# TIME OF MAXIMUM RESPONSE OF SINGLE-DEGREE-OF-FREEDOM OSCILLATOR TO EARTHQUAKE EXCITATION

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## SUMMARY

Two empirical regression models on the dependence of the relative time of maximum response of single-degree-of-freedom system subjected to recorded strong earthquake ground motion are presented. They enable the estimation of the time of maximum response in terms of (1) earthquake magnitude and epicentral distance, or (2) Modified Mercalli Intensity at the site. Both models also consider whether motion is horizontal or vertical, and the effects of the geologic conditions surrounding the site. The results are useful for the response spectrum approach in earthquake resistant design, as they provide information for superposition of different loads in time.

## INTRODUCTION

In response spectrum calculations, the times at which maxima occur are usually not considered. In a typical response spectrum approach, when computing an estimate of the maximum response of a multi-degree-of-freedom system by adding individual mode responses, it is assumed that it is possible, though not very likely, that all mode responses can experience maximum response at the same time. Depending on the type of the analysis, (a) the square root of the sum of the squared maximum responses of individual modes, or (b) the absolute sum of maximum mode responses, is used to represent the maximum response of the whole system. All of these methods may lead to poor estimates of the maximum response of the whole system.

For certain applications, it is advantageous to consider the times when maxima of mode responses occur in a multi-degree-of-freedom system. To this end, this paper presents two empirical models for estimation of this time in terms of (a) earthquake magnitude and epicentral distance, and (b) Modified Mercalli Intensity at the site. The analysis will be restricted to the characterization of the time of maximum response of the viscously damped single-degree-of-freedom system as excited by recorded strong ground motion.

## DATA BASE AND REGRESSION

The data base for this analysis consists of 186 records of strong ground motion in the Western United States, each consisting of two horizontal acceleration traces and one vertical trace. These accelerograms were recorded during the period between 1933 and 1971, and constitute, so far, the largest uniformly processed group of strong motion records any place in the world.

Fig. 1 is an example of a plot of the ordinates of Response Envelope

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Spectra (RES) for PSV [1] versus time and frequency. The heavy irregular line indicates the times of maximum responses, TMAX(T), for each of the oscillators at different periods.

For all 186 records, the RES at 91 periods between 0.04 to 15 sec and for damping $\zeta = 0.0$, 0.02, 0.05, 0.10 and 0.20 are calculated. From these RES spectra, the times of maximum response were extraced for analysis.

Scaling in Terms of M, $\Delta$, s, v and p: For damping values $\zeta = 0.02$, 0.05, 0.10 and 0.20, we write

$$TMAX(T)_{,p} = a(T)p + b(T)s + c(T)M + d(T)\Delta + e(T)v \quad , \tag{1}$$

where $TMAX(T)_{,p}$ is the relative time of maximum response of the single-degree-of-freedom, viscously damped, oscillator that will not be exceeded in 100p% cases. This relative time is measured with respect to the earliest time of all maximum responses considered in the frequency band between 0.07 and 25 cps [2]. a(T)p represents a linear approximation to the actual distribution of TMAX(T) about the regression model. s represents the site conditions s=2 for hard basement rock, s=0 for alluvium. M is the published earthquake magnitude. $\Delta$ is the epicentral distance in kilometers and v=0,1 for horizontal and vertical motion, respectively.

a(T) through e(T) are the scaling functions of T determined from regression analysis. Fig. 2 presents these is smoothed form. Fig. 3 shows an example of the average ($p = 0.5$) TMAX(T) computed from (1) for $M = 6.5$, and $\Delta = 10$ for s=0 and 2. In general, and for long period motions in particular, the times of maximum response increase with decreasing damping.

Scaling in Terms of MMI, s, v and p: The following scaling function is used [2]

$$TMAX(T)_{,p} = a(T)p + b(T)s + c(T)I_{MM} + d(T)v \tag{2}$$

Here, $I_{MM}$ is the Modified Mercalli Intensity. Fig. 5 gives TMAX(T) for $I_{MM} = VI$. Fig. 4 presents the above smoothed coefficients.

## CONCLUSIONS

Two simple regression models have been analyzed [2] for scaling the times of maximum response in terms of earthquake magnitude or Modified Mercalli Intensity at the recording station. The principal findings of this work can be summarized as follows:

(1) Horizontal response amplitudes reach their maxima within several seconds after the S-wave arrival for periods shorter than about 4 sec for stations located on hard rock and for epicentral distance less than 20 to 30 km. For every additional 50 km in epicentral distance, maximum response is delayed by approximately 1 additional second.

(2) The times of maximum vertical response typically occur later by 1 (for periods near 0.1 sec) to 5 (for periods near 1 to 2 sec) seconds after horizontal response has reached its maximum.

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FIGURES

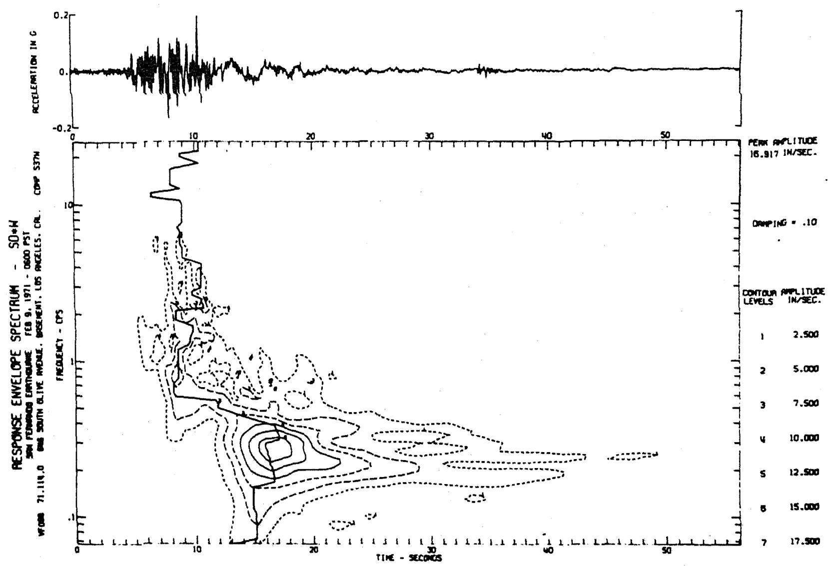


FIGURE 1

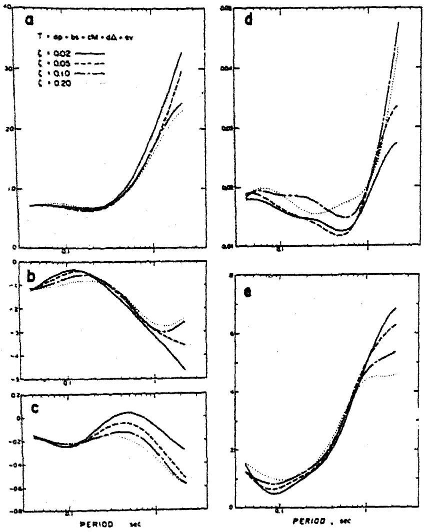


FIGURE 2

Functions a(T), b(T), ... and e(T) in
TMAX(T) = a(T)p + b(T)s + c(T)M + d(T)Δ + e(T)v

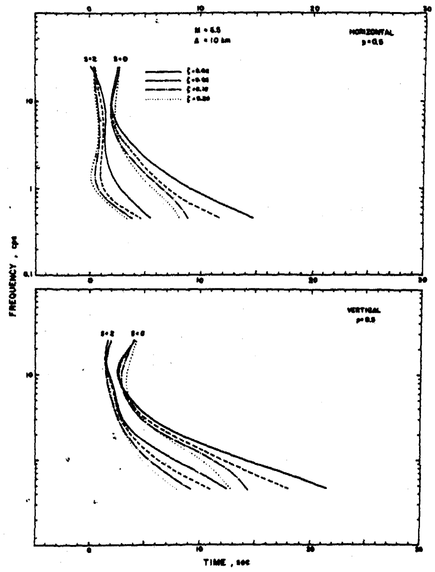


FIGURE 3

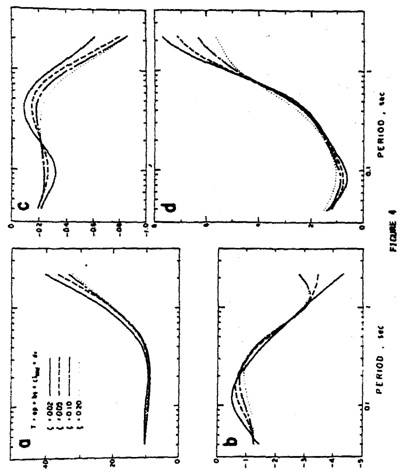


FIGURE 4

Functions a(T), b(T), ... and d(T) in
TMAX(T) = a(T)p + b(T)s + c(T)$I_{MM}$ + d(T)v

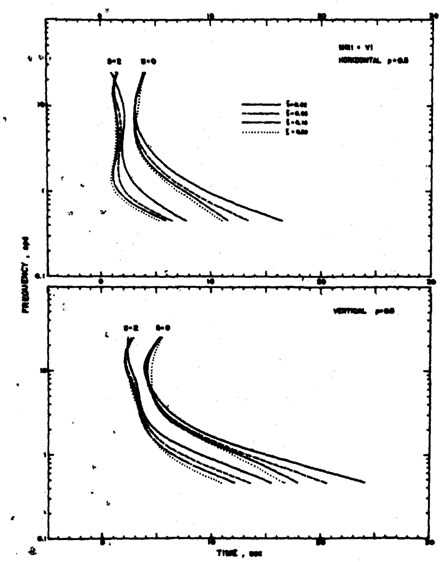


FIGURE 5