

# EARTHQUAKE FORCE PREDICTION CONSIDERING LOCAL SEISMIC SOURCE ACTIVITY

by

Teruo KAMADA<sup>I)</sup>

## SUMMARY

A practical method is presented for determining the seismic activity of the local seismic source area by the stochastic treatment of the instrumental and historical information on the earthquake occurrence. Supposing that the events of earthquake occurrence constitute a Poisson process, some parameters expressing the local seismic activity are evaluated based on Bayes' theorem and the seismic risk at a site in Kinki district in Japan is discussed as a case study.

## INTRODUCTION

The seismic risk analysis is a useful means of predicting the earthquake force and responses of structures. So that the analysis is reasonable, the locations of the potential earthquake source areas and their activities should be set up adequately based on the past earthquake data. The earthquake observations have been carried out in Japan since the first seismograph was installed in the 1870s. Although the data obtained through the systematic earthquake observation for the last several decades are very accurate, the observation period is still too short to evaluate the seismic activity of the local source area from these instrumental data only. On the other hand, the historical earthquakes which occurred in and near Japan were examined and the date of occurrence, location of epicentral region and magnitude of each earthquakes were determined. In order to estimate the local seismic activity reasonably, both instrumental and historical earthquake records should be taken into account.

## ESTIMATION OF LOCAL SEISMIC SOURCE ACTIVITY

According to Richter's law, the seismic activity of a source area can be expressed by:

$$v = e^{\alpha - \beta m} \quad (1)$$

where  $v$  is the mean number of earthquakes per unit area and per unit time having magnitude greater than  $m$ , and  $\alpha$  and  $\beta$  are zone dependent constants.

Fig. 1 shows the rectangular seismic source area of Kinki district in Japan. This area is not considered highly seismic, but several earthquakes of the highest magnitude occurred historically. Dividing this area into  $7 \times 7$  subareas by the longitude and the latitude with  $0.5^\circ$  width, the earthquake activities of each subareas are examined based on the earthquake data from 1961 to 1978 and are shown in Table 1, where  $n$  represents the number of earthquakes having magnitude greater than 4.0, and  $\alpha$  and  $\beta$  are calculated only for the subareas where more than five earthquakes were observed. The table also shows the values of the maximum magnitude  $m_m$  of the historical earthquakes observed in each subareas for the last 1200 years.

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I) Research Assistant of Faculty of Eng., Kyoto Univ., Kyoto JAPAN.

Because of relative shortness of the observation period, the number of earthquakes observed in this area is small and the estimation of  $\alpha$  and  $\beta$  are not always reliable. If the initial joint probability density function  $f(\alpha, \beta)$  is known, Bayes' theorem states that<sup>2)</sup>:

$$f'(\alpha, \beta | A) = \frac{f(\alpha, \beta)P[A|\alpha, \beta]}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta)P[A|\alpha, \beta]d\alpha d\beta} \quad (2)$$

where  $f'(\cdot)$  is the posterior probability density function when the event A is observed, and  $P[A|\alpha, \beta]$  is the probability of the event A for supposed parameters  $\alpha$  and  $\beta$ . Considering that the event A represents that an earthquake having magnitude greater than  $m$  occurred in the subarea for  $T$  years, the probability of the event A is expressed as follows:

$$\begin{aligned} P[A|\alpha, \beta] &= e^{-P_m \nu_0 T} P_m \nu_0 T, & \nu_0 &= e^{-\beta m_0} \\ P_m &= P[M > m] = 1 - F_m(m), & K_m &= \{1 - e^{-\beta(m_1 - m_0)}\}^{-1} \\ F_m(m) &= P[M \leq m] = K_m \{1 - e^{-\beta(m - m_0)}\} \end{aligned} \quad (3)$$

where  $m_0$  is the lowest magnitude whose contribution to risk is significant and  $m_1$  is the maximum feasible one.  $F_m(m)$  is the distribution function of magnitude.

Although the initial joint probability density function  $f(\alpha, \beta)$  is not always clear, it seems to obey the two dimensional normal distribution. Fig. 2 shows the distribution of  $\beta$  for the total subareas in and near Japan in the same period. The full line indicates the normal distribution which has the same mean and the same standard deviation. Rayleigh distribution having the same mean is also shown by dotted line. Supposing that  $f(\alpha, \beta)$  is expressed by the two dimensional normal distribution having the same mean and the standard deviation obtained from  $\alpha$ 's and  $\beta$ 's for the subareas in Kinki district ( $\bar{\alpha} = 5.957$ ,  $\bar{\beta} = 1.621$ ,  $\sigma_{\alpha} = 0.542$  and  $\sigma_{\beta} = 0.443$ ), and supposing that  $\alpha$  and  $\beta$  are mutually independent, the posterior joint probability density function is calculated and the expected values of  $\alpha$  and  $\beta$  are evaluated as shown in Fig. 3.

#### EVALUATION OF SEISMIC RISK

Several expressions for the peak earthquake ground motion at a site for a given magnitude  $m$  and focal or epicentral distance  $R$  are available. The peak acceleration  $Y$  in gal, proposed by Kanai, is:

$$\begin{aligned} Y &= b_1 e^{b_2 m} g(R) \\ b_1 &= 5/\sqrt{T_g}, & b_2 &= 0.61 \times \log 10 \\ g(R) &= 10^{-(1.66 + 3.60/R) \log_{10} R + (0.167 - 1.83/R)} \end{aligned} \quad (4)$$

where  $T_g$  is the predominant period of the ground at the site in second, and  $R$  is the focal distance in km.

Supposing that  $m$  and  $R$  are independent variables, the distribution function of  $Y$  is expressed as follows:

$$F_Y(y) = P[Y \leq y] = K_m \{1 - e^{-\beta m_0} \left(\frac{y}{b_1}\right)^{-\beta/b_2} E[g(R)^{\beta/b_2}]\} \quad (5)$$

where  $E[\cdot]$  represents the expectation. This expectation depends on the

shape of the earthquake source area and this takes more complex form for the rectangular area such as the case considered in this study. When the source area is relatively small, there is no significant difference between the correct value and the approximate one obtained supposing that earthquakes always occur at the center of the source area<sup>3)</sup>.

If the earthquake occurrences constitute a Poisson process, the distribution function of the annual maximum ground acceleration  $Y_{max}$  is:

$$F_{Y_{max}}(y) = \exp\{-\nu_0(1-K_{m1}) - \nu_0 K_{m1} e^{\beta m_0 (\frac{y}{b_1}) - \beta/b_2} E[g(R)^{\beta/b_2}]\} \quad (6)$$

When there exist many independent source areas (1,2,...,n) such as the case in this study, the probability that  $Y_{max}$  is less than y is the probability that  $Y_{max}$  from sources 1 through n are all less than y, or:

$$F_{Y_{max}}(y) = F_{Y_{max}}^{(1)}(y) \cdot F_{Y_{max}}^{(2)}(y) \cdot \dots \cdot F_{Y_{max}}^{(n)}(y) \quad (7)$$

in which  $F_{Y_{max}}^{(i)}(y)$  is the distribution function of  $Y_{max}$  from source area i.

Fig. 4 shows the distribution function of the annual maximum acceleration (represented by p) in Osaka for the variety of the predominant period of ground. The seismic activities of the subareas are determined from the results obtained above supposing that the historically maximum magnitude for 1200 years is  $m_m$  listed in Table 1. For the subareas where no earthquake was recorded or where  $m_m$  is less than 6.4, the minimal value of 6.4 is adopted. It is also supposed that the focal depth is 30 km and that earthquakes always occur at the centers of subareas. The expected maximum acceleration in Osaka for a hundred years is estimated to be about 250 gal.

#### CONCLUDING REMARKS

Applying Bayes' theorem to the instrumental and historical earthquake data, the local seismic source activity was evaluated and the seismic risk analysis was carried out. The method presented here seems to be fairly reasonable.

#### ACKNOWLEDGEMENTS

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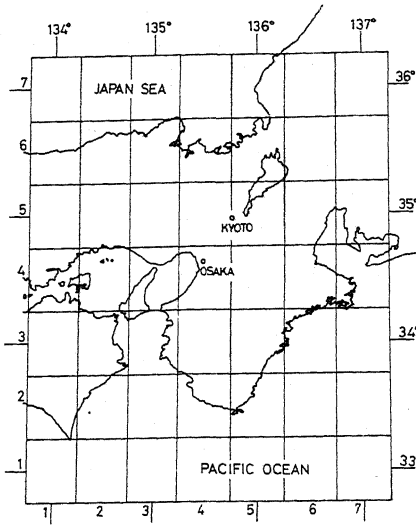


Fig. 1 Seismic Source Area in Kinki District.

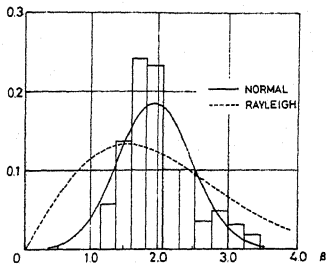


Fig. 2 Distribution of  $\beta$  in and near Japan.

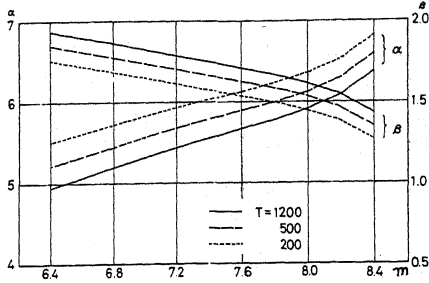


Fig. 3  $\alpha$  and  $\beta$  for the Maximum Magnitude for T Years.

Table 1 Seismic Source Activity in Kinki District.

	n	$\alpha$	$\beta$	$m_m$	6	7	23
7	-	-	1	-	6	7	23
	-	-	-	-	1.886	3.762	.880
	-	-	-	-	0.273	0.711	0.411
	-	-	5.3	-	7.3	6.9	7.9
6	2	1	-	4	10	7	18
	-	-	-	-	3.308	6.659	5.062
	7.4	6.5	7.5	7.0	0.570	1.307	0.957
	-	-	-	-	7.6	8.4	6.7
5	2	7	1	18	3	2	10
	-	2.886	-	4.648	-	-	3.759
	-	0.487	-	0.839	-	-	0.704
	-	5.9	7.1	7.0	6.9	7.4	7.4
4	-	1	2	2	5	2	2
	-	-	-	-	3.009	-	-
	-	-	-	-	0.567	-	-
	6.7	-	6.3	6.7	7.0	-	7.1
3	3	2	32	16	6	5	3
	-	-	6.925	3.238	5.667	3.219	-
	-	-	1.332	0.535	1.244	0.603	-
	5.6	6.0	5.6	7.0	7.6	7.0	7.0
2	5	5	11	9	1	1	5
	2.082	-	2.745	4.161	-	-	3.048
	0.376	-	0.448	0.806	-	-	0.505
	5.6	-	7.0	5.0	8.0	7.0	5.4
1	-	-	2	1	-	-	3
	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
	-	7.4	8.4	8.6	8.4	-	-
	-	-	-	-	-	-	-
	1	2	3	4	5	6	7

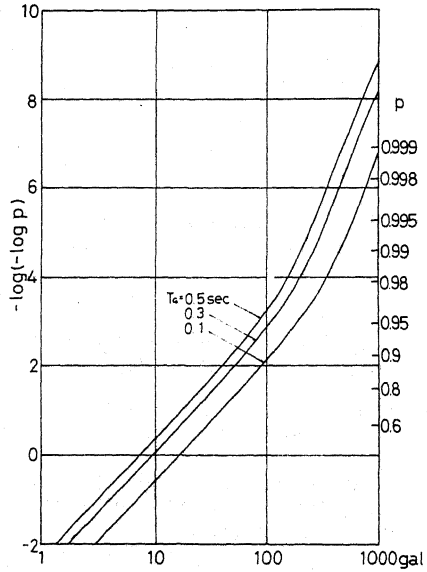


Fig. 4 Distribution Function of Annual Maximum Acceleration.