ASEISMIC DESIGN OF CYLINDRICAL AND SPHERICAL STORAGES FOR THEIR SLOSHING PHENOMENON

by
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SYNOPSIS

The authors have been developing the aseismic design of liquid storages through both theoretical and experimental works for these several years. They are trying to sumarize their experiances obtained through these experiments and analyses to a design procedure. The authors carried out a series of experiments and analyses: an experiment of earthquake response observation of a model cylindrical liquid storage, a response analysis of cylindrical storage, a response analysis of spherical storage and an experiment to analyze the effects of vertical motions on the side-slip of a cylindrical storage. These are reported in this paper. The results are summed up into formula and nomogram which are convienient and available for the response calculation of sloshing of liquid in the aseismic design of liquid storages.

INTRODUCTION

In recent years, many of industrial facilities have potential hazards to their environments including the human life under a destructive earthquake conditions. One of them is a problem of large liquid storage tanks, both cylindrical and spherical types. As already known, the sloshing phenomenon of inner liquid causes various types of damages to their shells(1)(2). Their eigen periods lie in the range of 2 sec to 13 sec. The ground motions of such range under a destructive earthquake are the most unkown area in Japan. The aseismic designs of most of oil storages in Japan are only covered by the law which refers only to their stability. From the view point of strength, we may say that they are almost non-design.

The authors have been developing their design through both theoretical and experimental works for these several years. Their main works are as follows:

i) to observe the response of inner liquid of a model cylindrical storage to natural earthquakes
ii) to establish the theory of response of sloshed liquid to the specified ground motion both through theoretical works and through experimental works. The response theories which are already known are insufficient, because they had done without sufficient knowledge on the ground motions in the range which we concern
iii) to develop the simple computer program of sloshing for arbitrary axisymmetrical vessels and to check it through experiments including a spherical vessel
iv) to make clear the effect of vertical ground motions on the side-slip of storages during strong earthquakes, for which the authors made two-axial vibration tests of a small size cylindrical vessel.

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EARTHQUAKE RESPONSE OBSERVATION OF A MODEL CYLINDRICAL LIQUID STORAGE

In Chiba Field Station of Univ. of Tokyo, 50 Km east of Tokyo, a model cylindrical liquid storage, of which diameter 4.0 m and height 2.0 m, was installed in Jan. 1973. This was equipped with the displacement meters to measure the liquid surface displacement at two points on diameter, dymamic pressure gauges to measure the pressure at two points on side wall and strain gauges to measure the strain at two points on the shell. The model was designed so as to set the fundamental period of sloshing of liquid equal to one of the dominant periodsof the displacement of ground motions. About fifty records have been obtained since Jan. 1973. An example of the records is shown in Fig. 1. The phase of each records usually can be devided into two: acceleration responding phase (acceleration type response) and displacement responding phase (displacement type response). The former phase coincides with the phase of an ordinary structure response. The wave forms of dymamic pressure and stress are similar to those of the input accelerations. About a minutes or more later, the sloshing phenomenon becomes dominant, and their patterns change to similar forms to those of the displacement of the ground. Some of the records lack one of the phases according to the magnitude and epi-center distance of earthquakes. These two types of responses can be understood in the relation with the design concept which Housner introduced in his papers(3). This concept, however, depend on ordinary response analysis based upon the acceleration records, in which the long period components are almost lost.

The response mechanism of sloshing of liquid can be devided into two. One is a resonanse of sloshing to the dominant ground motion. Another is expressed by ordinary displacement response curve S_d , that is, the tendancy of increase of response factor is nearly proportional to the period of a responding system. To the former mechanism, the authors found that the response analysis to transient sinusoidal wave is more effective than the usual response analysis. The response analysis to the latter mechanism can be done by a method modified from the ordinary response technique. However, the transient sinusoidal wave response technique is also applicable to the latter. The authors made clear the fact that the equivalent number of sinusoidal waves of natural earthquakes is usually three to four through the response observation in the field station and also through vibration tests of a 1 m diameter model storage set on a shaking table. The detail of this concept is shown in the next section.

As an example of response values, those of side wall pressure are shown in Fig. 2. The dispersions of response values of each type were calculated. It was found that the dispersion of displacement type response is comparable to that of ordinary structure while that of acceleration type response is very large.

RESPONSE ANALYSIS OF CYLINDRICAL STORAGE

A simple technique to evaluate the displacement type response of cylindrical liquid storage is proposed in this section. In order to evaluate the displacement type response of liquid storage, it is reasonable to treat the storage as a rigid container, because the period of sloshing is so long compared to the period of the vessel that the coupling effect between the sloshing and vibration of the container can be neglected.

Assuming that a cylindrical liquid storage, the diameter of which is $D(=2R)$ and the depth of liquid H , is applied ground motion $d_g(t)$, the

response of the storage are given as follows depending upon the velocity potential theory(4).
The liquid surface displacement η is

$$\frac{\eta}{R}=\frac{1}{g}\sum_{i=1}^{\infty}\frac{2}{\varepsilon_i^2-1}\frac{J_1\{\varepsilon_i(r/R)\}}{J_1(\varepsilon_i)}(\alpha_i-\alpha_g)\cos\theta \tag{1}$$

where r denotes the radius, $J_1(r)$ the Bessel function of the first order, ε_i the i-th root of the equation $J_1'(\varepsilon)=0$, $\alpha_g(=\ddot{d}_g)$ the acceleration of the ground and $\alpha_i(=\dot{v}_i)$ the nominal acceleration response of liquid of each mode and is given as the solution of the next velocity-base equation

$$\ddot{v}_i+2\zeta_i\omega_i\dot{v}_i+\omega_i^2 v_i=\dot{\alpha}_g \tag{2}$$

where v_i denotes the nominal velocity response of liquid of each mode, ζ_i the damping ratio of each mode and ω_i the eigen circuler frequency of sloshing of the i-th order.
The vibratory dynamic pressure p is

$$\frac{p}{\gamma_f H}=\frac{1}{g(H/R)}\sum_{i=1}^{\infty}\frac{2}{\varepsilon_i^2-1}\frac{J_1\{\varepsilon_i(r/R)\}}{J_1(\varepsilon_i)}\left[\frac{\cosh\{\varepsilon_i(z/R)\}}{\cosh\{\varepsilon_i(H/R)\}}\alpha_i-\alpha_g\right]\cos\theta \tag{3}$$

where z denotes the height from the bottom, γ_f the weight of liquid per unit volume and $\gamma_f H$ the static pressure at the bottom. The side wall pressure p_w and the bottom pressure p_b are obtained by putting $r=R$ or $z=0$ in above equation respectively.
The resultant horizontal force F_x is

$$\frac{F_x}{W_f}=\frac{1}{g}\sum_{i=1}^{\infty}\frac{2}{\varepsilon_i^2-1}\left[\frac{\tanh\{\varepsilon_i(H/R)\}}{\varepsilon_i(H/R)}\alpha_i-\alpha_g\right]\cos\theta \tag{4}$$

where $W_f(=\pi R^2 H\gamma_f)$ denotes the total weight of liquid in the storage.

As an example of response calculation, the response spectrum of the muximum amplitude of the displacement of liquid of the fundamental mode at the side wall h was calculated (Fig. 3). The input wave is the normalized El Centro NS,1940. This shows that the response spectrum of h/R is similar to the relative acceleration response spectrum of single-degree-of-freedom system. The response curve becomes flat with the increase of the period of sloshing of liquid.

Next, the response analysis to transient sinusoidal waves is examined. Assume that the storage is applied n times of period of sinusoidal wave. The period T_g and the amplitude D_g are equal to the dominant period and the muximum amplitude of the displacement of the ground motion respectively. Then the approximate muximum response of the liquid surface displacement $h(n)$ and the side wall pressure $P_w(n)$ can be calculated by the nomogran shown in Fig. 4.

Considering the resonant case ($\lambda_1=T_1/T_g=1$), the transient response factor of the displacement of the center of gravity of liquid to the displacement of the ground is given as follows

$$A_g(n)\simeq 1.21\frac{\tanh\{1.84(H/R)\}}{H/R}n \tag{5}$$

The relation between the transient response factor $A_g(n)$ and the response factor A_g obtained as the result from the response observation in the field station and vibration tests of a 1 m diameter model set on a shaking table is shown in Fig. 5. It is found from this figure that the equivalent number of sinusoidal waves of natural earthquakes is usually three to four.

This suggests that the responses during most of earthquakes can be estimated by using $n = 3$ or 4 except the case of extreme resonance. The survey(5) reported by one of the authors shows this fact.

RESPONSE ANALYSIS OF SPHERICAL STORAGE

A simple technique to evaluate the response of spherical storage to ground motion is proposed in this section.

As already known, the system of spherical storage can be regarded as a connection of a rigid system and a flexible system as shown in Fig. 6. The rigid system, the period of which is T_f, is composed of the container W_0, the fixed weight of liquid W_f and the supporting structure. The flexible system, the period of which is equal to the period of sloshing T_s, is composed of the free weight of liquid W_s. The other notations in use are: $S(h) = W_s / W$ = ratio of free liquid, $W = W_{max}\eta(h)$ = the weight of liquid, W_{max} = the full weight of liquid, $\eta(h) = h^2(3-2h)$ = ratio of liquid fullness and $h = H/D$ = ratio of liquid depth to the diameter.

The authors carried out an experiment by which a model spherical storage was shaken on a shaking table to evaluate its vibrational characteristics. The period of sloshing is shown in Fig. 7. The marks ⊗ represent the results of this experiment. The marks ○ represent the values obtained from the result of an experiment which was carried out by Akiyama using a model spherical storage of diameter 6.0 m in Tsukuba(6). Both show a good agreement in spite of the difference of their diameters. An semi-experimental formula which approximates the result of this experiment was formulated. First the spherical storage was approximated by a equivalent cylindrical storage which circumscribed the surface of liquid. The period of sloshing of this model T_c shows a similar feature as the result of experiment in general except the case of shallow vessel. Next, in the case of thin liquid, the motion of the liquid was modeled by a rigid pendulum model based upon the exprimental result. The period of this model T approximates the result of experiment well in the case of thin liquid. Finally, depending upon these results, the authors found that the period of sloshing of liquid in spherical storage T_s can be approximated by the next semi-experimental formula.

$$T_s = T_c(1.30-0.462h) \qquad (6)$$

This also approximates the result calculated by a general purpose program developed by the authors (illustrated by bouble chain line in Fig. 7)(7). This program can be used in a very small size computer. The transfer matrix method is employed in this program. These results are converted into a nomogram by which we can calculate the period of sloshing of liquid both in spherical and cylindrical storages (Fig. 8). This shows that the period of sloshing in a sherical storage changes more strongly than that of cylindrical storage in accordance with the change of the liquid depth.

Next, the ratio of free liquid and the base shear force was examined. Assuming that the system in Fig. 6 is applied n times of period of sinusoidal wave whose period is T_s and the muximum acceleration α_g, the muximum base shear forse $Q(n)$ is given by

$$Q(n) = n\pi\frac{W_s}{g}\alpha_g \qquad (7)$$

Therfore, if α_g and $Q(n)$ are measured through an experiment, free liquid $S(h) = W_S/W$ can be calculated. The ratio of free liquid $S(h)$ which was evaluated experimentally is shown in Fig. 9.

Once $S(h)$ is obtained, $Q(n)$ can be calculated by

$$Q(n) = n\pi K_g S(h)\eta(h) W_{\max} \tag{8}$$

where K_g (= α_g/g) denotes the seismic coefficient of the input ground motion.
Now the base shear coefficient

$$q(h) = \frac{Q(n)}{nK_g W_{\max}} = \pi S(h)\eta(h) \tag{9}$$

can be calculated by using $S(h)$ in Fig. 9. The result is also shown in Fig. 9. Once $q(h)$ is obtained, the muximum base shear force $Q(n)$ can be evaluated reversely by Eq. (9). Setting n =3 or 4 in this result, we can estimate the muximum base shear force for design during a certain earthquake.

EFFECTS OF VERTICAL MOTIONS ON THE SIDE-SLIP OF THE STORAGE

Effects of vertical motions on the side-slip of the cylindrical strage were examined by a small motor-driven shaking table set on another oil-driven shaking table. It was found that the effect of vertical acceleration is eminent to the side force caused by sloshing phenomenon compared to horizontal acceleration. The authers analyzed the side-slip based on the response analysis of cylindrical storage in the preceding section. The result showed a good agreement with the experimental results.

CONCLUSION

The authors have been developing the aseismic design of liquid storages through both theoretical and experimental works for these several years. They tried to sumarize their experiences obtained through a series of experiments and analyses. These results are summed up into formula and nomogram which are convienient and available for the response calculation of sloshing of liquid in the aseismic design of liquid storages.

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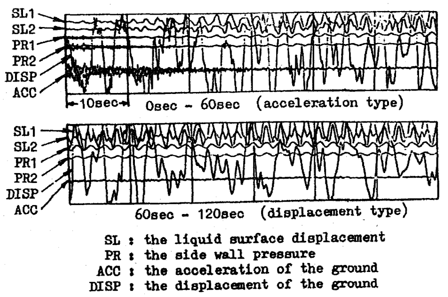

Fig.1 An example of the records

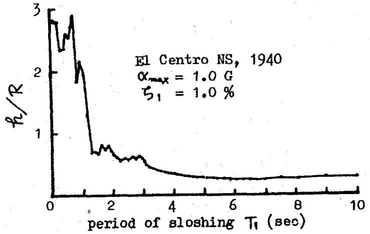

Fig.3 Response spectrum of the liquid surface displacement

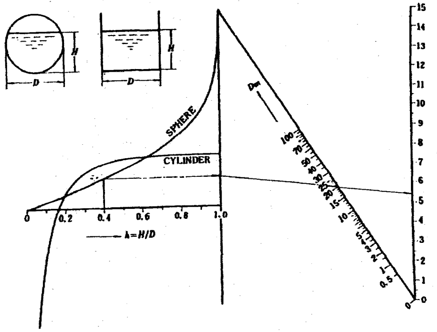

Fig.8 The nomogram to calculate the period of sloshing

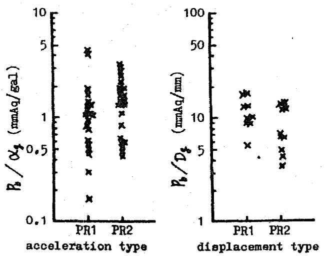

Fig.2 Response values of the side wall pressure

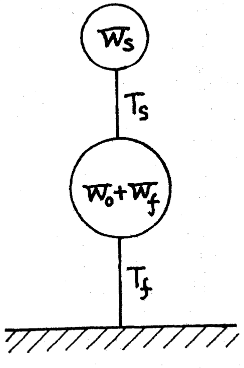

Fig.6 The vibrational model of spherical storage

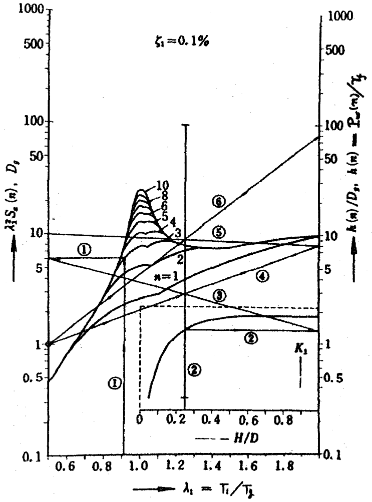

Fig.4 Nomogram to calculate $h(n)$ and $P_w(n)$

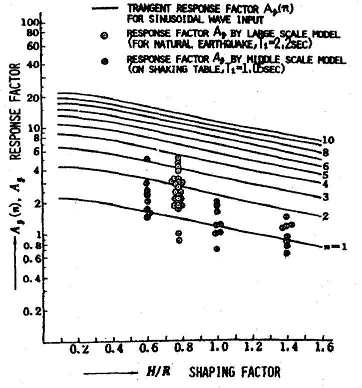

Fig.5 The relation between $A_g(n)$ and A_g

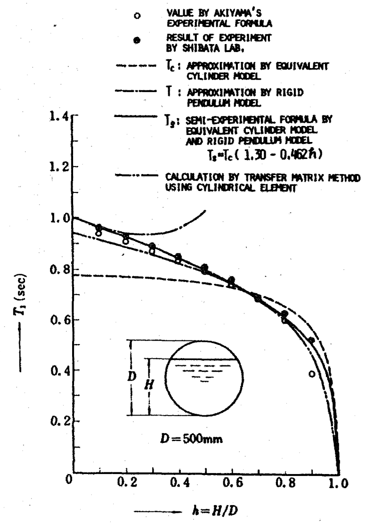

Fig.7 The period of sloshing

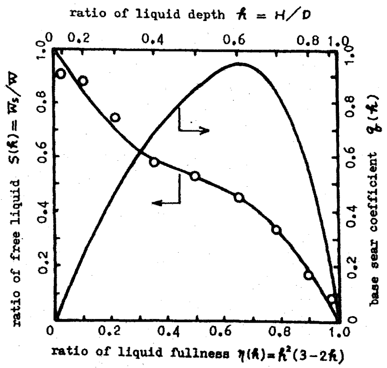

Fig.9 Ratio of free liquid and base shear coefficient

DISCUSSION

A.R. Chandrasekaran (India)

For evaluating impulsive and connective pressures in tanks containing fluids, the expressions proposed by Housner (Ref.1) are used extensively. The ratio of equivalent mass to actual mass of water in case of convective pressure according to Housner (Ref. 1) is

$$0.318 \frac{\text{Tanh}\ (1.84\ H/R)}{H/R} \qquad \ldots \qquad (1)$$

The dynamic amplification factor according to eq. 5 of author's paper is

$$3.63 \frac{\text{Tanh}\ (1.84\ H/R)}{H/R} \qquad \ldots \qquad (2)$$

if n is taken as 3 as proposed by author

Considering a circular water tank of10m dia and capacity of 400 cu. m, and applying a peak earthquake displacement of 1.53cm to the tank and using Housner's formula the ratio of amplitude of sloshing to displacement of tank would be 1.1. However, according to authors formula this ratio would be 3.33. Would be author's like to comment on this large difference ?

REFERENCE:

1. Housner, G.W., "Dynamic Pressures on Accelerated Fluid Containers", Bull. Seism. Soc. Amer., Vol. 47, No. 1, 1957.

Author's Closure

With regard to the question of Mr. Chandrasekaran, we wish to state that in your discussion, the comparison between the dynamic amplification factor calculated by Housner's formula and that calculated by our formula were made. However, it should be noted that both have different meaning. The dynamic amplification factor *according to Housner's formula is* that of the amplitude of sloshing to initial displacement of tank caused by some input acceleration, while the dynamic amplification factor according to Eq. (5) of the author's paper is that of the amplitude of horizontal displacement of the center of gravity of liquid by sloshing to the ground motion.

The discussions below are the authors' comments based on the above notion.

As was shown in their paper, they pointed out, as the result of earthquake response observation of a model cylindrical liquid storage, that there exist next two types of response in the response of liquid storage which are caused from different mechanisms to each other and that the response analyses for both of these two types should be taken into account in the aseismic design of liquid storages, that is <u>acceleration type response</u> and <u>sloshing or Displacement type response</u>. Both papers treated the latter type response problem, however, Housner's method (Ref. 1 in your discussion) is based on the assumption that the maximum displacement of the ground motion occurs in a very short period compared to the fundamental period of sloshing of liquid, and the consequent sloshing motion of liquid is a transient response to this initial displacement.

The authors' method is to estimate the probable maximum response of sloshing to semi-long period dominant ground motions whose dominant period is considerably near to the fundamental period of sloshing. Therefore, the response values calculated by our method should be larger than that calculated by Housner's method.

It should be also noted that the authors method were proven through their field experiment. In Fig. 1, the response instead of the acceleration records, the displacement records factors of sloshing water in a 4 m diameter tank to natural earthquakes are shown. Broken lines shown in the figure shows the calculated response factors by authors' method. The numbers on the lines are number of waves.