CONSISTENCY OF PROBABILISTIC SEISMIC RISK METHODS

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SUMMARY

For a part of California, the seismicity is estimated starting only with geological deformation rates. The "geological seismicity" is consistent with instrumental earthquake occurrence rates. The probabilistic seismic risk is found from this geological seismicity, and again from the historical rate of maximum Modified Mercalli Intensity occurrences. Ideally these two methods would give the same answer. Depending on frequency, spectral type, and probablity of exceedance, they give spectral amplitudes which differ by up to a factor of 4.5 in the cases checked.

ESTIMATION OF SEISMICITY FROM GEOLOGY

Occurrence rates of earthquakes may be modeled with the equation

$$N(M) = \begin{cases} 10^{a-bM} & M < M_{max} \\ 0 & M > M_{max} \end{cases} \qquad (1)$$

where $N(M)dM$ is the number of earthquakes per year occurring in a region, or on a fault, with magnitude between $M - \frac{1}{2}dM$ and $M + \frac{1}{2}dM$; a, b and M_{max} are regional constants. Often a and b are estimated from instrumental observations. M_{max} may be estimated from the largest historical earthquake [18] or from other geological criteria [23].

The parameters in Eq. 1 are related to the rate of geological deformation through the moment rate ($\dot{M}_0$) [3,20]. The moment of an earthquake (M_0) is a measure of the permanent deformation which it caused. The moment rate is estimated from the cumulative contribution of all the earthquakes in a region; for meaningful comparison with geological deformation, this rate must be averaged over at least tens of years.

Statistically, the moment is related to the magnitude. Thatcher and Hanks [25] and Kanamori [15] both found that on the average:

$$M_0(M) = 10^{16.0 + 1.5M} \qquad (2)$$

where the units of M_0 are dyne-cm. In Eq. 2, M is interpreted as M_L in California, M_S or M_W on a worldwide scale; it applies only to the M_W scale for great earthquakes where other scales have saturated. Hanks and Kanamori [13] proposed that Eq. 2 be used to define a new magnitude scale **M**. The data scatter about Eq. 2, however, is considerable and for some regions or magnitude scales, the coefficients '16.0' and '1.5' would have to be revised.

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Eq. 2 can be combined with Eq. 1 to obtain $\dot{M}_0$ [11,24]:

$$\begin{aligned} \dot{M}_0 &= \int_{-\infty}^{\infty} M_0(M)N(M)dM \\ &= \frac{(10^{16.0+1.5M_{max}})(10^{a-bM_{max}})}{(1.5-b)\ln_e 10} \qquad (b<1.5) \end{aligned} \tag{3}$$

The first term in the numerator is the moment of the maximum event; the second is its occurrence rate. The denominator corrects the numerator for the occurrence of smaller events.

The rate of geological deformation is related to $\dot{M}_0$. For a fault surface with area A,

$$\dot{M}_0 = \mu A s \tag{4}$$

where μ is the shear modulus (typically 3 to 4×10^{11} dyne-cm^{-2}), and s is the slip rate of the fault [9]. For a volume (V) being deformed with a strain rate ε, Anderson [3] showed that on the average

$$\dot{M}_0 = \left|\frac{2\mu V\varepsilon}{0.75}\right| \tag{5}$$

After $\dot{M}_0$ has been estimated from Eq. 4 or Eq. 5, the use of Eq. 3 gives a constraint on the parameters in Eq. 1. If some fraction of the deformation is aseismic, $\dot{M}_0$ can simply be revised downward proportionately [20].

When a, b and M_{max} are consistent with $\dot{M}_0$ derived from geological data, b is not a critical parameter in determining the risk. Changes in b imply greater or smaller numbers of small events, while the number of the largest events is nearly unaffected. Thus, the regional instrumental value is a reasonable choice. On the other hand, as M_{max} increases, a decreases so that $a + (1.5 - b)M_{max}$ remains constant. At the output of a risk calculation the amplitude of shaking at a selected probability level often decreases as M_{max} is increased.

ESTIMATING THE SEISMICITY IN SOUTHERN CALIFORNIA

Anderson [3] used the geological record to derive preliminary estimates for the slip rate of many important faults in southern California. M_{max} was selected by assuming that the maximum earthquake will rupture at most one-half the total fault length [23]. An approximate upper bound on the magnitudes which have been observed to rupture a fault of given length in the past was used [4], instead of an average relation; all estimated maximum earthquakes were rounded to the nearest one-half magnitude unit.

Fig. 1 shows major faults [14] in part of the region studied by Anderson [3]. The pair of numbers next to each fault gives the estimate for (slip rate, M_{max}). Fig. 2 shows N(M)dM (dM = 0.5) from this geological model, the instrumental record, and the historical record. Instrumental occurrence rates are from Anderson [3] except that the 1952 Kern County earthquake is treated as $M_L = 7.2$ rather than $M_L = 7.7$ [8,16].

The rate on Fig. 2 at M = 8 comes from the earthquake on the San

Andreas fault in 1857, and is probably overestimated. Sieh [21,22] used Holocene geology to estimate the occurrence rates at two sites, Pallett Creek and Wallace Creek, on the San Andreas fault (PC and WC on Fig. 1). The horizontal lines in Fig. 2 are at the mean rates which Sieh suggested, and the sloping lines show the earthquakes distributed in magnitude with the regional b-value. These occurrence rates agree closely with the model predictions.

The model overestimates the instrumental seismicity by an average of about 65%. This discrepancy is small from the viewpoint of the geological model. Within the range of uncertainty of the slip rates, depth of faulting, M_{max}, and other model parameters, one could select values to force the model to fit better. On the other hand, a great earthquake on the San Andreas fault with its aftershocks could significantly increase the average instrumental occurrence rates, resulting in better agreement.

While the discrepancy in Fig. 2 is small from the geophysical perspective, it is significant for estimates of the seismic risk. Even if the model represents the average seismicity, the occurrence rates over the most recent 50 to 100 years may yield more accurate risk estimates for the next 50 years [19]. Campbell [11] uses a Bayesian procedure to combine geological and historical seismicity rates, but does not demonstrate that the risk estimates are more accurate.

COMPUTER PROGRAM FOR RISK CALCULATIONS

Risk calculations were made by a FORTRAN computer program called EQRISK [2]. Based on the method of Cornell [12] it finds the probabilities of exceedance of spectral amplitudes, as suggested by McGuire [17] at up to eleven different periods, T. The following selection of correlation functions are programmed for earthquakes described by either their magnitude or Modified Mercalli Intensity: Fourier spectrum, FS [26,27,31]; absolute acceleration spectrum, SA [28]; pseudo-relative velocity spectrum, PSV [29, 32]; relative velocity spectrum, SV [30]. For FS and PSV, the correlations use either the depths of sediments at the site, or a simplified site condition parameter (rock, intermediate, alluvium). For SA and SV, only the site condition parameter is programmed. All response spectra have the choice of 0, 2, 5, 10 or 20 percent of critical damping.

Fig. 3 shows two paths from the distribution of intensities in a seismicity model, $D(I_0)$, to the probability $P(S(T))$ that a spectral amplitude $S(T)$ will be exceeded. One path is to use a correlation of epicentral intensity I_0, site intensity, I_S, and source-to-site distance (R) to obtain $D(I_S)$, the distribution of site intensities (step 1 on Fig. 3), and then a correlation of I_S and $S(T)$ to obtain $P(S(T))$ (step 2). This is the method used in EQRISK. An alternative is to proceed directly (step 3), using a correlation of I_0 and R with $S(T)$. Step 3 allows the spectral shape to change more with R, but the correlations are necessarily regionally dependent since the rate of decrease of Modified Mercalli Intensity with distance is regionally dependent [10]. In the two-step procedure, formally at least, the regional dependence is concentrated in step 1. Thus, the risk can be estimated anyplace this attenuation rate is known. The correlations used here between intensity and spectral amplitude [27,28,29,30, 31,32] might still have a regional bias because they are derived mainly from California data.

For step 1, program EQRISK uses the correlations of I_0, I_s, and R described by Anderson [1]. He found the distribution of the distance to the isoseismal bounding I_s for each I_0. Differences between the distributions give the probabilities of each I_s for any distance.

RISK ESTIMATES AND COMPARISONS

The rate of occurrence of earthquakes classified according to maximum Modified Mercalli Intensity was found for the study region in southern Califonria, and then this rate and the geological occurrence rate (Fig. 2) were used to calculate the risk at a site. Since one of these calculations used the magnitude correlation functions and theoretical seismicity, while the other used the intensity correlations and the observed seismicity, the two results are nearly independent. The only common aspect is that the correlation functions were derived from the same strong motion data.

The occurrence rates of events with various maximum Modified Mercalli Intensities is shown in Fig. 4. The divergence of the data from a straight line at intensities X and XI is not necessarily serious, since these points are based on one earthquake each; thus, extrapolation based on the line fit to the data in Fig. 4 were used. The inset in Fig. 4 shows the source region and a triangle where the risk is calculated. For both risk methods, earthquakes occur at random locations within the source region.

Fig. 5 shows Fourier spectra calculated according to the two methods. Each point on a spectrum has the probability of being exceeded, per year, given on Fig. 5. Fig. 6 shows the probabilistic FS amplitudes according to the two methods for probabilities down to 10^{-4} per year. At $T = 0.04$ sec, the two methods agree to within a factor of 1.5 over the entire range; at about $T = 0.7$ sec, the two curves differ by about a factor of 3 at small probabilities of exceedance. Fig. 7 shows the same information as Fig. 6 for PSV amplitudes with 0% damping. At $T = 0.04$ sec, these differ by up to a factor of 2.3, while at $T = 0.55$ sec, they differ by up to a factor of 4.5.

CONCLUSIONS

Anderson and Trifunac [5] concluded that probabilistic spectral amplitudes are uncertain within a factor of at least 3 from consideration of diferent seismicity models. From uncertainties in the model parameters and the seismicity, Anderson and Trifunac [6] again suggested that the uncertainties in the modeling process imply a factor of at least 2 uncertainty in spectral amplitudes at small (10^{-2} to 10^{-3} per year) probability of exceedance. In exploring the degree of consistency of two independent methods, this paper also finds disagreements of up to a factor of 4.5. The implication is that the results of probabilistic risk assessment should not be relied upon, except at the order of magnitude level.

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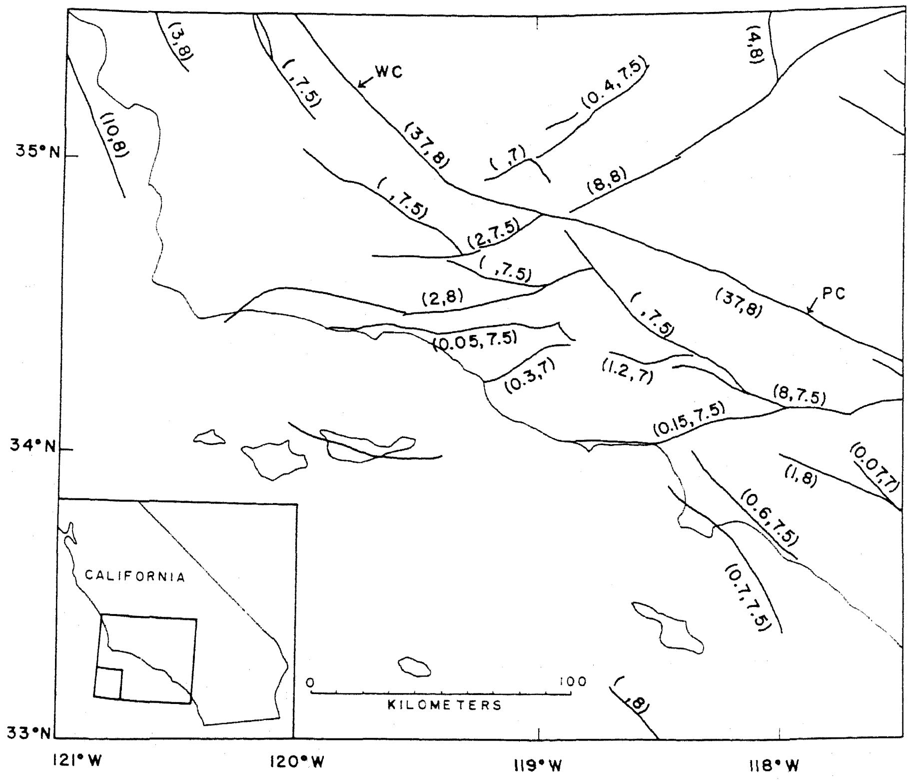

Fig. 1

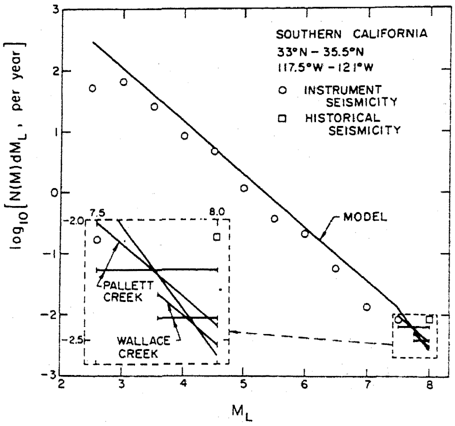

Fig. 2

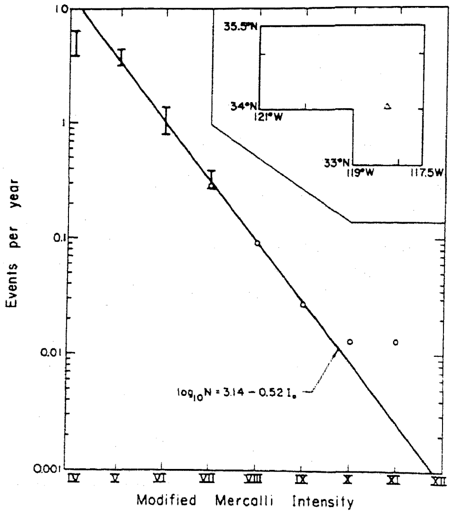

Fig. 4

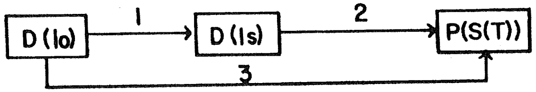

Fig. 3

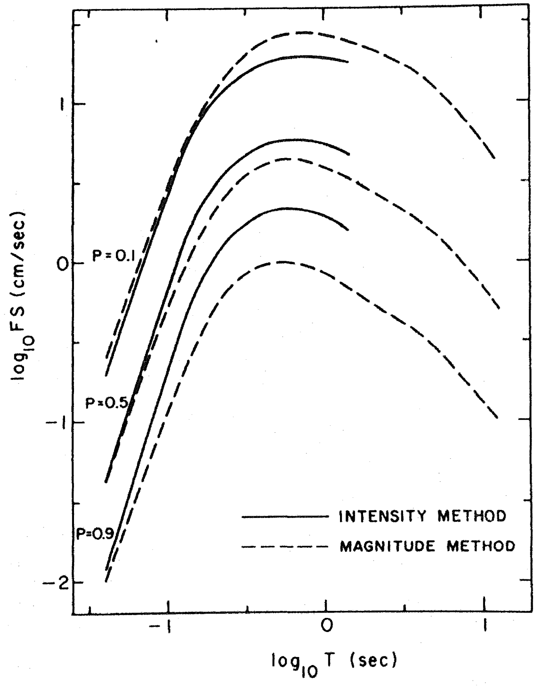

Fig. 5

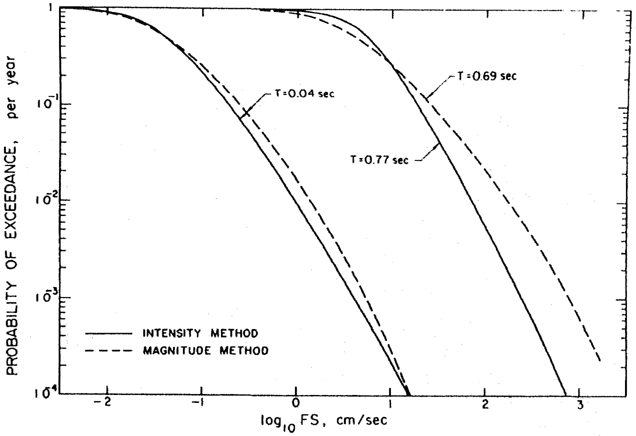

Fig. 6

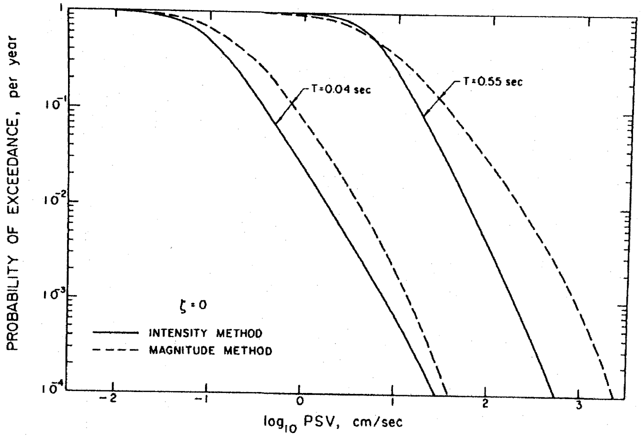

Fig. 7