PREDICTING THE EARTHQUAKE RESPONSE OF RESILIENTLY MOUNTED EQUIPMENT WITH MOTION LIMITING CONSTRAINTS

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SYNOPSIS

A method is presented whereby the response spectrum may be used to predict the response of an isolation system with nonlinear motion limiting constraints. The results of the approximate method are compared with the results obtained from direct numerical integration. Observations are made on the role of various system parameters in determining the response.

INTRODUCTION

Mechanical equipment used in building structures is often mounted on a resiliently supported base so as to minimize the transmission of mechanical vibration into the structure. If unconstrained, such equipment isolation systems will normally undergo very large relative displacements during a strong earthquake with the likelihood of broken connections, loss of isolation or other forms of failure. In order to minimize the displacement of such systems, motion limiting devices are frequently installed between the isolator base and the structure. The transient response of this type of nonlinear system cannot generally be analyzed except by means of numerical integration techniques which are costly to apply. Furthermore, the earthquake input information supplied to the isolation system designer is often in the form of design base or floor level response spectra, making the application of numerical integration techniques even more difficult.

This paper presents an approximate analytic technique whereby the response spectrum may be used to calculate the response of equipment isolation systems with motion limiting constraints. This is accomplished by defining of a set of "equivalent" linear support stiffnesses and equating the maximum stored energy of the linearized system to that of the actual system.

METHOD OF ANALYSIS

Consider the single-degree-of-freedom constrained isolation system shown in Fig. 1. For purposes of analysis, the equipment being isolated is assumed to be much stiffer than the isolation system. Hence, m is taken to be the total system mass. For small amplitudes of oscillation the supports have a stiffness k_1 which is normally low. When the system undergoes a relative displacement $|x| > \delta$ in either direction, it encounters a motion limiting constraint whose stiffness is k_2. In most practical applications the ratio $\kappa = k_2/k_1$ will be substantially greater than unity. In addition to the spring elements shown it is assumed that there is a viscous support damping with coefficient $c(x)$ which is a function of the displacement. Normally, $c(x)$ will increase as the support stiffness increases. It is assumed that the base of the system is excited by a time varying

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acceleration $a(t)$ with peak acceleration a_{max}. The differential equation describing the motion of the system will be

$$m\ddot{x} + c(x)\dot{x} + f(x) = ma(t) \ . \tag{1}$$

Following the equivalent linearization approach [1], it is assumed that

$$x = A(t) \cos[\omega t - \varphi(t)] = A(t) \cos \theta(t) \tag{2}$$

where $A(t)$ and $\varphi(t)$ are random functions which vary slowly compared to the effective frequency of the system. This assumption can be justified mathematically for systems with small nonlinearity but has been found to be useful even for systems with moderately large nonlinearity. With the assumptions of eqn. (2), a first-order approximation to the solution of eqn. (1) may be obtained by replacing the nonlinear restoring force $f(x)$ by a response dependent effective linear stiffness defined by

$$k_{eff} = E[A^2 \hat{k}(A)]/E[A^2] \tag{3}$$

where $E[\cdot]$ denotes the expected value and $\hat{k}(A)$ is the effective linear stiffness for harmonic excitation. The stiffness $\hat{k}(A)$ is defined as

$$\hat{k}(A) = (1/\pi A) \int_0^{2\pi} f(A \cos\theta) \cos\theta \, d\theta \ . \tag{4}$$

In order to evaluate the expression on the right hand side of eqn. (3), the probability density function $p(A)$ for the envelope variable $A(t)$ must be specified. For the stationary random response problem it is customary to assume that both the excitation and the response are Gaussian distributed. Then, for a lightly damped narrow-band process, the envelope will have a Rayleigh distribution. For a transient earthquake excitation there is presently no good theoretical basis for the definition of $p(A)$. However, it is clear that $p(A)$ must be zero for all A greater than the maximum response x_{max}. In the absence of any data to the contrary it will therefore be assumed that $p(A)$ has the particularly simple form

$$\begin{aligned} p(A) &= 1/x_{max} \ ; \quad 0 \le A \le x_{max} \\ &= 0 \ ; \quad A > x_{max} \ . \end{aligned} \tag{5}$$

Using eqn. (5) the effective stiffness k_{eff} as defined by eqn. (3) becomes

$$k_{eff} = (3/x_{max}^3) \int_0^{x_{max}} A^2 \hat{k}(A) \, dA \tag{6}$$

For the particular system under consideration, evaluation of the integrals in eqns. (4) and (6) yields

$$\begin{aligned} k_{eff} = k_1 \{ 1 + (2\kappa/\pi)[\cos^{-1}(\delta/x_{max}) - 2(\delta/x_{max})\sqrt{1 - (\delta/x_{max})^2} \\ + (\delta/x_{max})^3 \ln (x_{max}/\delta + \sqrt{(x_{max}/\delta)^2 - 1})]\} \ ; \quad x_{max} \ge \delta \end{aligned} \tag{7}$$

The effective stiffness k_{eff} defines an effective frequency and period of oscillation for the nonlinear system, ω_{eff} and $T_{eff} = 2\pi/\omega_{eff}$ where

$$\omega_{eff} = (k_{eff}/m)^{\frac{1}{2}} = \omega_0(k_{eff}/k_1)^{\frac{1}{2}} \ ; \qquad \omega_0 = (k_1/m)^{\frac{1}{2}} \tag{8}$$

It is also convenient to define an effective damping factor ζ_{eff} such that

$$\zeta_{eff} = \min_{\forall x \le x_{max}} \frac{c(x)}{2m\omega_{eff}} \tag{9}$$

The ζ_{eff} so defined will give a conservative measure of the actual damping in the system. Having defined the effective frequency and damping of the nonlinear system, eqn. (1) may be replaced by its effective linear analog

$$\ddot{x} + 2\zeta_{eff}\,\omega_{eff}\,\dot{x} + \omega_{eff}^2\,x = a(t) \ . \tag{10}$$

Due to the strong clipping action associated with the constraint nonlinearity, the maximum displacement of the effective linear system will not be an accurate indicator of the maximum displacement of the actual system. However, since the constraint has only a secondary influence on the energy dissipation of the system it might be assumed that the maximum energy absorbed by the nonlinear and effective linear systems is approximately the same. The maximum potential energy of the effective linear system will be

$$PE_{max} = \frac{m}{2}\,PSV^2(T_{eff}, \zeta_{eff}) \tag{11}$$

where $PSV(T_{eff}, \zeta_{eff})$ denotes the pseudovelocity spectrum value associated with the effective linear system (10). The maximum potential energy of the constrained system will be given by

$$\begin{aligned} PE_{max} &= k_1 x_{max}^2/2 \ ; \qquad x_{max} \le \delta \\ &= k_1 x_{max}^2/2 + \varkappa k_1 (x_{max} - \delta)^2/2 \ ; \qquad x_{max} > \delta \end{aligned} \tag{12}$$

Equating the expressions in eqns. (11) and (12) gives the maximum response displacement x_{max} as

$$\begin{aligned} x_{max} &= PSV(2\pi/\omega_0, \zeta_{eff})/\omega_0 \ ; \quad x_{max} \le \delta \\ &= \{\varkappa\delta + [(\varkappa+1)PSV^2(T_{eff}, \zeta_{eff})/\omega_0^2 - \varkappa\delta^2]^{\frac{1}{2}}\}/(\varkappa+1) \ ; \quad x_{max} > \delta \end{aligned} \tag{13}$$

Eqn. (13) may be used to predict the maximum response of the constrained system given the linear response spectrum of the excitation. The approximate method presented here may be extended to systems with more degrees-of-freedom and a number of nonlinear motion limiting constraints by means of modal analysis.

VERIFICATION OF THE METHOD

As an indication of the accuracy of the proposed method of analysis, the results of the approximate analysis may be compared with results of direct numerical integration of the equation of motion. This comparison can be made by calculating the actual value of x_{max} by numerical integration and then determining the velocity response spectrum value PSV which would be required to give this maximum response according to eqn. (13). The value of PSV so determined may then be compared with the actual velocity response spectrum value at the period T_{eff}. The difference in the two spectrum values will be a measure of the error in the approximate analysis.

Figure 2 shows the results obtained for the velocity response spectrum values PSV(T_{eff}, ζ_{eff}) along with the linear velocity response spectrum for the El Centro 1940 earthquake, N-S component. The small amplitude periods considered are 0.25, 0.5 and 1.0 sec and the stiffness ratios are κ = 10 and 50. The gap spacing δ has been varied so as to give a range of maximum normalized displacement x_{max}/δ from slightly greater than 1 to over 5. For all cases considered the damping coefficient c(x) has been taken to be proportional to the square root of the instantaneous stiffness such that ζ_{eff} = 2%.

It is seen from Fig. 2 that the PSV values required to give the exact maximum response amplitude (the data points) cluster fairly closely about the values which would have been used as input for the approximate analysis. The difference between the exact and approximate analysis values is, in fact, comparable to the local variations in the response spectrum due to small shifts in period. These local variations may be minimized by defining a smoothed upper bound and lower bound spectrum as indicated in Fig. 2. Except for the longest period cases, it is seen that the "exact" solution values generally fall between these two spectrum limits. In only a few cases does the exact value lie above the upper bound spectrum. Hence, the approximate analysis is generally conservative if based on the upper bound spectrum.

A more detailed comparison of the exact and approximate analysis results is given in Table I for a sample of eight different systems. The gap spacing has been selected so as to give maximum displacement ratios in the range of 1.5 to 3.0 where the effect of the nonlinearity is most pronounced. In all cases, use of the upper bound spectrum gives a conservative estimate of both the maximum displacement and the maximum acceleration or force. In five of the eight cases, the exact result lies between the upper and lower bound obtained by the approximate analysis. In only two cases is the approximate analysis clearly overly conservative. Both occur for a stiffness ratio κ = 10 and for maximum displacement ratios of the order of 1.6. Based on the results of Fig. 2 and Table I it is concluded that the approximate analysis is capable of providing maximum response displacement and acceleration estimates which are of sufficient accuracy to be useful in engineering applications.

RESPONSE OF A SIMPLE SYSTEM

In order to determine the response of a nonlinear isolation system by the method presented herein, it is first necessary to specify the amplitude and shape of the excitation response spectrum. This might be a ground acceleration spectrum or the spectrum at a particular location in a building structure. For systems attached at ground level, the excitation response spectrum may be specified as an existing earthquake spectrum, an average of several existing spectra or one of the available design spectra. A commonly used design spectrum is that given by Nuclear Regulatory Guide 1.60 [2]. Using this spectrum it is possible to make certain general observations about the influence of various system parameters on the response of a one-dimensional motion limited isolation system. This proves a basis for establishing design guidelines for such systems. The 2% damped horizontal spectrum is used in all cases presented here.

Figure 3 gives the maximum response ratio, x_{max}/SD, as a function of the gap spacing ratio, δ/SD, for four different combinations of stiffness ratio and small amplitude nominal frequency. SD is the value of the spectral displacement at the nominal frequency. The curves for $\kappa = 20$ would lie roughly half way between the curves for $\kappa = 10$ and $\kappa = 50$. It will be noted that the maximum response ratio is a monotonically increasing function of the gap spacing ratio but a decreasing function of κ. Hence, lower displacements are obtained by decreasing δ and increasing κ as anticipated.

In Fig. 4 the peak acceleration ratio, $\ddot{x}_{max}/a_{max}$, is given as a function of the gap spacing ratio for the same combinations of stiffness ratio and nominal frequency. The maximum acceleration is important in the design of equipment which is attached to the isolation system. In many applications this acceleration must be kept below 3g. Unlike the displacment, the acceleration exhibits a peak at values of gap spacing ratio in the range 0.2 - 0.6. In order to minimize equipment acceleration this peak should be avoided. This generally means making the gap smaller than the value for peak acceleration since the displacement increases if the gap is increased. In practice, the gap should be made as small as possible consistent with the mechanical tolerances involved.

As an example of the application of the results presented herein, consider the design of an isolator system with a nominal frequency of 1 Hz which is subjected to a 0.5 g earthquake (SD = 23.8 cm). Let the maximum allowable acceleration be 3 g and let the maximum allowable relative displacement be 3 cm. From Fig. 3 it is seen that the stiffness ratio must be greater than $\kappa = 10$. Hence, let $\kappa = 50$. Then, in order to maintain the prescribed displacement, the gap spacing ratio must be less than 0.054. However, as seen from Fig. 4, this gap ratio is too large to maintain the prescribed acceleration. Hence, the gap spacing ratio must actually be made less than 0.029. This corresponds to a gap spacing of 0.69 cm for the particular example. The resulting maximum displacement would be 2.1 cm.

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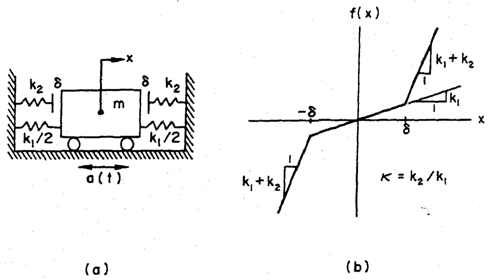

FIG. 1

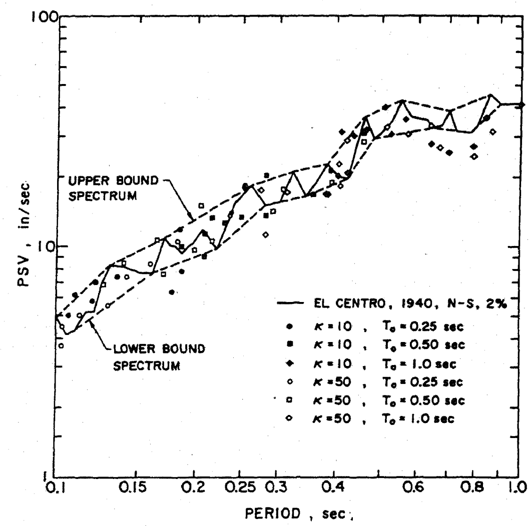

FIG. 2

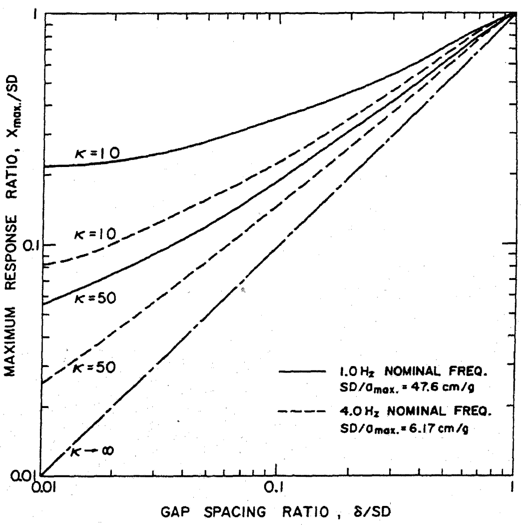

FIG. 3

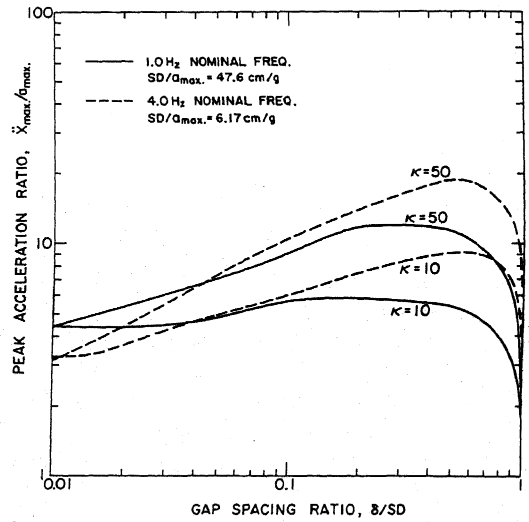

FIG. 4

Table I

Based on El Centro, 1940, N-S

System Stiff. Ratio, κ	Nominal Period (sec)	Maximum Norm. Displ. x_{max}/δ			Maximum Acceleration, g		
		Lower Bd. Spectrum	Upper Bd. Spectrum	Exact Num. Int.	Lower Bd. Spectrum	Upper Bd. Spectrum	Exact Num. Int.
10	0.25	1.72	1.91	1.56	1.55	1.92	1.25
10	0.25	2.35	2.70	2.52	1.11	1.37	1.23
10	1.00	1.77	2.07	1.57	1.65	2.22	1.27
10	1.00	2.70	3.08	3.01	1.37	1.67	1.61
50	0.25	1.31	1.38	1.34	2.92	3.60	3.16
50	0.25	1.51	1.59	1.55	1.90	2.16	2.03
50	1.00	1.36	1.45	1.39	3.35	4.17	3.60
50	1.00	1.58	1.69	1.56	2.13	2.52	2.06