A GALERKIN FINITE ELEMENT MODEL FOR TSUNAMI RUN-UP

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SUMMARY

Prevention of destruction due to tsunamis on the coasts require prediction of the tsunami behaviour when it moves over the continental shelf and then on the beaches or protective structures. A numerical model based on the finite element method has been prepared and tested for analysing such a situation. This model is particularly useful in designing protective levees whose front slope is such that the tsunami wave climbs without breaking.

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INTRODUCTION

An important aspect of earthquakes is their capacity to generate Tsunami waves. These waves can be quite destructive to ocean and coastal structures. It is, therefore, beneficial to be able to predict their motion and interaction with structures. The wave induced motion carries water up structures such as levees and dykes meant to prevent damage to the coastal regions. Such a motion is commonly referred to as wave run-up. Several studies in the past have produced numerical solutions to this problem under restrictive assumptions. One of the important restrictions in those studies is that the bottom is gently sloping--such as 1 on 10 or flatter slopes.

The traditionally used shallow water equations cannot be applied to regions where the vertical accelerations become significant. Such accelerations are significant when water moves over dykes or levees with fairly steep slopes. In the present study an approach has been made to develop the appropriate momentum equation which will account for the vertical acceleration and hence replace the shallow water equation for the situation mentioned above.

In order to solve this system the Finite Element technique is adopted for the space domain. The Finite Difference Euler Predictor-Corrector is used to step up the variable in the time domain. Such a combination, known as the semi-discrete method, is found to produce a powerful model for the wave run-up. An important aspect of this problem is that the problem domain changes with time. In the present model an interesting way of tackling this aspect is developed using a time dependent boundary element.

THEORY

The governing equations for the situation of waves rushing up a sloping structure are the Eulerian momentum equations and the continuity equation. Considering a vertical plane, these are written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{1}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

where u = the horizontal particle-velocity
w = the vertical particle-velocity
p = pressure
g = acceleration due to gravity

x,z = the independent space coordinates in the horizontal and vertical directions respectively

t = time

However, in shallow waters it can be assumed that the velocity component u remains uniform over the depth; let this be denoted by U. By integrating the momentum equations (i.e. Eqs. 1 and 2) over the range $-h$ to η (Fig.1) and carrying out certain substitutions it can be shown that the following equation represents the situation:

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = -g\frac{\partial \eta}{\partial x} + f_v \qquad (4)$$

The term f_v here stands for the effects in the pressure variation over the depth due to vertical accelerations. This is arrived at primarily from the momentum equation in the vertical direction. (The steps are explained in Ref.2).

The continuity equation (Eq.3) can be brought to the following form:

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\left\{U(h+\eta)\right\} = 0 \qquad (5)$$

Thus, the governing equations are Eqs.4 and 5.

The initial conditions are assumed to be still water conditions as the tsunami moves into the coast. The forcing function is the rate of change of water level (i.e. $\partial\eta/\partial t$) at the point 1 in Fig.2. This serves as the upstream boundary condition. The downstream boundary condition is somewhat complicated and will be explained under the moving boundary aspect of this problem.

THE NUMERICAL TECHNIQUE

The finite element technique is used to numerically solve the situation of wave run-up. Application of the technique is explained in detail in Ref.2. The technique discretises the problem domain into line elements with nodes at their extremities (Fig.2). The variables, namely U and η are described using the Hermitian Cubic shape function. With these approximations the finite element technique reduces the governing differential equations into a system of algebraic equations with the time derivatives of U and η as the unknowns:

$$\left[\; C \;\right]\left\{\frac{\partial U}{\partial t}\right\} = \left\{B_1\right\} \qquad (6)$$

$$\left[\; C \;\right]\left\{\frac{\partial \eta}{\partial t}\right\} = \left\{B_2\right\} \qquad (7)$$

The variables U and η are then stepped up in time using a time integration algorithm; here the Euler Predictor-corrector is used. Thus, a complete space-time history of the velocity and water level as the tsunami crosses can be obtained.

The Moving Boundary Aspect: An interesting aspect of the problem of wave run-up is that the problem-domain is not fixed; the tip of the body of water on the beach keeps moving and hence the resolution of U and η are further complicated. This aspect is tackled in the present study using a mobile finite-element at the tip (Fig.2). The last node, denoted by E in Fig.2 is allowed to move with the tip and hence unnecessary approximations at this region of expansion are avoided. Such a mobility of the node is hydrodynamically substantiated by considering the Lagrangian acceleration of water at the tip while at all other nodes we consider the Eulerian acceleration. Figs. 3 and 4 show how the last node moves with the up-rushing water tip. The beach slope brings about a relationship between the velocity of the tip and the rate of rise of the tip.

This condition is provided as the downstream boundary condition. For details of this approach Ref.2 is cited.

RESULTS

Numerical experiments using the finite element model described above were run on the IBM 370 for various beach and structure-front configurations. Results were tested against experimental findings of earlier investigators. The agreement is found to be quite satisfactory. An important natural phenomenon accurately reproduced by this model is that the tsunami like waves climb the structures without breaking if the slope is steeper than 0.18 as concluded by Street and Camfield (4) from experiments.

The results of the present study are given in Figs. 5 to 8.

ACKNOWLEDGMENT

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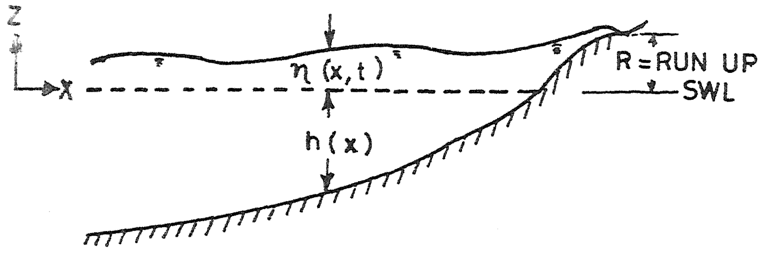

FIG.1 WAVE SHOALING AND RUN-UP

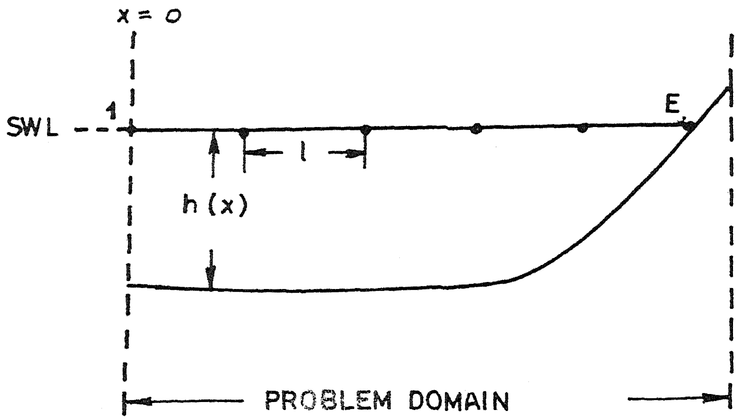

FIG.2 SPATIAL DISCRETIZATION

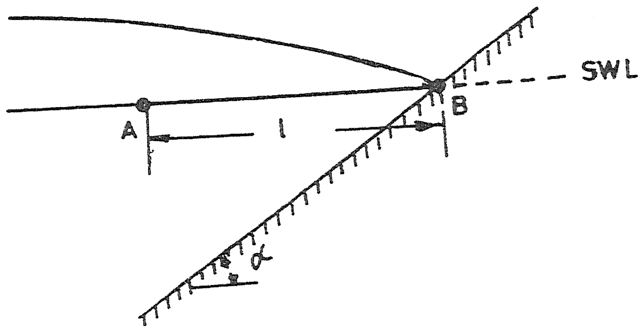

FIG. 3 WATER PROFILE AND END-ELEMENT AT TIME 't'

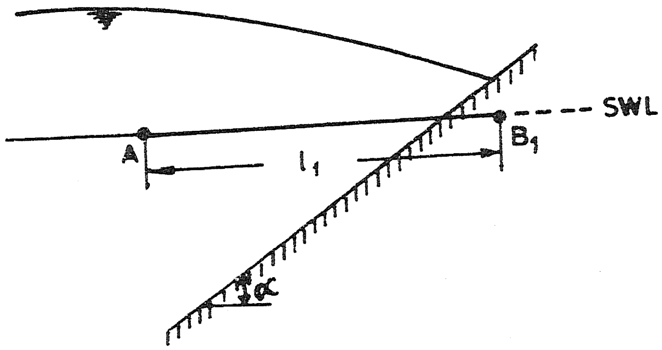

FIG. 4 WATER PROFILE AND END-ELEMENT AT TIME t+Δt

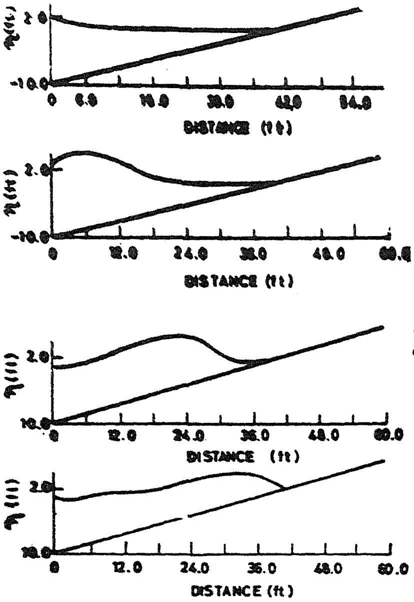

FIG.5 RUN UP OF TSUNAMI

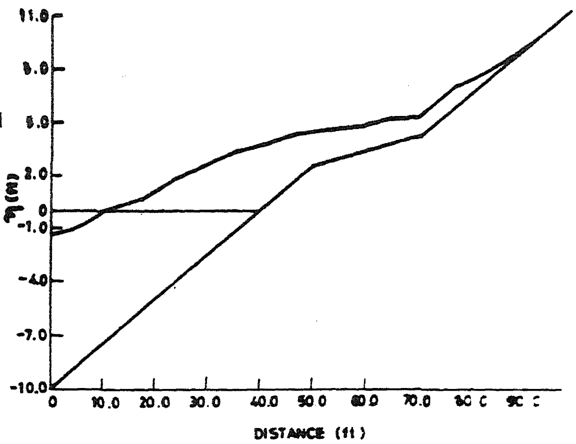

FIG. 6 RUN-UP ON COMPOSITE SLOPE

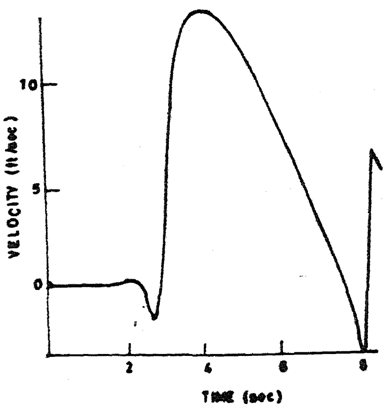

FIG 7 VELOCITY OF TIP

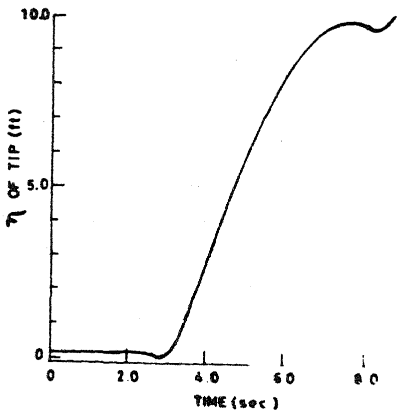

FIG.8 LEVEL OF TIP