# ON A SIMPLE METHOD ESTIMATING THE APPENDED SYSTEM RESPONSE SPECTRUM FROM A STATISTICALLY SIMULATED SPECTRUM

H. Sato<sup>I</sup>, K. Suzuki<sup>II</sup>, M. Komazaki<sup>III</sup> and M. O-hori<sup>III</sup>

#### SYNOPSIS

Response spectra of single-degree-of-freedom system for earthquake motions show the behaviour with peaks and troughs as the natural period varies. It was shown in the previous work that this could be simulated by assuming basically two ground predominant periods for an artificial earthquake. However, complicated computation which would be practically impossible to carry out for multi-ground predominant system was required for the simulation. A new simple method of the estimate based on the basic spectrum with single ground predominant period and on the method of square root of sum of squares is proposed.

### INTRODUCTION

In the process of dynamic aseismic design averaged response spectrum for many earthquakes has been used to provide the seismicity, meanwhile it is still important to pay attention to the spectrum for a specific earthquake motion in case that the earthquake motion is recorded at a construction site as it describes the ground-characteristics. Then it was attempted to simulate the response spectrum by assuming basically the two ground predominant periods in a statistical estimate of the amplification factor based on random vibration theory for extreme probability density function. The showed that simulated spectrum agrees well with that by earthquake motion not only for the single-degree-of-freedom system corresponding to building structure, but for the appended system to this which simulates equipment or machine structure system on the floor of the building structure.

A simple and new method presented by this paper adopt the simulated response spectrum of aforementioned building and building-appendage structure system for single ground predominant period. The effect of a few ground predominant periods is expressed in terms of the square root of sum of squares of the response for the respective predominant period. The results by this analysis show very good agreement with the spectrum for El Centro (May 18, 1940).

## ESTIMATE OF THE SPECTRUM FOR BUILDING SYSTEM

Fig.1 shows an example of simulated response spectrum of the building structure system for the single ground predominant period. The earthquake motion is represented by random vibration with gaussian distribution and its power spectrum is even between 0.05Hz and 20Hz at bottom of the ground which are given due to  $\psi_1$  and  $\psi_2$  in the figure. The characteristic of the ground is assumed by single-degree-of-freedom system. <sup>6</sup> The amplification factor is obtained by a statistical evaluation based on probability density

I Professor of Mechanical Engineering and III Engineering Assistant, Institute of Industrial Science, University of Tokyo, Tokyo.

II Associate Professor of Mechanical Engineering, Tokyo Metropolitan University, Tokyo.

function of extreme. The shape of the spectrum is simple. It has a peak where the natural period of the structure coincides with ground predominant period.

The response spectrum for El Centro which is shown by the solid line in Fig.2 is made objective to be estimated by the new method. If the peak of the spectrum shown in Fig.1 is adjusted to one of the peaks for El Centro, the former covers the latter at the peak and does not follow the rise at the another peak. Assuming two ground predominant periods in the simulated earthquake the shape of the peaks and the estimate of the amplification factor could be fit well with those by El Centro. In this procedure  $\lambda$  which controls the power ratio of the second ground predominant period to the first one, was chosen so that both characteristic might coincide by trial and error.

Fig.2 suggests that the appropriate addition of the basic response spectrum for single ground predominant system might be able to generate another simulated spectrum. Method which has been adopted for the analysis of structural response is to evaluate the square root of sum of squares of the response for evaluating the effect of the respective mode of vibration. The same approach is taken for the new method.

For  $T_b=0.2s$  where  $T_b$  is natural period of the building structure this approach is attested. Since the ground predominant period  $T_g$  can be taken as  $T_{g\,1}=0.2s$  and  $T_{g\,2}=0.5s$  for the spectrum of El Centro, the amplification factors  $A_{b\,1}$  and  $A_{b\,2}$  to be looked up from the spectrum shown in Fig.l are obtained as follows,

$$T_b/T_{\frac{1}{2}1}=1.0$$
  $A_{b1}=2.96$   $T_b/T_{\frac{1}{2}2}=0.4$   $A_{b2}=1.64$  (1)

The damping ratio of the structure  $h_b$  is now given as  $h_b=0.07$ . If the contribution of a factor to the total sum of the respective mode of vibration is represented by  $\eta$ , the amplification factor  $A_{b_0,2}$  at  $T_b=0.2s$  can be estimated by the method of the square root of sum of squares,

$$A_{b0,2} = \sqrt{(A_{b1})^2 + \eta^2 (A_{b2})^2} / \sqrt{1 + \eta^2}$$
 (2)

The same procedure can be followed for  $T_b=0.5s$ . The amplification factors for the basic spectrum are given as

$$T_b/T_{g_1}=2.5 : A_{b_1}=1.41$$
  
 $T_b/T_{g_2}=1.0 : A_{b_2}=2.96$  (3)

so that the factor taking the sum of those  $A_{b0.5}$  is

$$A_{b0.5} = \sqrt{(A_{b1})^2 + \eta^2 (A_{b2})^2} / \sqrt{1 + \eta^2}$$
(4)

Refering to the spectrum for El Centro earthquake, the amplification factors at peaks are equal, then

$$A_{b0.2} = A_{b0.5}$$
 (5)

 $\eta$  can be obtained by solving this equation. The estimate of the spectrum is made using this evaluation of  $\eta.$ 

Fig.3 compares thus estimated spectrum with that for the earthquake motion. This depicts that the former derived by the newly proposed method fits well with the latter. It is feasible to take the effect due to more than the third ground predominant period into consideration by this method.

The estimated spectrum assuming  $T_{33}$ =0.9s is overlapped in Fig.3, too. The tendency in the shorter natural period range shows little change in comparison with that of two ground predominant periods. In the range between 0.6s and 0.9s the spectrum for earthquake motion is smaller than the estimated one. This would be based on that the power spectrum characteristic around the ground predominant period might be slender than theoretically assumed one.

Fig.4 performs the same procedure for Kushiro earthquake (Dec. 24, 1961), which is known as that has unusually large amplification factor at the peak. Although the effect of the second ground predominant period is taken into consideration, the shape of the spectrum is slender and scarecely change from the original shown in Fig.1. It is interesting to note that the spectrum for Kushiro can be thus simulated by the most basic spectrum and the unusually large amplification factor is explained by this characteristic.

Fig.5 is a result for a motion of Off Izu Peninsula (May 19, 1974) which contains rather longer period components. The spectrum for displacement which is obtained by performing the approximate integral of the acceleration spectrum is also compared. The characteristic that the spectrum increases as the natural period of the structure increases is well represented by the simulated spectrum. The estimate is also very close to the spectrum for the earthquake motion. The difference observed in the short period range of the acceleration spectrum is degenerated in the displacement spectrum.

### ESTIMATE OF THE SPECTRUM FOR APPENDAGE SYSTEM

It is often required to estimate the response behaviour of the appended machine structure system to the building structure. The number of the system parameters increase for the system and the corrected seismicity method which was effective as a practical one of dynamic design for the building structure system is really impossible to apply extensively. For such cases the so called floor response spectrum has been referred for the practical aseismic design.

The simple method to estimate the response spectrum for the building system is attempted to develop for the building-appendage system. Statistically estimated response spectrum of the building-appendage system for a single ground predominant period which was given as a table for various system parameters as well as the case of only the building system is the basis of the extended method. Assuming two ground predominant periods and carrying out the statistical computation, the spectrum for El Centro could be well simulated in the magnitude and the shape of the spectrum.

Again the spectrum by El Centro is taken as an example for the development of the new method. The natural period of the appended system  $T_m$  is varied around that of the building system  $T_b$ . Now the spectrum of  $T_b$ =0.2s in which  $T_m$ =0.2s makes the maximum acceleration amplification factor is discussed. If the mass ratio, the damping ratio of the building and the appendage system are given as  $\gamma$ =0,  $h_b$ =0.07 and  $h_m$ =0.02 respectively, since  $T_{91}$ =0.2s and  $T_{92}$ =0.5s,

$$A_{\text{M1}} = 37.2$$
 for  $T_b/T_{91} = 1.0$   
 $A_{\text{M2}} = 14.5$  for  $T_b/T_{92} = 0.4$  (6)

are given by the tabledue to the statistical approach assuming the single ground predominant period.

If the amplification factor of the building system is represented by  $A_{b1}$  for  $T_b/T_{g1}$  and  $A_{b2}$  for  $T_b/T_{g2}$ , the allocated amplification factors to the appendage system  $A_{abp1}$  and  $A_{abp2}$  for the respective combination of the period which can be considered as the independent mode of vibration are described as follows,

$$A_{abb1} = A_{m1}/A_{b1} \qquad A_{abb2} = A_{m2}/A_{b2} \tag{7}$$

The effect of two ground predominant period is evaluated by the method of square root of sum of squares, so that the synthesized amplification factor  $A_{app}$  is expressed as

$$A app = \sqrt{(A_{app_1})^2 + \eta^2 (A_{app_2})^2} / \sqrt{1 + \eta^2}$$
(8)

where  $\eta$  is the aforementioned factor of the contribution of the mode by the second ground predominant period.

Fig.6 especially looks at the spectrum of  $T_b=0.2s$ . The amplification factor of the total system is obtained by multiplying  $A_{abb}$  with  $A_{b\,T_b}$ . This makes it obvious that the maximum amplification factor at  $T_{w}=0.2s$  agrees very well with the result by the simulation where the appendage system is in pseudo resonance. The existence of the second ground predominant at  $T_{32}=0.5s$  push the foot of the spectrum for the earthquake motion up. This characteristic is also well simulated by the spectrum due to the developed method. Fig.7 compares the spectra provided by the method aforementioned with those by El Centro.

## CONCLUSION

It is shown that the effect of a number of the ground predominant periods is easily taken into account by the method and the results agree well with the characteristic due to the earthquake motion. It is also shown that the method can be easily extended to the case of building-appendage structure system. This means that the time historical computation of the floor response spectrum can be saved. The effect of mass ratio which has not been taken in the floor response method can be introduced. The estimate of the appendage system partly does not agree with the characteristic to be explained.

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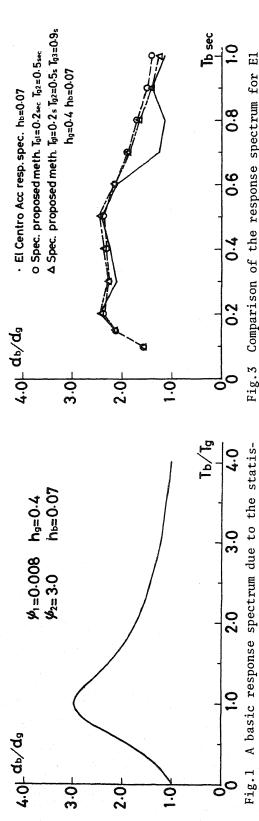


Fig.3 Comparison of the response spectrum for El Centro with that by the proposed method

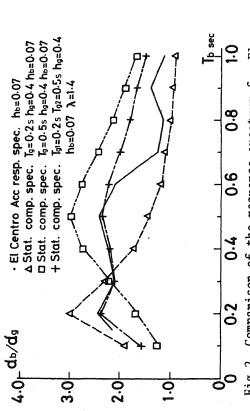
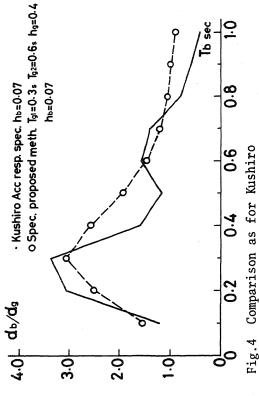


Fig.2 Comparison of the response apectrum for El Centro with those by the statistical computation



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