DISSIPATION AND DISPERSION OF SEISMIC WAVES IN WATER-SATURATED STRATA

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SUMMARY

Dissipation and dispersion of seismic waves in water-saturated strata are studied in some detail, based on the theory of wave propagation in porous, fluid-saturated media presented several years ago by the writer. The numerical results show that, besides the frequency, other three main factors having evident effect are the permeability of the media, the ratio of elastic wave reactance of the solid phase to that of the pore water and the initial porosity. Finally some conclusions are drawn and the significance of considering these effects in practical problems is emphasized.

INTRODUCTION

It is well known that more than 70 percent of the earth surface is covered with oceans. In continents, there is, generally, a ground water sphere in which water fills in, or flows through pores of soils and cracks, fissures and joints of rocks besides rivers, lakes and etc. on the surface. Though the depth of lower bound of saturated strata is not yet known exactly up to the present, it may be sure from a modern viewpoint of the earth and of hydrogeology that there may be possibility of water circulation, at least, in the whole domain of the earth crust. And within the upper part of the crust there may be more than several km of rocks containing pores and cracks, caused by weathering and cracking, through which water seepage flow can occur. So it is quite clear that seismic waves should propagate through the ground water sphere and/or sea water, the latter will again apply pressure disturbance to the former, before their reaching to the ground surface. So investigation of wave propagation in water-saturated media seems to be necessary.

But, up to the present, elastic wave theory is still predominatingly used as a basis in Geophysics and Earthquake Engineering. For materials of essentially identical with multi-phased, fluid-saturated media, M. A. Biot (1,2) had presented a series of famous papers including those on dynamic problems in 1956 to 1962. Since then, a few papers on related questions have been published, based on Biot's theory. In 1961, Tsien Hsue-shen (3) established a set of basic eqs. of soil dynamics by using a different means, taking into account the interaction between the two phases. But each of these two theories also has its disadvantage.

Extracting both the advantages of Biot's and Tsien's theory, a simplified and practicable theory was presented by the writer (4-6) in 1965 for incompressible and in 1978 for compressible pore fluid cases, the solid skeleton having been regarded as an elastic or viscoelastic one. In this article, as a first approximation, the water-saturated strata are regarded as composed of water-saturated elastic materials and dissipation

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and dispersion of seismic waves in the strata are studied on the basis of numerical computations in order to show how the pore water influences the wave propagation and what deviation should thus be resulted from the classical elastic wave theory. A further work, using a more complicated constitutive relation, is being made.

THEORETICAL BASIS

The model that a continous solid skeleton contains many inter-connected porous channels, saturated with water whose flow obeys Darcy's law, is concerned herein. By regarding the earth materials as an ideal porous elastic medium and the pore water as a compressible ideal fluid, the systems of eqs. of motion have been derived for two cases as follows in terms of displacement potentials defined later.

Case 1. Compressibility of solid grains is small enough than that of the solid skeleton.

$$\Box_1^2 \phi_1 = \left(-\sigma + \rho_2 \ddot{\phi}_2\right) \frac{1}{\lambda + 2\mu} \tag{1}$$

$$\nabla^2 \dot{\phi}_1 = \frac{f}{1-f}\left(-\nabla^2 \dot{\phi}_2 + \frac{1}{f E_w} \dot{\sigma}\right) \tag{2}$$

$$\sigma + b(\dot{\phi}_1 - \dot{\phi}_2) - \rho_2 \ddot{\phi}_2 = 0 \tag{3}$$

$$\Box_3^2 \vec{\psi}_1 = \frac{\rho_2}{\mu} \ddot{\vec{\psi}}_2 \tag{4}$$

$$b(\dot{\vec{\psi}}_1 - \dot{\vec{\psi}}_2) - \rho_2 \ddot{\vec{\psi}}_2 = 0 \tag{5}$$

Case 2. Compressibility of solid grains is of the same order as that of the solid skeleton.

$$\Box_1^2 \phi_1 = \left(-\sigma + \rho_2 \ddot{\phi}_2\right) \frac{1}{\lambda + 2\mu} \tag{1'}$$

$$\nabla^2 \dot{\phi}_2 + \left(\frac{1-f}{f} - \frac{K}{f E_s}\right) \nabla^2 \dot{\phi}_1 - \frac{1}{f}\left(\frac{1}{E_w} + \frac{1}{E_s}\right) \dot{\sigma} = 0 \tag{2'}$$

$$\sigma + b(\dot{\phi}_1 - \dot{\phi}_2) - \rho_2 \ddot{\phi}_2 = 0 \tag{3'}$$

$$\Box_3^2 \vec{\psi}_1 = \frac{\rho_2}{\mu} \ddot{\vec{\psi}}_2 \tag{4'}$$

$$b(\dot{\vec{\psi}}_1 - \dot{\vec{\psi}}_2) - \rho_2 \ddot{\vec{\psi}}_2 = 0 \tag{5'}$$

The displacement vector of the solid and of the fluid phases, $\vec{u}$ and $\vec{U}$, may be determined from the dilatational and the rotational displacement potential, ϕ and $\vec{\psi}$ as follows.

$$\vec{u} = \mathrm{grad}\,\phi_1 + \mathrm{rot}\,\vec{\psi}_1 \tag{6}$$

$$\vec{U} = \mathrm{grad}\,\phi_2 + \mathrm{rot}\,\vec{\psi}_2 \tag{7}$$

with $\quad \mathrm{div}\,\vec{\psi}_1 = \mathrm{div}\,\vec{\psi}_2 = 0$

In all above Eqs.

$\Box_j^2 = \nabla^2 - \frac{1}{C_j^2}\frac{\partial^2}{\partial t^2} \quad (j = 1, 2, \cdots)$ is d'Alembert's operator;

∇^2, Laplace operator; $C_1^2 = \frac{\lambda+2\mu}{\rho_1}$, $C_2^2 = \frac{\mu}{\rho_1}$, $\dot{\phi} = \frac{\partial \phi}{\partial t}$,
λ, μ, Lame elastic coef. of the solid skeleton, σ, the mean pore water pressure; f, the initial porosity of the medium; k, the coef. of permeability of the medium; $b = f^2/k$, E_s, the elastic modulus of the solid grains; K, the elastic bulk modulus of the solid skeleton, E_w, the elastic bulk modulus of water; $\rho_1 = \rho_s(1-f)$, $\rho_2 = \rho_f f$ where ρ_s, the density of solid phase; ρ_f, the density of water.

It has shown that there may be two P waves and one S wave in general cases.

DISSIPATION AND DISPERSION OF PLANE WAVES

To study this topic for a random plane wave, one needs first paying attention to that of harmonic plane waves in order to see how the phase velocity, c, and dissipation constant, β, vary with frequencies.

<u>P wave dispersion</u> Setting harmonic plane waves

$$\phi_j = A_j e^{i(l\vec{n}\cdot\vec{r} - \omega t)} \qquad (j = 1, 2)$$

in Eqs. 1-3, one can obtain that

$$c_p = 1/Re(l/\omega)_p \qquad \beta_p = \omega Im(l/\omega)_p$$

$$(l/\omega)_p = \pm\sqrt{\left(-B \pm \sqrt{B^2 - 4AC}\right)/2A}$$

where for case 1, defined in the above section

$$A = (\lambda+2\mu)\omega$$
$$B = -\left[ib\left(\frac{1}{f} + \frac{\lambda+2\mu}{fE_w}\right) + \left(\rho_1 + \frac{\lambda+2\mu}{fE_w}\rho_2\right)\omega\right]$$
$$C = \frac{1}{fE_w}\left[ib(\rho_1+\rho_2) + \omega\rho_1\rho_2\right]$$

and for case 2

$$A = (\lambda+2\mu)\omega$$
$$B = -\left\{ib\left[\frac{1}{f} - \frac{K}{fE_s} + \frac{\lambda+2\mu}{f}\left(\frac{1}{E_w} + \frac{1}{E_s}\right)\right] + \left[\rho_1 + \frac{\lambda+2\mu}{f}\left(\frac{1}{E_w} + \frac{1}{E_s}\right)\rho_2\right]\omega\right\}$$
$$C = \frac{1}{f}\left(\frac{1}{E_w} + \frac{1}{E_s}\right)\left[ib(\rho_1+\rho_2) + \omega\rho_1\rho_2\right]$$

<u>S wave dispersion</u> Setting harmonic plane waves

$$\vec{\psi}_j = \vec{B}_j e^{i(l\vec{n}\cdot\vec{r} - \omega t)} \qquad (j = 1, 2)$$

in Eqs. 4-5, one can get that

$$c_s = 1/Re(l/\omega)_s \qquad \beta_s = \omega Im(l/\omega)_s$$

$$(l/\omega)_s = \sqrt{X + iY}$$

$$X = \frac{\rho_2\omega^2 + (1+\rho_2/\rho_1)b^2}{C_{s0}^2\left[(\rho_2\omega)^2 + b^2\right]} \qquad Y = \frac{\rho_2^2\omega b}{C_{s0}^2\rho_1\left[(\rho_2\omega)^2 + b^2\right]} \qquad C_{s0}^2 = \frac{\mu}{\rho_1}$$

For many groups of data of b, $c_1 = \sqrt{(\lambda+2\mu)/\rho_1}$, $c_w = \sqrt{E_w/\rho_f}$, $c_K = \sqrt{E_s/\rho_1}$, ρ_1, ρ_2, f and ω, a detailed numerical computation has been made. In Table 1 and Fig. 1, 1a some extracts are shown to give a general concept.

MAIN RESULTS

The following trends may be gained from the computations.

1. The relations of phase velocities, dissipation constants of P waves to the frequency depend also on such factors as the permeability of the media, the ratio of the elastic wave reactances of the two phases, the initial porosity as well as the difference of compressibility between the solid grains and skeleton besides the frequency itself.

2. The two P waves in water-saturated media that independently take the solid wave (with $c_p = c_1$) and water wave (with $c_p = c_w$) as their limit under certain particular conditions are coupled with each other. One is dispersive and dissipative in a more extent and the other in a less extent, depending upon the elastic wave reactance of the solid and water phases. The wave is the less dispersive and dissipative, the larger the corresponding reactance.

3. Under the same other conditions, the two phase velocities of P wave, c_{p1} and c_{p2} , approach c_1 and c_w, respectively, and the dissipation constants, β , tend to zero, when the permeability, k , is quite large, say, 10^{-2} m/sec or more. Whereas, in contrast with this, both c_{p1} and c_{p2} become smaller and smaller, but with different rate, as k tends in succession to smaller values, and, at last, as k reaches an infinitecimal value, say, 10^{-8} m/sec or less, instead of them, a third value should be resulted from the fact that the relative motion between the two phases would not occur, and the medium behaves as a single-phased one. When k is smaller than 10^{-4} m/sec or so, only one P wave will be dominant and the other one will be damped out within a short distance.

4. The initial porosity, f, is also an important factor affecting the wave propagation. As an example, for the case where c_1=5000 m/sec, c_w=1500 m/sec, ρ_s =270 kg.s /m , ρ_f =100 kg.s /m , b =10 kg.s/m and ω = 1rad/sec the dominant c_p =2664 m/sec when f=0.05; while c_p =4335 m/sec when f =0.40, showing that the difference is quite large.

5. The error due to neglecting the compressibility of the solid grain and considering only that of the skeleton does not exceed 1-2%.

6. As to what extent of k Darcy's law holds under the dynamic conditions could not be answered clearly, following a comprehensive experimental investigation, a testing project and equipment is being devised by the writer and his coworkers in the institute.

7. S wave has a slight dispersion and dissipation. The phase velocity at the utmost may reach to $\sqrt{\rho_1/(\rho_1+\rho_2)}$ times of $c_{s0} = \sqrt{G/\rho_1}$, and the dissipation constant is quite small having an order of 10^{-3} 1/m or less. A more detailed description cannot be made here. Those who are interested in this respect may refer to (7).

CONCLUSIONS

Bearing the above and related results in mind, some conclusions may be drawn as follows.

1. The presence of pore water in the strata may cause a quite different kind of dispersion and dissipation arising from an interaction between the two phases in contrast with the results of the classical elastic wave theory.

2. Dispersion and dissipation due to variation of the frequency will be more evident when the permeability is low.

3. Depending on whether the solid phase or the water phase has a greater value of elastic wave reactance, a dominant P wave will approach the solid wave or the water wave, suffering slight dissipation, whereas the other one will have a greater dissipation and be damped out along a shorter travel path, especially, at low permeability. This trend is, in general, coincident with that predicted from Biot's theory.

4. As there are many factors affecting the wave propagation as pointed out above, it may happen that the **wave** first arrived is the water wave or may even be S wave (see Fig.2) in certain range of the permeability and frequency and for certain specific material properties of the two phases. For instance, it may be got that c_{p1} =429 m/sec, c_{p2}=1 m/sec, while c_s =2500 m/sec for the case where c_1 =5000 m/sec, c_w=1500 m/sec, ρ_1 =243 kg·s /m , ρ_2=10 kg·s /m , f=0.10, k =0.16 $\times 10^{-5}$ m/sec and ω = 1rad/sec. Therefore, it is necessary to carefully consider the pore water effect in the seismic wave analysis in Seismology and in the explanation of seismic measurement data in Geophysical Prospecting as well as in the treatment of dynamic problems of earth materials in Geomechanics. Otherwise some error must appear, which can clearly be seen from the above example, showing that what great error would be led if the first wave with c_s =2500 m/sec has been regarded as one P wave An another case can be seen from Table 1 that a dominant c_p of 1146 m/sec for the case 1.4 due to b =10^6 and ω =1rad/sec is markedly deviating from c_w=1500 m/sec and c_1 =500 m/sec, which means that the pore water effect cannot be neglected.

5. Except inhomogenity, anisotropy of earth materials, only the variation of permeability or porosity may also make the wave guide to occur, leading a little small disturbance to reach a long distance. And when k is large enough the pore water can even not make any contribution to wave dissipation, and it only leads an unfavorable result to aseismic designers because a hydraulic liquefaction may occur when the dynamic pore water pressure exceeds a certain value, or at least, a far reaching pressure may reach everywhere from the disturbing source and lowers the stability of the bearing strata.

6. A few words would be mentioned here about the role of the air or gas content in water. It may evidently influence wave propagation, especially, when the permeability is low enough. Due to the space limitation it will be discussed elsewhere.

Finally, a further comprehensive study seems to be necessary both in theoretical and in experimental to improve this preliminary investigation.

ACKNOWLEDGEMENT

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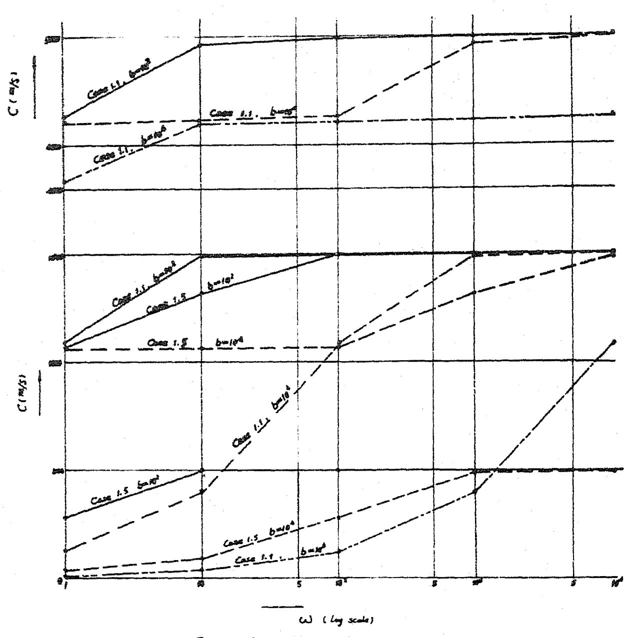

Fig 1. An Example of P wave Dispersion Curves

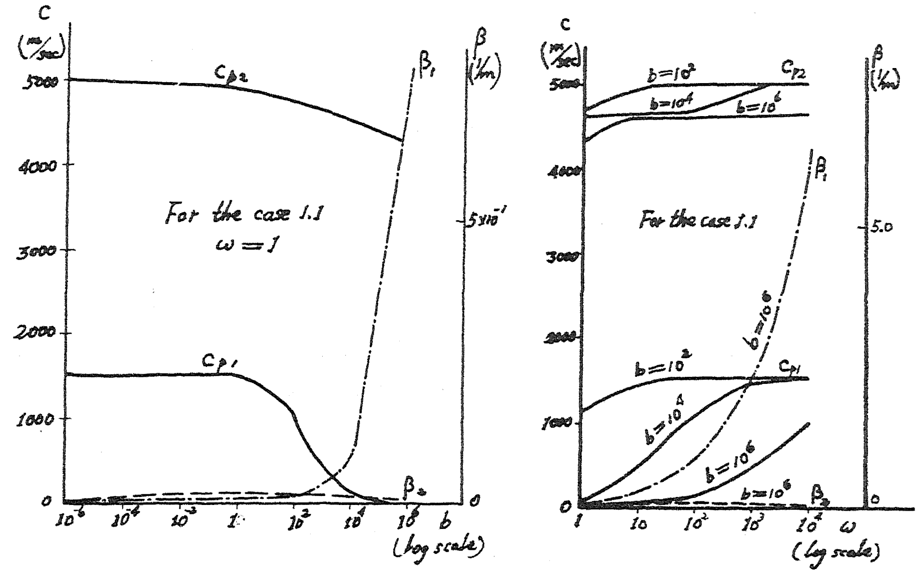

Fig. 1a Dispersion curves

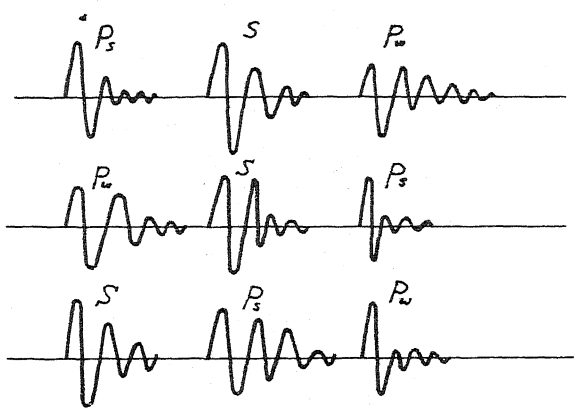

Fig. 2 Three possible orders of the wave arrival

Table 1 An Extract of Computation Results

ω C, β b ($s \cdot Kg/m^6$)	1		10		50		10^2		10^3	
	C (m/s)	β (1/m)	C (m/s)	β (1/m)	C (m/s)	β (1/m)	C (m/s)	β (1/m)	C (m/s)	β (1/m)
P wave Case 1.1 ($C_1=5000$, $C_w=1500$, $\rho_1=162$, $\rho_2=40$, $f=0.4$)										
10^6	13	0.77×10^{-1}	41	0.24	82	0.48	130	0.76	405	2.38
10^4	130	0.76×10^{-2}	405	0.24×10^{-1}	766	0.45×10^{-1}	1088	0.64×10^{-1}	1488	0.86×10^{-1}
10^2	1088	0.64×10^{-3}	1488	0.86×10^{-3}	1499	0.86×10^{-3}	1499	0.86×10^{-3}	1500	0.86×10^{-3}
1	1499	0.86×10^{-5}	1500	0.86×10^{-5}	1500	0.86×10^{-5}	1500	0.86×10^{-5}	1500	0.86×10^{-5}
10^{-6}	1500	0.86×10^{-11}	1500	0.86×10^{-11}	1500	0.86×10^{-11}	1500	0.86×10^{-11}	1500	0.86×10^{-11}
10^6	4335	-0.77×10^{-4}	4597	-0.82×10^{-4}	4600	-0.82×10^{-4}	4600	-0.77×10^{-4}	4600	0.48×10^{-3}
10^4	4600	0.39×10^{-6}	4600	0.60×10^{-5}	4606	0.89×10^{-4}	4635	0.51×10^{-3}	4964	0.48×10^{-2}
10^2	4635	0.51×10^{-5}	4964	0.48×10^{-4}	4997	0.52×10^{-4}	4999	0.53×10^{-4}	5000	0.53×10^{-4}
1	4999	0.53×10^{-6}	4999	0.53×10^{-6}	4999	0.52×10^{-6}	4999	0.53×10^{-6}	5000	0.53×10^{-6}
10^{-6}	5000	0.53×10^{-12}	4999	0.53×10^{-12}	4999	0.52×10^{-12}	4999	0.53×10^{-12}	5000	0.53×10^{-12}
P wave Case 1.4 ($C_1=500$, $C_w=1500$, $\rho_1=162$, $\rho_2=40$, $f=0.4$)										
10^6	5	0.19	16	0.60	33	1.20	52	1.90	161	5.88
10^4	52	0.19×10^{-1}	161	0.59×10^{-1}	300	0.11	411	0.15	499	0.17
10^2	411	0.15×10^{-2}	499	0.17×10^{-2}	499	0.17×10^{-2}	500	0.17×10^{-2}	500	0.17×10^{-2}
1	500	0.17×10^{-4}	500	0.17×10^{-4}	500	0.17×10^{-4}	500	0.17×10^{-4}	500	0.17×10^{-4}
10^{-6}	500	0.17×10^{-10}	500	0.17×10^{-10}	500	0.17×10^{-10}	500	0.17×10^{-10}	500	0.17×10^{-10}
10^6	1146	0	1146	0.71×10^{-5}	1146	0.14×10^{-4}	1146	0.78×10^{-4}	1146	0.81×10^{-2}
10^4	1146	0.18×10^{-5}	1146	0.82×10^{-4}	1152	0.13×10^{-2}	1182	0.72×10^{-2}	1477	0.46×10^{-1}
10^2	1182	0.72×10^{-4}	1477	0.46×10^{-3}	1498	0.48×10^{-3}	1499	0.49×10^{-3}	1500	0.49×10^{-3}
1	1499	0.49×10^{-5}	1500	0.49×10^{-5}	1500	0.48×10^{-5}	1500	0.49×10^{-5}	1500	0.49×10^{-5}
10^{-6}	1500	0.49×10^{-11}	1500	0.49×10^{-11}	1500	0.48×10^{-11}	1500	0.49×10^{-11}	1500	0.49×10^{-11}
S wave Case 1 ($C_s=3000$, $\rho_1=162$, $\rho_2=40$)										
10^6	2686	0.15×10^{-8}	2686	0.15×10^{-7}			2686	0.15×10^{-6}	2686	0.15×10^{-5}
10^4	2686	0.15×10^{-5}	2687	0.15×10^{-4}			2722	0.13×10^{-3}	2977	0.96×10^{-4}
10^2	2722	0.26×10^{-3}	2977	0.19×10^{-3}			2999	0.21×10^{-4}	3000	0.21×10^{-5}
1	2999	0.31×10^{-4}	3000	0.31×10^{-5}			3000	0.31×10^{-6}	3000	0.31×10^{-7}
10^{-6}	3000	0.41×10^{-10}	3000	0.41×10^{-11}			3000	0.41×10^{-12}	3000	0.41×10^{-13}