GROUND MOTION ON RIDGES UNDER INCIDENT SH WAVES

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SUMMARY

A boundary method is applied to solve the scattering and diffraction of harmonic SH waves by a ridge wich constitutes an irregularity on the surface of an elastic half-space. Two domains are defined; in the exterior one the scattered field is represented in terms of a linear combination of Green's functions for the Newmann problem in the half-space; the refracted field in the interior region is constructed using Green's functions for the whole space. Singularities or sources are located outside the regions of interest. Boundary conditions are satisfied in a least-squares sense. Results are presented for a semi-circular ridge for different frequencies and angles of incidence.

INTRODUCTION

The influence of local topography and geology on the characteristics of strong ground motion has been recognized as an explanation of the spatial distribution of seismic damage (10). It is of interest to know the surface motion at a given site due to incoming and scattered seismic waves in order to assess the potential ground motion.

Many problems of scattering and diffraction of elastic waves have been solved using suitable wave functions when the geometry allows use of separation of variables. The excellent monography by Mow and Pao on the subject presents many such solutions for infinite domains (13). The same method of separation of variables has been applied to solve the diffraction of harmonic SH waves by semi-circular and semi-elliptical cylindrical canyons and alluvial deposits on the surface of an elastic half-space (23, 24, 26, 27).

For arbitrarily shaped surface cavities and harmonic incident SH waves a formulation in terms of a singular Fredholm integral equation of the second kind has been presented and applied to study the effects of the topography on ground motion at the Pacoima dam site due to the San Fernando earthquake in 1971 (25). This last method has been extended to deal with the case of a deposit overlying a semi-infinite soil medium and applied to modeling the measured displacements during a full-scale low-amplitude wave propagation test (28).

A similar approach in terms of singular integral equations has been presented by Sills (20) for solving the problem of scattering of SH waves by ridges or other surface irregularities on the surface of an elastic half-space. Sills describes in full the surface motion on a semi-circular cylindrical ridge.

Using matched asymptotic expansions a solution for arbitraily shaped scatterers has been obtained (15). This approximation is valid for long wave

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lengths. Other methods assume small slope scatterers (11) and/or periodic form for the shape of the scatterer (5). Finite difference and finite element methods have been used (4, 21) in which the definition of artificial boundaries may introduce spurious waves. This difficulty can be minimized by using the so called efficient-active boundaries (3). With a finite differences scheme Boore (4) found that topographic effects may be significant when the incident wavelengths are comparable with the characteristic size of the irregularity.

Recent years have seen the rise of boundary methods which have gained increasing popularity (2). This "rediscovering" of the tools of classical analysis is due mainly to the availability of high speed computers. Moreover, in many problems the reduction of the dimensionality by one may lead to a considerable economy in numerical work. A boundary method which employs multipole expansions of Hankel functions has recently been developed and applied to solve the scattering of SH waves by surface cavities (8) and alluvial valleys (14). The coefficients of the expansions were obtained by least-squares treatment of boundary conditions. A recent survey (29) shows how boundary methods can be used in a finite element context.

In this work a boundary method is presented for solving the scattering and diffraction of harmonic SH waves by an arbitrarily shaped cylindrical ridge on the surface of an elastic half-space. This alternative approach makes use of the superposition principle. The scattered field in an exterior region is represented in terms of a linear combination of Green's functions for the Neumann problem in the half-space; the refracted field in the interior region which includes the ridge itself is constructed using Green's functions for the whole space. The coefficients of the linear forms are determined in such a way that, for each given frequency of the excitation, boundary conditions at the free surface of the hill and those of continuity at the common boundary between the interior and the exterior regions, are satisfied in a least-squares sense. Singularities or sources are located in each case outside the region of interest in order to get regular functions. The idea is similar to Copley's (6) and has been applied by De Mey (7) to the solution of Laplace's interior problem.

Millar (12) has shown for a related problem that the least-squares criterion leads to a representation which converges uniformly in the mean to the exact solution of the problem, provided a complete set of functions is chosen.

The method has been applied to solve scattering and diffraction of P, SV and SH waves in canyons (16, 17) as well as in alluvial valleys (19) and underground cavities and tunnels (9, 18) for incident SH waves.

Numerical results are presented for a semi-circular cylindrical ridge. The surface motion is computed for different frequencies and angles of incidence. In general, agreement with Sills' solution (20) is good.

THE PROBLEM

In the propagation of horizontally polarized harmonic shear waves, the antiplane displacement w in z-direction satisfies the reduced wave equation

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + k^2 w = 0 \quad , \qquad (1)$$

where $k = \omega/\beta$ = wave number, ω = circular frequency, $\beta = \sqrt{\mu/\rho}$ = velocity of propagation of shear waves, μ = shear modulus, and ρ = mass density of the medium. The time factor *exp* $i\omega t$ is assumed through the paper.

The traction-free boundary condition implies that

$$\frac{\partial w}{\partial n} = 0 \quad , \qquad (2)$$

on the free surface (Fig. 1). Here n = vector normal to the ridge and the half-space surface.

Considering a plane SH wave of unit amplitude which propagates toward the surface of the half-space, it can be shown that, if there are no irregularities, the free-field solution can be written (17) as

$$w^{(0)} = 2 \cos\left(ky \sin\theta\right) \exp\left(-ikx \cos\theta\right) \quad , \qquad (3)$$

where θ = angle of incidence (Fig. 2). Obviously the amplitude of surface displacements in the free-field is two.

Then, under incident SH waves a scattered field due to the ridge should exists in addition to the free-field solution. Such a scattered field must satisfy Eq. 1, the traction-free boundary condition, and the Sommerfeld radiation condition (22).

FORMULATION OF THE SOLUTION

Let the entire domain be divided in two regions as shown in Fig. 3. The solution in the exterior region is given by

$$w_1 = w^{(0)} + w^{(s)} \quad , \qquad (4)$$

where $w^{(s)}$ = displacement due to scattered waves. The displacement w_2 in the interior region -the ridge itself- is caused by the refracted waves.

Assume that $w^{(s)}$ and w_2 can be written as linear combinations of Green's functions for the half-space and for the whole space, respectively.

The Green's function for the half-space is given (1) by

$$G_1(P,Q) = \frac{i}{4} \{ H_0^{(2)}(kr_1) + H_0^{(2)}(kr_2) \} \quad , \qquad (5)$$

where P and Q are points in the half-space, $H_0^{(2)}(\cdot)$ = Hankel function of the second class and order zero, $r_1 = \overline{PQ}$, and r_2 is the distance between P and Q', the *image* point of Q outside the half-space. This solution satisfies the traction-free boundary condition $\partial G_1/\partial y = 0$ at $y = 0$ and the radiation condition.

The solution given by Eq. 5 represents cylindrical SH waves which propagate towards infinity, as can be shown using the asymptotic expansion of the Hankel function for large arguments (1). The first part represents the solution for a line *source* at point Q, and the second (another line source) the reflection by the free surface.

For the whole space the Green's function is given by

$$G_2(P,Q) = \frac{i}{4} H_0^{(2)}(kr_3) \quad , \tag{6}$$

where $r_3 = \overline{PQ}$ (the difference in the notation for r_1 and r_3 exphasizes that Q for *each* solution will be different).

For the scattered field we write

$$w^{(s)} = \sum_{m=1}^{M} a_m \, G_1(P,Q_m) \quad , \tag{7}$$

where Q_m, m = 1,2,..., M are the points of line sources located outside the exterior region (see Fig. 4a), a_m = unknown complex coefficient to be determined from boundary conditions, and M = number of sources. P is any point in the exterior region and its boundary.

The refracted field is given by

$$w_2 = \sum_{n=1}^{N} b_n \, G_2(P,Q_n) \quad , \tag{8}$$

here Q_n, n = 1,2,..., N are points located outside the interior region (Fig. 4b), b_n = unknown complex coefficient, and N = number of sources.

The boundary conditions to be satisfied are

$$\begin{aligned} w_1 = w_2 \quad , \quad \frac{\partial w_1}{\partial n} = \frac{\partial w_2}{\partial n} \qquad & \text{in } S_1, \text{ and} \\ \frac{\partial w_2}{\partial n} = 0 \qquad & \text{in } S_2 \end{aligned} \tag{9}$$

We will seek the solution to the problem in a least-squares sense by obtaining the minimum of the mean square error, which is defined as

$$E = \int_{S_1} \{\alpha_1 |w_1 - w_2|^2 + \alpha_2 \left|\frac{\partial w_1}{\partial w} - \frac{\partial w_2}{\partial w}\right|^2\} \, dS + \int_{S_2} \alpha_3 \left|\frac{\partial w_2}{\partial w}\right|^2 dS \quad , \tag{10}$$

where $\alpha_1, \alpha_2, \alpha_3$ = normalization factors.

Taking into account Eqs. 4, 7, and 8 a straightforward derivation, to obtain the mininum mean square error, leads us to a system of linear equations given, in matrix form, by

$$\begin{aligned} & \left[F^{(1,1)}_{\ell m}\right] \{a_m\} - \left[F^{(1,2)}_{\ell n}\right] \{b_n\} = -\{F^{(1,0)}_{\ell 0}\} \\ & -\left[F^{(2,1)}_{qm}\right] \{a_m\} + \left[F^{(2,2)}_{qn} + f_{qn}\right]\{b_n\} = \{F^{(2,0)}_{q0}\} \, , \end{aligned} \tag{11}$$

where ℓ, m = 1,2,..., M and q, n = 1,2,..., N,

$$F_{rs}^{(i,j)} = \alpha_1 \int_{S_1} G_i^* (P,Q_r) \cdot G_j (P,Q_s) \, dS_P + \alpha_2 \int_{S_1} \frac{\partial G_i^*(P,Q_r)}{\partial n_P} \frac{\partial G_j(P,Q_s)}{\partial n_P} \, dS_P \quad , \tag{12}$$

$i = 1,2,\ j = 0,1,2,\ r = \ell,\ q,\ \ s = m,\ n,\ 0,\ \ G_0(P,Q_0) = w^{(0)}(P)$, and

$$f_{qn} = \alpha_3 \int_{S_2} \frac{\partial G_2^* (P,Q_q)}{\partial n_P} \frac{\partial G_2(P,Q_n)}{\partial n_P} \, dS_P \tag{13}$$

In these equations G^* is the complex conjugate of G. The system in Eqs. 11 is of order (M + N) x (M + N) and the coefficient matrix is hermitian. Integrals in Eqs. 12 and 13 will be evaluated by numerical integration.

Once the system is solved, Eqs. 4, 7, and 8 allow us to compute the displacement w at any point of the domain and its boundary.

EXAMPLE

Displacement amplitudes have been calculated on part of the surface for a semicircular ridge for several incidence angles and normalized frequencies. Let the normalized frequency η be given by $\eta = k\,a/\pi = 2a/\lambda$, where λ = incident wave length, a = radius or height of the hill. Then η is the ridge-width to wave-length ratio.

Results are presented in Figs. 5a to 5c for $\eta = 0.25$, 0.5 and 1.0, respectively. Four incidence angles were chosen: θ = 0°, 30°, 60° and 90°. The curve S_1 selected was a semi-circumference and the sources were located, equally spaced, on circumferences with radii $0.7a$ and $1.3a$ for the exterior and the interior solutions, respectively. Several values of M + N were used, ranging from 26 to 50. Numerical integrations were performed with the trapezoidal rule. Comparisons are provided with Sills' results at some points, for which Sills described in full the displacement amplitude spectra. Agreement is good, although the points from Sills' graphs were read with three digit accuracy.

At the frequencies considered amplifications on the ridge range from, say, 25 per cent to 100 per cent. Important reductions and large variations of displacement amplitudes were found at the sides of the hill.

CONCLUSIONS

Very large amplification of displacement were found on the ridge. Results show strong dependence on the incidence angle and the normalized frequency. Although the SH motion is only a part of the incident seismic waves and the shape of the semi-circular hill is not representative of common ridges, results suggest that the influence of these topographic irregularities on ground motion cannot be disregarded.

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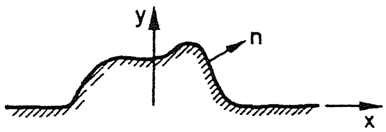

Fig 1. Ridge of arbitrary shape

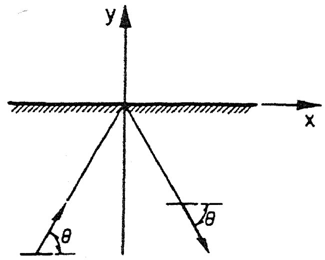

Fig 2. Incident and reflected SH waves

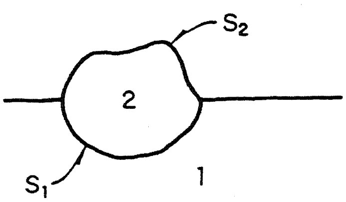

Fig 3. Interior and exterior regions

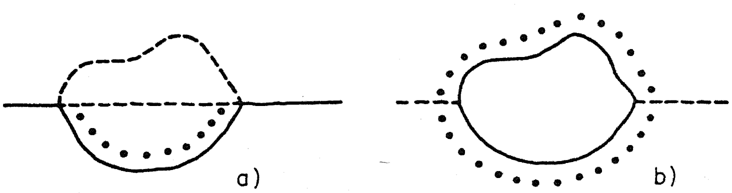

Fig 4. Location of sources for a) the exterior region; b) the interior region

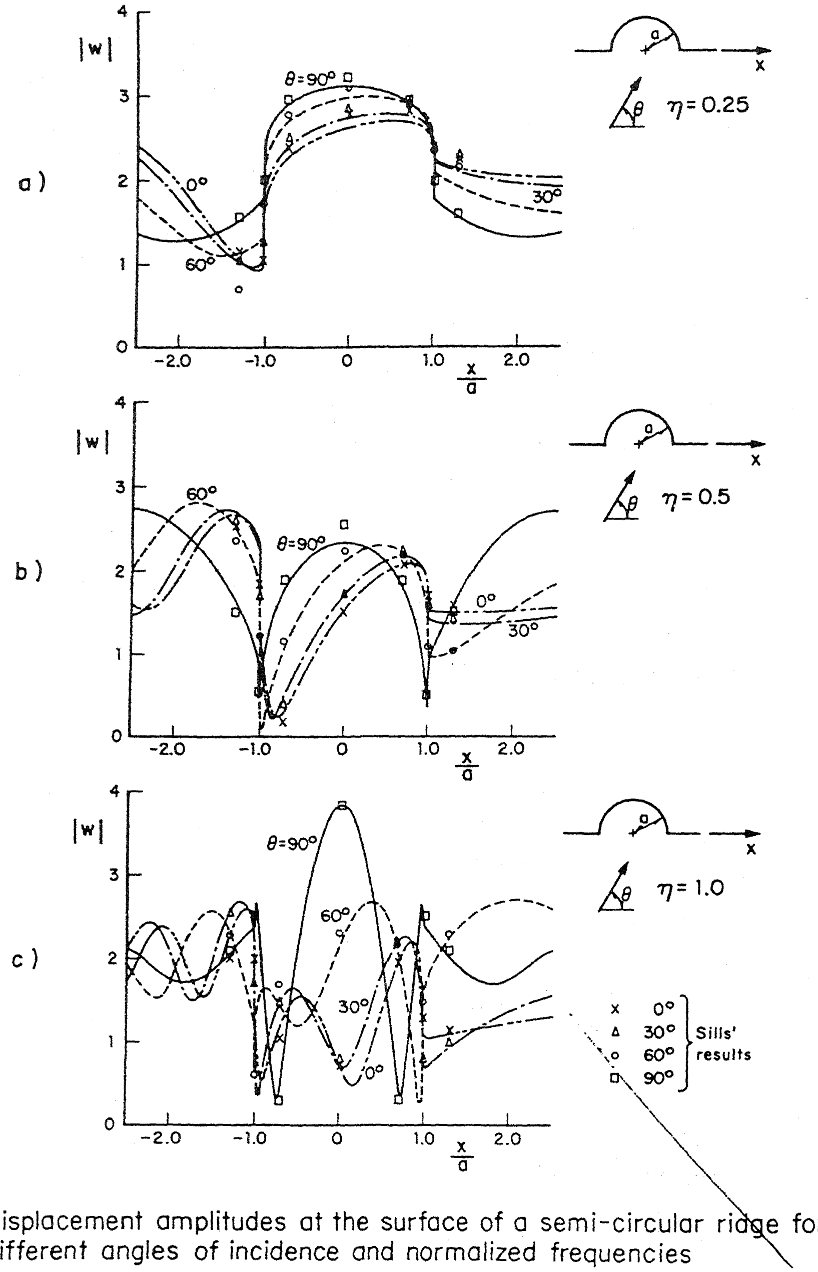

Fig 5. Displacement amplitudes at the surface of a semi-circular ridge for different angles of incidence and normalized frequencies