1 | Struct. 2 | Struct. 1 | Struct. 2 |
| 1934 | 1.16 | 1.19 | 1.16 | 1.30 |
| 1938 | 0.26 | 0.33 | 0.27 | 0.29 |
| 1938 | 0.35 | 0.42 | 0.26 | 0.38 |
| 1940 | 0.71 | 1.18 | 0.61 | 1.03 |
| 1942 | 1.37 | 0.86 | 1.46 | 0.94 |
| 1951 | 0.80 | 0.46 | 0.94 | 0.49 |
| 1953 | 1.13 | 0.73 | 1.02 | 0.68 |
| 1954 | 0.50 | 0.57 | 0.43 | 0.60 |
| 1955 | 0.80 | 0.89 | 0.83 | 0.84 |
| 1956 | 1.18 | 0.92 | 1.07 | 0.92 |
| 1966 | 0.34 | 0.41 | 0.33 | 0.40 |
| 1968 | 1.18 | 0.89 | 1.19 | 0.77 |
| Average | 0.82 | 0.74 | 0.80 | 0.72 |
| Std. Dev. | 0.39 | 0.30 | 0.41 | 0.30 |

## Table 7

Probability of Time-History Maximum Response Exceeding Smooth Spectra SRSS Response

| | Base Shear | | Drift | |
|---|---|---|---|---|
| | Struct. 1 | Struct. 2 | Struct. 1 | Struct. 2 |
| $P(T/H > \overline{SA})$ | 0.39 | 0.32 | 0.35 | 0.31 |
| $P(T/H > \overline{SA} + \sigma)$ | 0.25 | 0.17 | 0.24 | 0.15 |