VIBRATION CHARACTERISTICS OF CABLE SYSTEMS

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Suspended cable roof systems are flexible structures and may develop large deformations under dynamic forces. In order to estimate the dynamic response of such systems, it would be desirable to have closed form formulae for computing natural frequencies eventhough these may be approximate. Such formulae developed for cable nets and trusses using Rayleigh-Ritz procedure are described in the following paragraphs.

The equilibrium equation for a translational surface of two parabolic curves with horizontal components of tensile forces in the cables as $(H_x + h_x)$ and $(H_y + h_y)$ in x and y directions is given by,

$$\bar{h}_x \frac{\partial^2 z}{\partial x^2} + \bar{h}_y \frac{\partial^2 z}{\partial y^2} + \bar{H}_x \frac{\partial^2 w}{\partial x^2} + \bar{H}_y \frac{\partial^2 w}{\partial y^2} = -m \frac{\partial^2 w}{\partial t^2} \qquad \ldots(1)$$

in which w denotes displacement at any point (x,y,z) on the surface, $\bar{H}_x$ and $\bar{H}_y$ are the horizontal component of tensile force per unit length and m is the mass per unit area. $\bar{h}_x, \bar{h}_y$ and changes in $\bar{H}_x$ and $\bar{H}_y$ respectively and are obtained in terms of extensional rigidity (EA) and strain of the cables. The deflection pattern is assumed to be given by,

$$w = \sum^{i} \sum^{j} C_{ij} \sin \frac{i\pi x}{l_x} \sin \frac{j\pi y}{l_y} \qquad \ldots(2)$$

Eq. 1 is used to compute the strain energy of the net which on equating with maximum kinetic energy and considering only the first term of the series in Eq. 2 gives the following result for frequency (ω)

$$\omega_{11}^2 = \frac{\pi^2}{m}\left[\frac{H_x}{l_x^2} + \frac{H_y}{l_y^2} + \frac{8}{\pi^4}\left(K_x^2 EA_x + K_y^2 EA_y\right)\right] \qquad \ldots(3)$$

in which K corresponds to the curvature of the cable. Results for a flat net computed from Eq. 3 as well as from conventional analysis indicate close comparision, the former approach giving 2-3% higher values of frequency (ω).

Using similar approach for cable trusses, approximate formulae for natural frequencies of trusses are derived for a single wave symmetric mode and double wave antisymmetric mode as follows:

$$\omega_{11}^2 = \frac{\pi^2}{m l^2}\left[(H_u + H_l) + \frac{4 l^2}{\pi^4} K_u^2 (EA)_u (1 + \gamma^2 \phi)\right], \qquad \ldots (4)$$

$$\omega_{22}^2 = \frac{4\pi^2}{m l^2}\left[H_u + H_l\right] \qquad \ldots (5)$$

in which H_u, H_l are Horizontal Component of tension in upper and lower cable, m is mass per unit length, $\gamma = K_l/K_u$ and $\phi = (EA)_l/(EA)_u \cdot K$

Results for a few cases of truss systems obtained from Eq.4 and 5 show close comparision with the values obtained from the lumped mass system approach.

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