RISK ASSESSMENT OF RUNNING VEHICLES AGAINST RANDOMLY OCCURING STRUCTURAL FAILURES DURING EARTHQUAKES

by

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SUMMARY

Probabilistic methods are presented for assessing the risk of earthquake-induced traffic accidents caused by failures of structures on the traffic routes. An analytical procedure for an idealized model and a Monte Carlo simulation technique for complicated practical systems are developed. An example application is made for a high-speed railroad system.

INTRODUCTION

Losses caused by structural damage to traffic facilities during earthquakes include economic loss arising directly from structural failure, possible economic and human loss that may occur if running vehicles hit upon failed structures on their routes before they safely stop, and losses due to malfunction of the facility. This study deals with the second of these three items. High speed railroads and elevated highways are of particular concern in discussing the subject.

Safety of running vehicles during earthquakes clearly depends on the stability of structures supporting these vehicles. Motorvehicles may be endangered by falling of bridges, and failures of railroad tracks can be a serious source of danger to high-speed trains. The risk of these earthquake-induced traffic accidents will depend on failure probability of the supporting structures as well as on the average number of vehicles per unit distance and the distance they must run before they stop or their speeds reduce to a safe level. This requires seismic design criteria for traffic facilities with various safety levels depending on the state of the traffic; heavy or light, high speed or low speed, consequence of a crash, etc.

In this view of the problem, this study develops a methodology for assessing the risk of earthquake-induced traffic accidents caused by structural failures. Since the time of earthquake occurrence and the locations of structural failures can not be predicted deterministicly, probabilistic methods are employed. First, analytical methods are developed using an idealized model. Then for practical cases with more complicated structural and traffic conditions, a Monte Carlo simulation technique is used. The major results are presented in terms of the probability that vehicles will be endangered by failed structures during earthquakes, and the conditional probability distribution of the speed of the vehicle when it hits upon a failed structure. The simulation technique is applied to an example problem of a high speed railroad system, and their results are compared with equivalent analytical models.

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ANALYTICAL METHOD

Basic Risk Function

Risk of earthquake-induced traffic accidents is analyzed using a model so idealized that it will enable theoretical analysis. First, consider a one-way traffic route. Suppose a section of the route with a length L is affected by an earthquake, structural failures occuring in a Poisson process along the route with an occurrence rate ν. The stopping distance, namely the distance the vehicle must run before it comes to a complete stop, is denoted by d. Then there is a possibility that the vehicle may hit a failed structure if it is passing through the section with a length L+d as shown in Fig.1.

The basic risk function $R_b(\nu,L,d,v)$ that will play a central role in the subsequent analysis is defined as the probability that a vehicle passing through the section in Fig.1 will encounter a structural failure at a speed no less than v. A closed form solution for the basic risk function is derived below.

Let the random variable representing the locations of the i-th structural failure caused on the route by an earthquake be denoted by X_{fi}, i=1,2,····,k, its realized value being represented by x_{fi} in Fig.1. Here k is a realized value of a random variable K representing the number of structural failures. The location of a vehicle at the moment structural failures occur is also treated as a random variable denoted by X_t, whose realized value is represented by x_t in Fig.1. Let C represent the event that a vehicle is passing through the section in Fig.1, and let S denote the event that the vehicle will reduce its speed to below v or come to a stop before it reaches any of the failed structures. Then the basic risk function is represented by

$$R_b(\nu,L,d,v) = \sum_{k=0}^{\infty} \{1-P(S|C\cap K=k)\}P(K=k) \qquad \cdots\cdots (1)$$

where $P(S|C\cap K=0)=1$, and for $k\neq 0$,

$$P(S|C\cap K=k) = k!\{\sum_{i=1}^{k} P[(X_{f1}<X_{f2}<\cdots<X_{f,i-1}<X_t)\cap(X_t+d_f(v)<X_{fi}<\cdots <X_{fk})|C\cap K=k] + P(X_{fk}<X_t|C\cap K=k)\} \qquad \cdots\cdots (2)$$

in which $d_f(v)$=portion of the stopping distance d which the vehicle will run before its speed is reduced to v. Under the assumption that the location of the vehicle is uniformly likely over the (L+d) section and that structural failures occur in a Poisson process within the L section, the closed form solution for the basic risk function is obtained as

$$R_b(\nu,L,d,v) = \begin{cases} p_1(\nu,L,d,v) \ ; & L>d_f(v) \\ p_2(\nu,L,d,v) \ ; & L\leq d_f(v) \end{cases} \qquad \cdots\cdots (3)$$

where

$$p_1(\nu,L,d,v) = \frac{1}{1+d/L}\left[1+\frac{d_f(v)}{L}-\frac{2}{\nu L}-\exp\{-\nu d_f(v)\}\left\{1-\frac{d_f(v)}{L}-\frac{2}{\nu L}\right\}\right] \qquad \cdots\cdots (4)$$

$$p_2(\nu,L,d,v) = \frac{1}{1+d/L}\left[1+\frac{d_f(v)}{L}-\frac{2}{\nu L}+e^{-\nu L}\left\{1-\frac{d_f(v)}{L}+\frac{2}{\nu L}\right\}\right] \qquad \cdots\cdots (5)$$

Relation between Speed, Distance and Time

The distances d and $d_f(v)$ will depend on the efficiency of emergency brake systems. Herein it is assumed that the stopping distance d is represented by

$$d=(v_b/\alpha)^\gamma\ ;\quad \alpha>0,\ \gamma>1 \qquad \cdots\cdots (6)$$

in which v_b=vehicle speed at which emergency brake is applied, and α,γ=constants. If the speed reduces to v(t) after running a distance y(t) taking a time t, Eq.6 leads to

$$y(t)=d-(v(t)/\alpha)^\gamma=d\{1-(v(t)/v_b)^\gamma\} \qquad \cdots\cdots (7)$$

Differentiation of Eq.7 with respect to t yields a differential equation for v(t), whose solution is obtained as

$$v(t)=v_b(1-t/t_b)^{1/(\gamma-1)} \qquad \cdots\cdots (8)$$

in which t_b=time needed for the vehicle to stop from a speed v_b, and is represented by

$$t_b=(d/v_b)\{\gamma/(\gamma-1)\} \qquad \cdots\cdots (9)$$

From the above formulation, the stopping distance d_o for a cruising speed v_m and the corresponding stopping time t_o are given by

$$d_o=(v_m/\alpha)^\gamma\ ,\quad t_o=(d_o/v_m)\{\gamma/(\gamma-1)\} \qquad \cdots\cdots (10)$$

There can be a delay between the time the brake is applied responding to earthquake motions and the time structural failures actually occur. This time lag, called an allowance time t_a, will also affect the risk of accidents. Consideration of t_a is particularly important in the risk assessment for high speed railroad trains with automatic emergency brake systems linked with seismometer stations located along the route or near causative faults (Ref.1). In such cases the vehicle speed at the moment of structural failures will be reduced to

$$v_r=v_m(1-t_a/t_o)^{1/(\gamma-1)} \qquad \cdots\cdots (11)$$

Then the reduced stopping distance d_r and the reduced stopping time t_r are given by

$$\left.\begin{aligned} d_r&=d_o(v_r/v_m)^\gamma=d_o(1-t_a/t_o)^{\gamma/(\gamma-1)} \\ t_r&=(d_r/v_r)\{\gamma/(\gamma-1)\}=t_o(1-t_a/t_o)^{1/(\gamma-1)} \end{aligned}\right\} \qquad \cdots\cdots (12)$$

The distance $d_f(v)$ used in Eqs.2--5 is represented by

$$d_f(v)=d_r-(v/\alpha)^\gamma=d_r-d_o(v/v_m)^\gamma=d_o\{(v_r/v_m)^\gamma-(v/v_m)^\gamma\} \qquad \cdots\cdots (13)$$

For a given value of d_f, the value of v is determined from

$$v=v_m\{(v_r/v_m)^\gamma-d_f/d_o\}^{1/\gamma}=v_m\{(d_r-d_f)/d_o\}^{1/\gamma} \qquad \cdots\cdots (14)$$

The parameters α and γ will depend on the efficiency of braking systems. Particularly, the value of α will vary greatly with the type of vehicles. However, if normalized expressions such as Eqs.8--14 are used, α can be excluded from formulation. The value of γ will vary in a range of some $\gamma=1$ --3. Action of a constant frictional force during braking gives $\gamma=2$. In the numerical computaions in this study, $\gamma=2.55$ is used. This value has been determined on the basis of emergency brake systems of high-speed railroad trains which will be dealt with later in the example application.

Risk of Earthquake-Induced Accident

A vehicle running at a cruising speed v_m has a possibility of being endangered by a failed structure, if it is passing through the section with a length $L+d_o$ affected by an earthquake, the case with $d=d_o$ in Fig.1, at the moment of earthquake occurrence. Considering the possibility that the vehicle may leave the section within the allowance time t_a, the probability of an event D that this vehicle will hit upon a failed structure before it stops is expressed in terms of the basic risk function; i.e., noting that a case with $v=0$ and $d=d_f=d_r$ is being discussed, we have

$$P(D;\nu,L,d_o,t_a) = \{(L+d_r)/(L+d_o)\}R_b(\nu,L,d_r,0)$$

$$= \begin{cases} \dfrac{1}{1+d_o/L}\left\{1+\dfrac{d_r}{L}-\dfrac{2}{\nu L}-e^{-\nu d_r}\left(1-\dfrac{d_r}{L}-\dfrac{2}{\nu L}\right)\right\}; & L>d_r \\ \dfrac{1}{1+d_o/L}\left\{1+\dfrac{d_r}{L}-\dfrac{2}{\nu L}+e^{-\nu L}\left(1-\dfrac{d_r}{L}+\dfrac{2}{\nu L}\right)\right\}; & L\leq d_r \end{cases} \quad \cdots\cdots (15)$$

Eq.15 gives the risk of an earthquake-induced accident. If n_p denotes the mean number of vehicles passing through the $(L+d_o)$ section and if statistical independence can be assumed between the earthquake-induced accidents of these vehicles, then the mean number n_d of vehicles being in danger is represented by

$$n_d = P(D;\nu,L,d_o,t_a)n_p \quad \cdots\cdots (16)$$

Fig.2 shows the risk of an earthquake-induced accident determined from Eq.15 for a case with $t_a=0$. Observe that P(D) decreases with decreasing νd_o and νL. The parameters ν and d_o can be reduced through engineering practice, whereas L is primarily determined by the earthquake magnitude and distance which are beyond our control. It should, therefore, be aimed to keep the parameter νd_o at a satisfactorily low level in order to assure a small value of the risk P(D).

Fig.3 shows the risk P(D) plotted against the allowance time t_a. The influence of t_a/t_o on P(D) varies with νd_o. When $\nu d_o<2$, the risk linearly decreases with t_a/t_o, whereas t_a/t_o has effects on P(D) only in the range $t_a/t_o>0.5$ when $\nu d_o>5$.

Probability Distribution of Hitting Speed

The degree of damage caused by an earthquake-induced accident will be affected by the hitting speed V_f at which the vehicle encounters a failed structure. Therefore, it is important to estimate the probability distribution of V_f. The distribution function of V_f is represented by

$$F_{V_f}(v_f) = P(V_f \leqq v_f | D) = 1 - P(V_f \geqq v_f)/P(D)$$

which is rewritten in terms of the basic risk function in the following form.

$$F_{V_f}(v_f) = 1 - R_b(\nu,L,d_r,v_f)/R_b(\nu,L,d_r,0)$$

$$= \begin{cases} 1-p_1(\nu,L,d_r,v_f)/p_1(\nu,L,d_r,0) \ ; & L>d_r \\ 1-p_1(\nu,L,d_r,v_f)/p_2(\nu,L,d_r,0) \ ; & d_r \geqq L > d_f(v_f) \\ 1-p_2(\nu,L,d_r,v_f)/p_2(\nu,L,d_r,0) \ ; & d_f(v_f) \geqq L \end{cases} \quad \cdots\cdots\cdots\cdots (17)$$

for $0 \leqq v_f \leqq v_r$, and $F_{V_f}(v_f)=1$ for $v_f > v_r$, in which $d_f(v_f)$ is determined from Eq. 13 with $v=v_f$. On differentiation of Eq.17 with respect to v_f, the probability density of V_f is obtained as

$$f_{V_f}(v_f) = \begin{cases} (1/v_m)p_3(\nu,L,d_r,v_f)/p_1(\nu,L,d_r,0) \ ; & L>d_r \\ (1/v_m)p_3(\nu,L,d_r,v_f)/p_2(\nu,L,d_r,0) \ ; & d_r \geqq L > d_f(v_f) \\ (1/v_m)p_4(\nu,L,d_r,v_f)/p_2(\nu,L,d_r,0) \ ; & d_f(v_f) \geqq L \end{cases} \quad \cdots\cdots (18)$$

for $0 \leqq v_f \leqq v_r$, and $f_{V_f}(v_f)=0$ for $v_f > v_r$, where

$$p_3(\nu,L,d_r,v_f) = \left(\frac{v_f}{v_m}\right)^{\gamma-1} \frac{d_o}{L} \frac{\gamma}{1+d_r/L}\left[1-\exp\{-\nu d_f(v_f)\}\left\{1-\nu L\left(1-\frac{d_f(v_f)}{L}\right)\right\}\right] \cdots (19)$$

$$p_4(\nu,L,d_r,v_f) = \left(\frac{v_f}{v_m}\right)^{\gamma-1} \frac{d_o}{L} \frac{\gamma}{1+d_r/L}(1-e^{-\nu L}) \quad \cdots\cdots\cdots\cdots\cdots\cdots (20)$$

Fig.4 shows the probability density $f_{V_f}(v_f)$ for various values of νd_o with t_a=0and d_o/L=0.05. It has proved from the numerical results that the values of d_o/L have little effects on $f_{V_f}(v_f)$ within a range $d_o/L<0.1$. Observe that the probability density is concentrated toward the maximum value of $v_f/v_m=1$ as d_o increases, suggesting that with a large mean number of structural failures within the stopping distance the vehicle is likely to hit upon a failed structure without appreciable reduction of its speed. The probability of V_f not exceeding v_f is read from Fig.5 showing the distribution function $F_{V_f}(v_f)$. Finally, Fig.6 shows the effect of the allowance time t_a on the distribution function $F_{V_f}(v_f)$. The value of v_f at which $F_{V_f}(v_f)$ reaches unity for each value of t_a/t_o coincides with v_r given by Eq.11. It may be observed that increasing the allowance time is quite effective in reducing the probability of a high hitting speed.

MONTE CARLO SIMULATION WITH APPLICATION TO HIGH-SPEED RAILROAD TRAINS

Applicability of the analytical method developed in the previous chapter is examined by comparing it with the results of a Monte Carlo simulation performed to provide a realistic model of high-speed railroad trains. Actual railroad tracks are supported by structures of various types including embankments, bridges, etc., with different local soil conditions. The intensity of earthquake motions will also vary along the route. From these, the occurrence rate ν of structural failures will not be uniform. Besides, actual railroad trains are operated according to a certain schedule, so that their relative locations are not independent. The train speed also varies along the route, particularly in relation to stopovers at stations. Monte Carlo simulation would be the only method to make a risk assessment considering these all complicated factors. Herein a model embodying situations similar to those of an actual Japanese high-speed railroad has been introduced, and a number of simulation runs have been made to generate the risk of earthquake-induced accidents.

Simulation Model

Earthquakes and Affected Sections.—Two types of earthquakes have been considered: (A) an earthquake of magnitude M=6.4, and (B) an earthquake with M=8.0. The length of the affected sections are 48km and 338km for the earthquakes A and B, respectively, out of which, sections with 12km and 149 km, respectively, are assumed to be contained in epicentral regions subjected to high intensity ground motions. One station is located in the center of the epicentral region for the earthquake A, and eight stations are included in the section affected by the earthquake B. Train schedules have been arranged, and the train speeds have been determined for either cruising, accelerating, or braking state.

Occurrence Rate of Structural Failures.—The occurrence rate ν is determined from $\nu=\bar{\nu}\nu_1\nu_2\nu_3$, in which $\bar{\nu}$=standard value, ν_1,ν_2,ν_3=reduction factors accounting for the type of structures, local soil conditions, and intensity of earthquake motions, respectively. Four cases of $\bar{\nu}$=0.1,0.5,1.0,4.0 km^{-1} are compared. The values of ν_1 and ν_2 used are listed in Table 1. The value of ν_3 is equal to unity within the epicentral regions and reduces linearly so that it will vanish at the edges of the affected sections. Table 1 also lists the proportion of the types of structures and soil conditions composing the affected sections . These values to determine ν have been employed tentatively to deal with an example problem. Extensive future studies are required for determination of their actual values.

Risk Estimation.—Random times of earthquake occurrence are generated. It is assumed that the emergency brake begins to operate immediately, and the speed v_t and the location of each train after an allowance time t_a are determined. The number of structural failures are generated by using Poisson random numbers for each subsections within which the values of ν are constant, and the random locations of the failures are uniformly distributed over these subsections. If the minimum distance d_t between a train and structural failures ahead of it is smaller than a stopping distance determined from Eq.5 with $v_b=v_t$, the train is in danger of an earthquake-induced accident. The hitting speed is determined from Eq.13 with $v_r=v_t$ and $d_f=d_t$.

Result of Simulation

Examples of simulation results are shown in Figs.7 and 8. They are for a case with a zero allowance time, t_a=0. Fig.7 shows the mean ratio of the number of trains in danger of earthquake-induced accidents relative to the total number of trains passing through the affected section,plotted against the mean total number νL of structural failures. The dashed lines are analytical results obtained from Eq.15 with $d_r=d_o$, using the values of d_o/L determined so that it will agree with the simulated result for small values of $\bar{\nu}$.

It may be observed for the earthquake A that the analytical and simulated results agree well for the values of $\bar{\nu}$ except for $\bar{\nu}$=4.0. The low risk level for the simulated result relative to the analytical result at a large value of $\bar{\nu}$=4.0 can be explained as an effect of trains close to the station running at speeds lower that the cruising speed, which is not considered in the analytical model. Same arguments can be made for the case of the earthquake B.

Fig.8 shows the frequency diagram of the simulated hitting speed and the corresponding probability density determined from Eq.18. Observe that the analytical result agrees well with the simulated result for the earthquake B, Fig.8(b), in which case the endangered trains are widely distributed along the affected section. In the case of the earthquake A, Fig.8(a), on the other hand, the effect of the station located in the center of the epicentral region makes the simulated relative frequency for v_f/v_m=0.9--1.0 much smaller than the analytical value.

CONCLUDING REMARKS

The analytical method and the simulation technique developed in this study have proved quite useful. Studies are underway to make numerical estimation of relevant parameters, and on that basis it is aimed to make practical risk evaluation to provide a rational basis for structural design of traffic facilities.

ACKNOWLEDGMENT

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REFERENCE

(1) Tamura, K., Ashida, Y., and Okamoto, S., (1976), "Control of Train Operation on the New Trunk Lines on the Occation of Earthquake," Proceedings of the U.S.-Japan Seminar of Earthquake Engineering Research with Emphasis on Lifeline Systems, Tokyo, Nov. 1976, pp.399-415.

Table 1. Outline of Earthquake-Affected Sections Used in Simulation.

		structure					soil condition					
		embank-ment	cut	viaduct	bridge	tunnel	rock	dilu-vial	allu-vial	soft	very soft
ν_1, ν_2		1.0	0.1	0.01	0.01	0.001	0.9	1.0	1.1	1.2	1.5
proportion (%)	e.q. A	26	3	54	2	15	15	36	45	0	5
	e.q. B	52	11	15	5	17	14	38	26	7	15

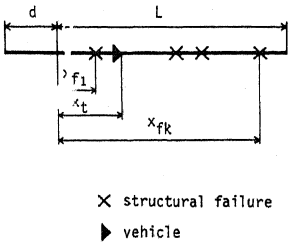

Fig.1 Analytical Model

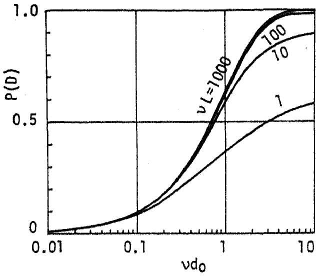

Fig.2 Risk of Earthquake-Induced Accedents (t_a/t_o=0)

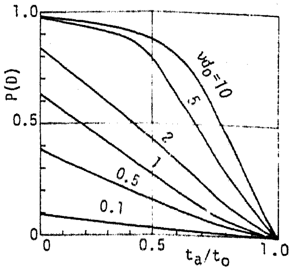

Fig.3 Risk of Earthquake-Induced Accidents (νL=100)

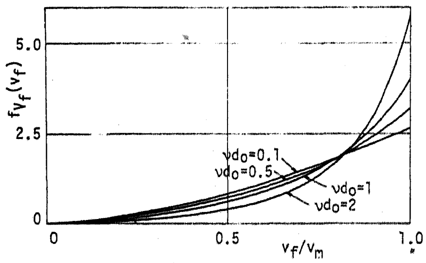

Fig.4 Probability Density of Hitting Speed (t_a/t_o=0, d_o/L=0.05)

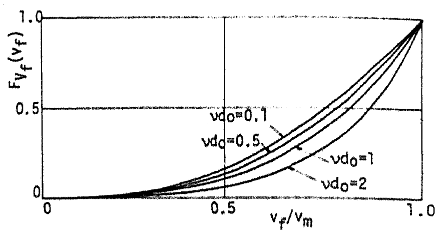

Fig.5 Cumulative Distribution of Hitting Speed (t_a/t_o=0, d_o/L=0.05)

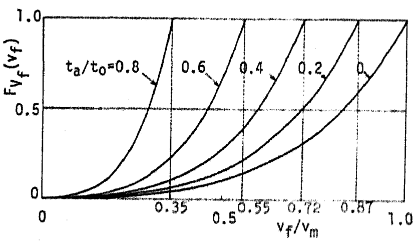

Fig.6 Cumulative Distribution of Hitting Speed (νL=10, νd_o=1)

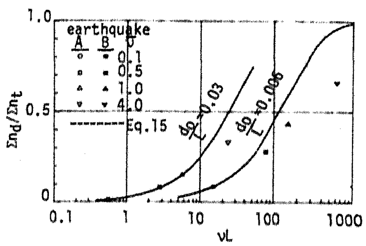

Fig.7 Simulated Risk of Earthquake-Induced Accidents

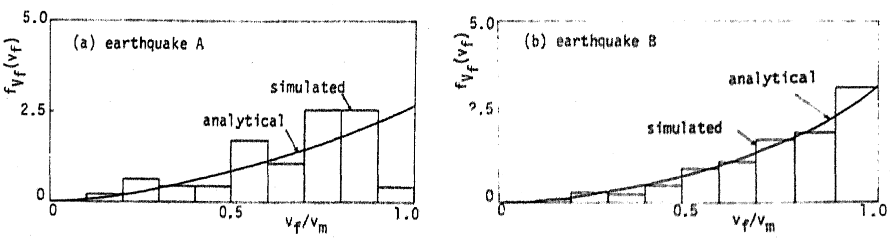

Fig.8 Frequency Diagram of Simulated Hitting Speed ($\bar{\nu}$=0.5)