

WAVE PROPAGATION PROPERTIES OF A MULTI-PILE REINFORCED SOIL LAYER  
BY A CONTINUUM MIXTURE THEORY

BY

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ABSTRACT

In the problem of a pile foundation, it is necessary to estimate the lateral resistance of a pile as well as the bearing capacity. By applying the continuum mixture theory, the governing equations of the wave motion propagating in a multi-pile reinforced soil layer are derived. Applying the finite element method to the equations, the wave propagation properties of a pile and a soil layer, i.e., the magnification factor and the mode shape are obtained and discussed.

INTRODUCTION

For the construction of structures on a soft layer, a pile foundation is much used to transmit the load of a superstructure to a rigid layer. It is necessary to estimate the lateral resistance of a pile to investigate the behavior of the pile foundation during earthquakes. For this purpose, we obtain the wave propagation properties, which are the overall averaged properties of the pile and the soil layer, for the P-wave and S-wave propagating in the vertical direction. Regarding the pile ground as the composite of the pile and the soil layer and applying the continuum mixture theory to the composite, we derive the governing equations. And by the finite element method we obtain the numerical solution of the equations. We discuss the wave propagation properties in a viewpoint of the interaction between the pile and the soil layer.

MODEL AND GOVERNING EQUATIONS

The pile ground system is composed of a reinforced concrete pile and a soft soil layer on a base soil layer. These materials are homogeneous, isotropic and elastic. Let's consider a periodic array of multi piles in the soft soil layer as shown in Fig.1. The pile ground is idealized to two phase. In a cross sectional element as shown in Fig.1(c), the governing equations are derived according to the reference(1).

1) THE GOVERNING EQUATION (P-WAVE)

$$C_{11} \frac{\partial^2 U^1 a}{\partial x^2} + C_{12} \frac{\partial^2 U^2 a}{\partial x^2} - \rho_1 \frac{\partial^2 U^1 a}{\partial t^2} = F(U^1 a - U^2 a)$$

$$C_{12} \frac{\partial^2 U^1 a}{\partial x^2} + C_{22} \frac{\partial^2 U^2 a}{\partial x^2} - \rho_2 \frac{\partial^2 U^2 a}{\partial t^2} = -F(U^1 a - U^2 a)$$

where  $C_{11} = n_1(\lambda_1 + 2\mu_1) - \lambda_1^2/E$ ,  $C_{12} = \lambda_1 \lambda_2 / E$ ,  $C_{22} = n_2(\lambda_2 + 2\mu_2) - \lambda_2^2/E$ .

$$E = \frac{(\lambda_1 + \mu_1)n_2 + (\lambda_2 + \mu_2)n_1}{n_1 n_2}, \quad F = \frac{3\mu_1 \mu_2}{a_2 b_2} \left[ \frac{b_1}{a_1 \mu_2 + 3Q_1 \mu_1} + \frac{a_1}{b_1 \mu_2 + 3Q_2 \mu_1} \right]$$

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## 2) THE GOVERNINR EQUATION (S-WAVE)

$$C_1 \frac{\partial^2 V^1 a}{\partial x^2} - \rho_1' \frac{\partial^2 V^1 a}{\partial t^2} = F(V^1 a - V^2 a)$$

$$C_2 \frac{\partial^2 V^2 a}{\partial x^2} - \rho_2' \frac{\partial^2 V^2 a}{\partial t^2} = F(V^1 a - V^2 a)$$

$$\text{where } C_1 = n_1 \mu_1, C_2 = n_2 \mu_2, \quad F = \frac{1}{a_2 b_2} \left[ \frac{3b_1 (\lambda_1 + 2\mu_1) (\lambda_2 + 2\mu_2)}{a_1 (\lambda_2 + 2\mu_2) + 3Q_1 (\lambda_1 + 2\mu_1)} + \frac{3a_1 \mu_1 \mu_2}{b_1 \mu_2 + 3Q_2 \mu_1} \right]$$

$$\text{and } Q_1 = \frac{2a_1^2 b_1 - a_2 b_1 (a_1 + a_2) + 3a_2 b_2 (a_2 - a_1)}{6(a_2 b_2 - a_1 b_1)}, Q_2 = \frac{2a_1 b_1^2 - a_1 b_2 (b_1 + b_2) + 3a_2 b_2 (b_2 - b_1)}{6(a_2 b_2 - a_1 b_1)}$$

$$n_1 = a_1 b_1 / a_2 b_2, n_2 = (a_2 b_2 - a_1 b_1) / a_2 b_2, \quad \rho_1' = n_1 \rho_1, \rho_2' = n_2 \rho_2,$$

$\lambda, \mu$  are Lamé constants.  $U$  and  $V$  denote the vertical displacement and the horizontal displacement, respectively. The superscript 1 and 2 mean a pile and a soil layer, respectively.  $( )^a$  expresses the averaged value.

An interaction is a mutually acted force between the pile and the soil layer. The interaction force, which is indicated in the right part of the governing equations, is represented by the product of the coefficient of the interaction  $F$  and the difference of the averaged displacement of two constituents. The coefficient is represented by the material constants and the geometrical configuration of the pile ground. The dimensions and the material constants of the pile ground are shown in Table 1(a). In Table 1(b), the dimensionless parameters applying to the analysis are shown.

## WAVE PROPAGATION PROPERTIES

We investigate the wave propagation properties of the pile ground by the finite element analysis. In figures, the abscissa represents the dimensionless frequency  $f$  ( $f = fH/V$ , where  $f$ ,  $H$  and  $V$  ( $V_p$  or  $V_s$ ) denote frequency, layer depth and phase velocity, respectively.) and the ordinate represents the magnification factor, namely the ratio of the amplitude of the surface to the amplitude of the incident wave.

### 1) THE MAGNIFICATION FACTOR OF THE PILE GROUND

The magnification factor obtained by employing the value as shown in Table 1(b) for P-wave and S-wave is shown in Fig.2 and 3, respectively. In these figures, the magnification factor of the soil layer without piles is also shown. A)P-wave In the frequency range considered in earthquake engineering practice, the pile and the soil layer apparently behave like a mono-phase medium due to the interaction. B)S-wave The magnification factor of the pile ground is much smaller than that of the soil layer without piles. The predominant frequency of the pile ground is much larger than the soil layer without piles. So that it is mentioned that apparently the stiffness of the soil layer increases by the existance of the pile.

### 2) THE COEFFICIENT OF THE INTERACTION

The coefficient of the interaction is under the influence of the area ratio  $\alpha (= a_1/a_2 = b_1/b_2)$  and the aspect ratio  $\gamma (= a_2/H)$ , while it is not under the influence of the stiffness ratio  $\eta (= \mu_1/\mu_2)$  as shown in Fig.4. The amplitude of the coefficient for S-wave is larger than that for P-wave.

### 3) THE MODE SHAPE OF THE PILE GROUND

We investigate the mode shape for the phase velocity of the soil layer. The shapes of the first mode and the second mode are shown in Fig.5. The mode shapes of the pile and the soil layer become equal due to the interaction. According to the increase of the stiffness ratio, the mode shape of the pile

becomes equal to the shape of the rigid body for P-wave. For S-wave the mode shapes of the pile and the soil layer do not change under the variation of the stiffness ratio.

#### 4) THE MAGNIFICATION FACTOR FOR THE SHORT PILE

We investigate the effect of the pile on the surface response for the variation of the pile length against the constant depth of the soil layer. The magnification factors for P-wave and S-wave are shown in Fig.6 and 7, respectively. The shorter the pile is, the smaller the effect of the pile on the soft soil layer is. Hence, the magnification factor becomes large gradually. It is shown that the effect reinforcement of piles is more remarkable for S-wave than for P-wave.

### CONCLUSIONS

The wave propagation properties of the pile ground for P-wave and S-wave are obtained. Some results are summarized as follows.

- 1) The interaction force is represented by the product of the coefficient of the pile and that of the soil layer. The coefficient of the interaction is under the influence of the area ratio and the aspect ratio, namely the influence of the geometrical configuration of the pile ground.
- 2) Under the influence of the interaction, the responses of the pile and the soil layer become equal and the pile ground apparently behaves like a mono-phase medium.
- 3) The smaller the ratio of the pile length to the soil layer depth is, the smaller the effect of the reinforcement of the pile on the soft layer is. The effect is larger for S-wave than for P-wave.

### ACKNOWLEDGEMENTS

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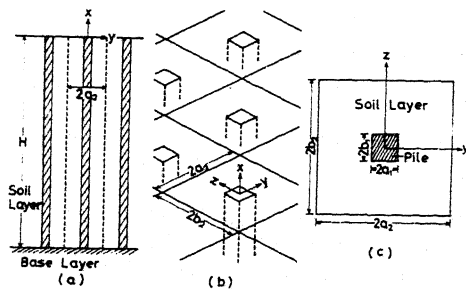


Fig.1 Model of Pile Ground

Table 1 Constants of Pile Ground

	DIMENSION		MATERIAL CONSTANTS				
	CROSS SECTION	LENGTH	MASS DENSITY	VELOCITY (P-WAVE)	VELOCITY (S-WAVE)	SHEAR MODULUS	POISSON'S RATIO
NOTATION	a, b, a <sub>1</sub> , b <sub>1</sub>	H	$\rho$	$V_p$	$V_s$	$\mu$	$\nu$
(DIMENSION)	(m)	(m)	(TON/M <sup>3</sup> )	( $\frac{m}{sec}$ )	( $\frac{m}{sec}$ )	(TON/M <sup>2</sup> )	
PILE	0.5	50	0.24	2600	2000	$1.0 \times 10^8$	0.16
SOIL LAYER	2.5	50	0.18	500	100	$2.0 \times 10^7$	0.485
BASE LAYER	—	—	0.18	1800	600	$1.0 \times 10^8$	—

(a) Dimensions and Material Constants of Pile Ground

	STIFFNESS RATIO	AREA RATIO	ASPECT RATIO	IMPEDANCE RATIO	DENSITY RATIO
FUNDAMENTAL PARAMETER	$\mu_1/\mu = 500$	$a_1/a = 0.1, \rho_1/\rho = 0.2$	$a_1/H = 0.05$	$\mu_1/\mu_2 / \rho_1/\rho_2 = 0.5$	$\rho_1/\rho = 1.5$
VARIABLE PARAMETER	125, 500, 2000	0.1, 0.2, 0.5	0.025, 0.05, 0.1	0.15, 0.5, 0.64	1.0, 1.5, 1.6

(b) Nondimensional Parameters

\*P-WAVE \*\*S-WAVE

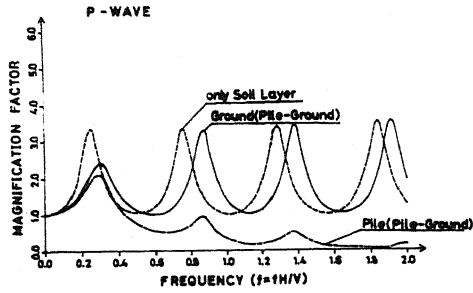


Fig. 2 Magnification Factor (P-wave)

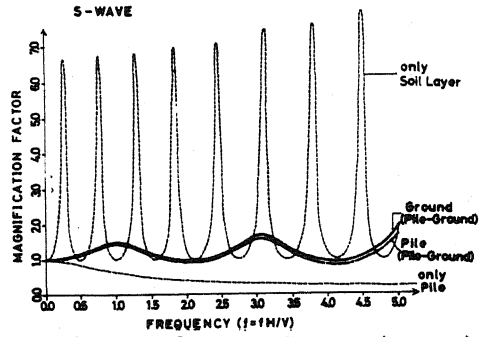


Fig. 3 Magnification Factor (S-wave)

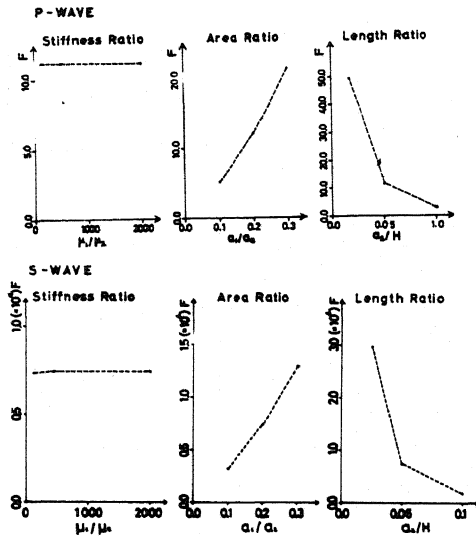


Fig. 4 Coefficient of Interaction

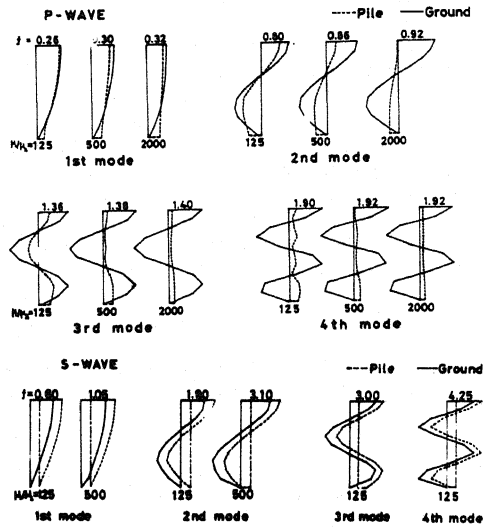


Fig. 5 Mode Shape

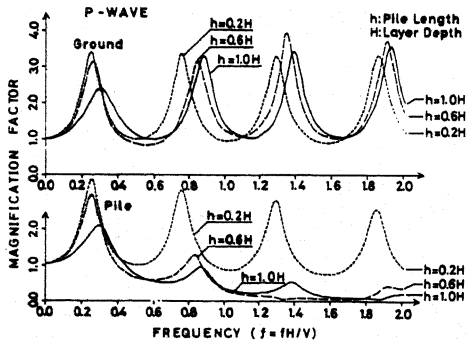


Fig. 6 Magnification Factor (P-wave) (parameter:  $h/H$ )

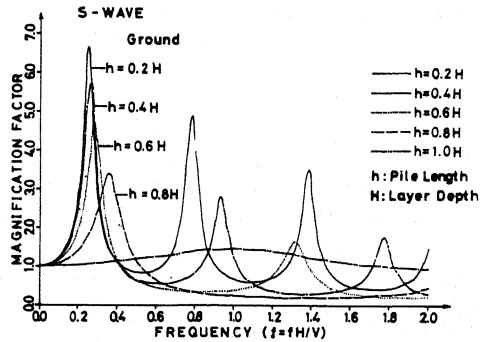


Fig. 7 Magnification Factor (S-wave) (parameter:  $h/H$ )