A HOMOGENEOUS ALTERNATING MARKOV MODEL
FOR EARTHQUAKE OCCURRENCES

by

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SUMMARY

The objective of this paper is to present a probabilistic model for earthquake occurrences with spatial and temporal memory. Alternating Markovian processes are used to characterize both the spatial and temporal dependencies of seismic occurrences along a fault. Currently, only homogeneous space and time transitions are considered. The resulting process however is evolutionary and depends on the specific sequence of events over a period of time. The model provides estimates on the cumulative activity of a fault over a future time period. In addition, probabilities of occurrences of individual events along a geologic fault at some specified future time are obtained. The information from these evaluations is particularly useful in engineering seismic hazard computations and for social and engineering seismic risk assessments. The model is applied to the northern part of the San Andreas fault from Point Arena to San Juan Bautista.

INTRODUCTION

Probabilistic models developed with the intent of representing the frequency and magnitude of earthquakes, have been based on the Poisson process (e.g., Cornell, 1968; Shaw et al., 1974; Der Kiureghian and Ang, 1975). The main assumptions in these models are that seismic events are independent in magnitude and occur independently in time and space, which is contrary to observations of clustering and seismic gaps (Shlien and Toksoz, 1970; and Esteva, 1976). Other models have used strain energy release mechanisms (Hagiwara, 1975), ultimate strain mechanisms (Rikitake, 1975), and foreshock and aftershock sequences (Knopoff and Kagan, 1977). Spatial and temporal considerations for seismic events were included by Veneziano and Cornell (1974) in a simulation procedure. A semi-Markov model is used by Patwardhan et al. (1978) to describe the recurrence of great earthquakes based on the time between their occurrence. Each of these models represents some aspect of the earthquake phenomenon, however some of the models are developed on the basis of specific regional data and are valid only for their corresponding locations.

The model presented in this paper is general and can be adopted for any seismic region. For demonstration purposes, the model is applied to a section of the San Andreas fault, however it is not limited to that fault. The current approach does consider seismic gaps and spatial and temporal dependencies of earthquakes with various magnitudes. Foreshock and aftershock

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sequences are not included in this paper, however their mechanism is currently under investigation and will be included subsequently.

MODEL DESCRIPTION

The sequence of large earthquake occurrences along a fault depends on the rate at which energy is accumulated along the fault and the capacity of the fault to retain this energy. Observations on past earthquake data reveal that considerable time elapses between major earthquakes. In comparison, small magnitude events are believed to be due primarily to localized stress-strain concentration and redistribution (Nur, 1978) and thus can be ignored for the current model.

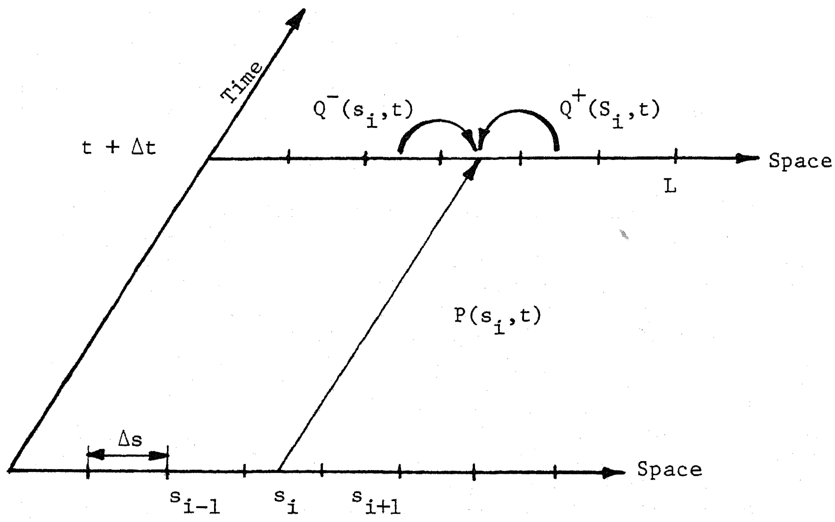

Figure 1. Time and space transition of process.

Figure 1 shows a linear fault of length L divided into segments of length Δs. The space coordinate is taken along the fault. The process is observed at every segment s_i along the fault as it progresses in time at increments of Δt. At any time t, a segment s_i along the fault can release an amount of energy E_j, where E_j is the energy from a given size earthquake per unit legnth. The various levels of energy release form the state vector of the process are defined as

$$\{E_j\} = \{E_0, E_1 \dots E_n\} \tag{1}$$

in which n is the number of different energy release values and E_0 denotes no energy release or release below some threshold value. The probability that an energy E_j is released at segment s_i at time t is given by $\pi_i(s_i, t)$. Then $\{\pi(s_i, t)\}$ = state probability vector for segment s_i at time t.

During a time increment Δt energy release at segment s_i is related to activity at the same segment which occurred during the previous time increment. At the end of the time increment Δt, the influence of the fault segments adjacent to s_i is examined. These transitions are next discussed.

Time Transition. The probability that E_k amount of energy is released in time t to $t + \Delta t$ given that at time t there was E_j energy released at segment s_i is denoted by $p_{jk}(s_i, \Delta t)$. The one step time transition matrix is given by $\underset{\sim}{P}[s_i, \Delta t]$ = n x n matrix for segment s_i whose elements $p(s_i, \Delta t)$ describe the probability of energy release of any E_k given that E_j was released in the previous time step. The symbolic movement of the $\underset{\sim}{P}$ matrix is shown in Fig. 1 for segment s_i. The matrix $\underset{\sim}{P}$ need not be the same for all segments along a fault and in general it is likely to differ over several portions of the fault. Currently only time homogeneous transition matrices are considered. Possible time-dependent representations are currently under investigation.

Space Transition. At the end of the time $t + \Delta t$, segment s_i can have an energy release of E_β given that E_α energy was released at s_{i+1} at time t. The probability of that event occurring is denoted by $q^+_{\alpha\beta}(s_i, \Delta t)$. The influence of the segment s_{i-1} on s_i is given by $q^-_{\alpha\beta}(s_i, \Delta t)$. These probabilities form the elements of the space transition matrices:

$$\underset{\sim}{Q}^+(s_i, \Delta t) = \text{n x n matrix for space transitions from } s_{i+1} \text{ to } s_i$$

and $$\underset{\sim}{Q}^-(s_i, \Delta t) = \text{n x n matrix for space transitions from } s_{i-1} \text{ to } s_i.$$

Segments at the end of the fault have only one-sided transitions.

Initial State Space. Since future energy releases depend on the previous energy releases, it is important to consider the time when the last major event occurred and the location where it occurred. Thus every segment along the fault is traced back in time to the most recent major event. It is assumed that the process is renewed after each major event. The process for the overall fault is started from time $t^- = \max_i\{t_i^-\}$, where t_j^- is the last event on segment i. At time $t^- = t^-_{max}$, segments which have no energy releases are assumed to start with a state probability vector $\{\pi(s_i, t^-_{max})\} = \{1, 0, 0, \ldots 0\}$. Generality is not lost by this assumption. When the process arrives at the time $t_i^- < t^-_{max}$ corresponding to the time of the last major occurrence for segment s_i, the state probability vector for s_i is reset to describe that energy release. At time $t = t_0$ (the present), the process has reconstructed the recent history of major earthquakes along the fault.

Forecasting Process. Given that the process is at time $t = t_0$, forecasts are made on the basis of time and space transitions from one energy release level to another. At time $t_0 + \Delta t$, the transition in time alone is described by

$$\{\pi^1(s_i, t_0 + \Delta t)\} = \underset{\sim}{P}(s_i, \Delta t)\{\pi(s_i, t_0)\}. \quad (2)$$

At the end of the time $t_0 + \Delta t$ the effect on segment s_i due to activity (or inactivity) at adjacent segments is found by

$$\{\pi^+(s_i, t_0 + \Delta t)\} = Q^+(s_i, \Delta t)\{\pi^1(s_{i+1}, t_0 + \Delta t)\} \tag{3}$$

and $$\{\pi^-(s_i, t_0 + \Delta t)\} = Q^-(s_i, \Delta t)\{\pi^1(s_{i-1}, t_0 + \Delta t)\} . \tag{4}$$

The state probability vector for segment s_i at some future time t is

$$\{\pi(s_i, t)\} = \frac{1}{3}\left[\{\pi^1(s_i, t)\} + \{\pi^+(s_i, t)\} + \{\pi^-(s_i, t)\}\right] \tag{5}$$

in which

$$\{\pi^+(s_i, t)\} = Q^+(s_i, \Delta t)\{\pi^1(s_{i+1}, t)\} \tag{6}$$

$$\{\pi^-(s_i, t)\} = Q^-(s_i, \Delta t)\{\pi^1(s_{i-1}, t)\} \tag{7}$$

For engineering seismic risk analysis purposes it is important to obtain the cumulative probabilities of energy release over a time t. These are given by the vector $\{v(s_i, t)\}$ whose elements are determined in the recursive equation form:

$$v_j(s_i, 1) = \pi_j(s_i, 1)$$

$$v_j(s_i,N) = v_j(s_i,N-1)+[1-v_j(s_i,N-1)][\pi_j(s_i,N)-\pi_j(s_i,N-1)h_{jj}(s_i,1)] \tag{8}$$

in which $N = t/\Delta t$.

DEVELOPMENT OF TIME TRANSITION MATRICES

The element $p_{jk}(s_i, t)$ of the time transition matrix $P(s_i, t)$ was defined as the probability of energy release of E_k at segment s_i at time t given that there was an energy release of E_j at the same segment s_i at time Δt before t. From earthquake occurrence data, the average time to have a transition from E_j to E_k is T_{jk}. It is hypothesized that for homogeneous time transitions the probability $p_{jk}(s_i,t)$ will be

$$p_{jk}(s_i, t) = \frac{c_j}{T_{jk}} \tag{9}$$

where c_j is a proportionality constant given as

$$c_j = 1./(1. + \sum_k \frac{1}{T_{jk}}) \tag{10}$$

and $c_0 = 1/n$
where n is the number of energy release levels.

The proposed method provides initial estimates for the homogeneous time transition probabilities. With homogeneous time transition matrices, the process reaches steady state after some time. How fast steady state is reached depends on the values of T_{jk}.

DEVELOPMENT OF SPACE TRANSITION MATRICES

The elements $q^{+}_{jk}(s_i, t)$ and $q^{-}_{jk}(s_i, t)$ of the space transition matrices $Q^{+}(s_i, t)$ and $Q^{-}(s_i, t)$ were defined as the probabilities of energy release E_k at segment s_i given that the segment respectively from the right (s_{i+1}) or from the left (s_{i-1}) has an energy release of level E_j. Since the amount of energy released from a large earthquake will emanate from several segments along a fault, then the effect of one segment on an adjacent segment will depend on that energy release level and on the directivity of the rupture of the fault. For simplicity, it is assumed that an event originating at some segment along the fault will propagate the rupture in both adjacent directions along the fault with equal likelihood. If the directivity of a fault section is known, it can be easily incorporated in the computations of $\underset{\sim}{Q}^{+}$ and $\underset{\sim}{Q}^{-}$. These transition matrices are developed next.

Let $U = L/\Delta s$ be the number of segments in a fault of total length L, and let $u_j = \ell/\Delta s$ be the number of segments corresponding to a rupture length ℓ_j resulting from an event with energy release of E_j. For segments along the fault with

$$i = 1, 2, \ldots, u_j - 2$$

$$q^{+}_{jj} = i/(i+1) \quad \text{and} \quad q^{+}_{j\ j-1} = 1/(i+1); \tag{11}$$

for $\quad u_j - 1 \leq i \leq U - u_j$

$$q^{+}_{jj} = (u_j - 1)/u_j \quad \text{and} \quad q^{+}_{jj-1} = 1/u_j \ ; \tag{12}$$

for $\quad i = U-u_j + 1, U-u_j + 2, \ldots U$

$$q^{+}_{jj} = 1.0 \tag{13}$$

and all other q^{+}_{jk} are zero.

In a similar manner the form of $\underset{\sim}{Q}^{-}$ matrix is found to be:

for $\quad i = 1, 2, \ldots u_j - 1$

$$q^{-}_{jj} = 1.0 \tag{14}$$

for $\quad u_j \leq i \leq U - u_j + 1$

$$q^{-}_{jj} = (u_j - 1)/u_j \quad \text{and} \quad q^{-}_{jj-1} = 1/u_j \tag{15}$$

for $\quad i = U - u_j + 2, \quad U - u_j + 3, \ldots U$

$$q^{-}_{jj} = (U - i + 1)/(U - i + 2) \quad \text{and} \quad q^{-}_{jj-1} = 1/(U - i + 2) \tag{16}$$

and all other q^{-}_{jk} are zero.

ILLUSTRATIVE EXAMPLE

For purposes of preliminary testing of the model, a section of the San Andreas fault spanning from Point Arena to San Juan Bautista has been selected ..d is shown in Figure 2.

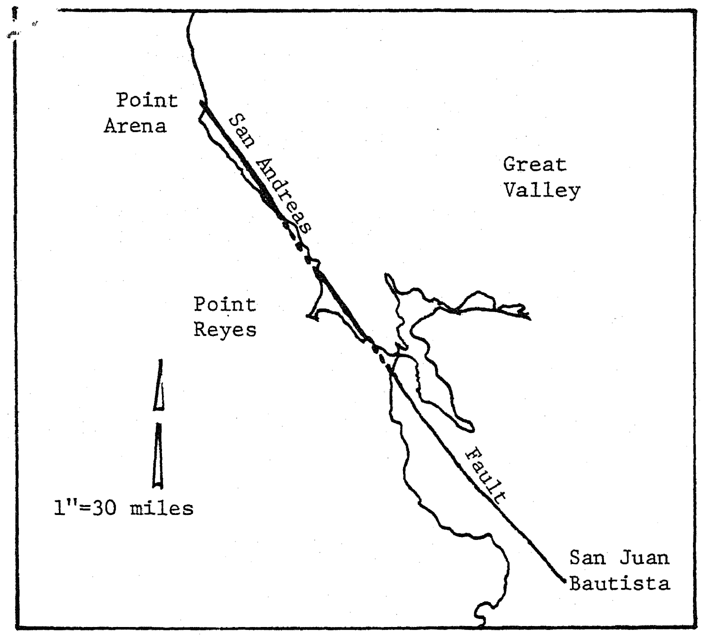

Figure 2. Section of San Andreas Fault considered in this study.

Earthquake data from 1906 to 1976 for the study fault were obtained from the National Earthquake Information Center, Boulder, Colorado. These data account for six earthquakes of magnitudes greater than or equal to 6.0. As a first trial Richter magnitude is used as the earthquake strength measuring parameter, however other parameters such as stress drop, seismic moment, and strain energy release rates are being reviewed for potential use in the model. Information on earthquake events before 1906 was obtained from Coffman and Von Hake, (1977)and was used in this example. Similarly, preliminary data obtained from carbon dating of displacements along the study segment of the San Andreas fault as reported, by Cotton et al. (1979) were also utilized. The total section of the San Andreas considered in this example is discretized into segments of Δs equal to 5 km. This corresponds to earthquake rupture from a magnitude 6.0 event. Thus only events with magnitudes greater than or equal to 6.0 are considered. Since a discrete state space {E} is used in the model, the magnitude scale is discretized into increments of 0.5, starting at a magnitude of 6.0.

The time step Δt selected, is a one year interval. Table 1 shows the amount of energy released per 5 km segment and the approximate number of segments ruptured from a given magnitude earthquake (after Slemmons, 1978). The energy levels E_j, are defined in units of 10^{20} ergs as listed in Table 1.

With these definitions of {E}, Δt, $\Delta \ell$, tests on the model were

Table 1. Relationship Between Magnitude, Fault Rupture and Energy Release (After Slemmons, 1978)

	M	ℓ (km)	E (10^{20} ergs)	$E/\Delta\ell$ (10^{20} ergs)	$u_j = \ell/\Delta\ell$
E_1	6.0	3.2	6.3	1.3	~ 1
E_2	6.5	7.2	35.5	7.1	~ 2
E_3	7.0	16.6	200.0	40.0	~ 4
E_4	7.5	39.1	1,120.0	220.0	~ 8
E_5	8.0	91.4	6,310.0	1,300.0	~ 19
E_6	8.5	214.4	35,500.0	7,100.0	~ 43

conducted with initial conditions starting at magnitude 8.3 in 1906 and retracing the magnitude 6.0 and greater earthquakes at more recent times. The transition matrices were developed on the basis of the u_j values from Table 1 and U = 65. Time transition matrices were developed from the average time between events from all of the data. Tests were run with homogeneous time and space transition matrices for future time periods of 10 and 25 years. The probabilities $\{\pi(s_i, t)\}$ and $\{v(s_i, t)\}$ for the 25 year time period are shown in Figs. 3 and 4 respectively. Both figures are for central segments of the fault.

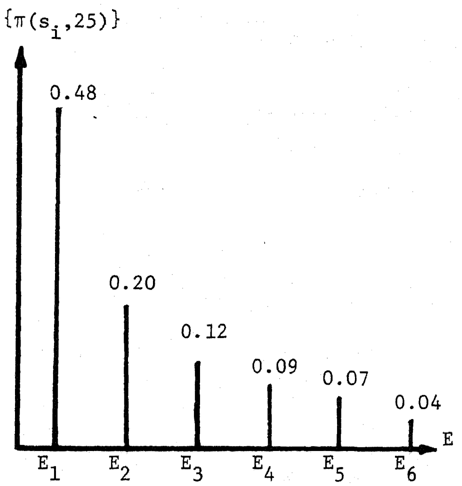

Figure 3. Probabilities of energy release E_j at t=25 yrs.

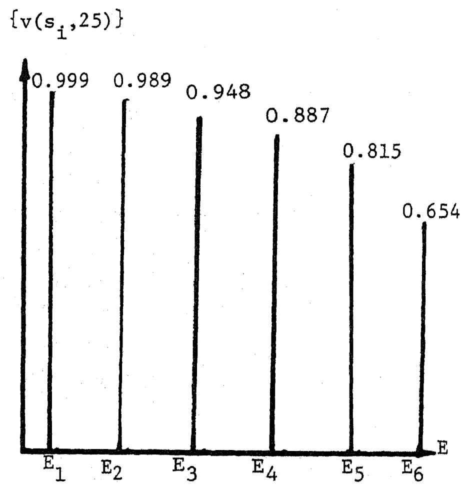

Figure 4. Probabilities of at least one E_j in 25 yrs.

For engineering seismic risk analysis purposes, the cumulative probabilities of events $v(s_i, t)$ are of greater interest. These probabilities can be used to obtain the probability of ground motion at some distance from the fault which provides the basis for evaluating the risk of damage to a structure.

ACKNOWLEDGEMENTS

The authors express their deep gratitude to Haresh C. Shah for many helpful suggestions. Our thanks to Tim Hall for providing the carbon dating information on the San Andreas fault. This research is partially supported by the U.S. Geological Surveys, Grant No. 14-08-0001-17766.

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