ELASTO-PLASTIC EARTHQUAKE RESPONSE OF FRAMES WITH SHEAR WALL

by

TAKUJI KOBORI[I], YUTAKA INOUE[II] and MASAHIRO KAWANO[III]

SYNOPSIS

A structural model of a wall-frame system is assumed simply as a single-story and single-bay frame filled with a wall. It is divided into elements for modeling applied to the finite element method. All elements are supposed to be applicable to the yield condition of von Mises type. The responses of a system having masses concentrated at nodal points and stiffness matrix varied with time are evaluated to earthquakes. The results are compared for the stiffness and strength ratios of wall to frame.

INTRODUCTION

It has been recognized that a shear wall filled in a structural frame is effective to resist against seismic forces, and many investigators have dealt with the analysis of a structure containing shear walls. The analytical researches have been made mainly to clear the static stress distribution or the static deformation of wall-frame system, but usually limited in the elastic range on account of the difficulty for the analysis. Most of experimental studies have treated reinforced concrete shear wall models for the quasi-static loading over the yielding or to the failure state. And the results showed that the failure of wall occurs suddenly with the falling down of strength of the structure and its stiffness and strength are more degraded gradually according to the repeated loading cycles succeeding the initial failure of wall. Consequently, the dynamic behavior of such a shear wall system as required strongly for the safety against the earthquake excitations to have ductility without brittle failure is not enough to be explained up to its final state yet.

The object of this research is to investigate precisely the dynamic responses of a frame with ductile shear wall in the elasto-plastic range, when excited by the ground motion. The responses are analyzed and discussed from the point of view how the deflection, the stress distribution and the expansion and transition of the plastic zone of a structure are influenced after the yielding took place at a frame or a wall. For such an analysis of a wall-frame system, the finite element method is a suitable and successful numerical procedure.

STRUCTURAL MODEL

A two-dimensional, finite element model is assumed in this study to a portal frame filled with a shear wall to make clear in detail the dynamic characteristics of a wall-frame system beyond the elastic limit of the material. A structural model is subdivided into triangular elements and it

---

I Professor of Fuculty of Eng., Kyoto Univ., Kyoto, JAPAN

II Associate Professor of Fuculty of Eng., Osaka Univ., Osaka, JAPAN

III Lecturer of Fuculty of Eng., Osaka Univ., Osaka, JAPAN

consists of 42 nodal points and 60 elements as shown in Fig. 1. Mass of each element is assumed to be lumped at nodal points and a consistent mass matrix is used here. It is considered that the stiffness of each element is proportional to its thickness, but the strength of each element is independent to its stiffness. The stiffness and strength ratios of a wall element to a frame one are important quantities to be designed and are varied to some extent in the present study.

Since the damping effect caused by the plastic energy dissipation is mainly interested here, a small viscous damping is considered for the model. The damping matrix is assumed to be proportional to the stiffness matrix and the damping ratio for the fundamental vibration mode of the elastic system is 0.01. The dynamic response is analyzed for the structural model subjected to the base excitation as the earthquake; El Centro, 1940, N-S component. Adjusting the reference values of mass and stiffness, the fundamental period of the system is fixed in relation to the frequency characteristic of the earthquake in spite of the difference of wall thickness.

ELASTO-PLASTIC CHARACTERISTICS OF ELEMENT

The elastic stress-strain relation of an element on the plane stress condition is written as follows:

$$\{\sigma\} = [D]\{\varepsilon\}, \quad \{\sigma\} = \{\sigma_x \ \sigma_y \ \tau_{xy}\}^T, \quad \{\varepsilon\} = \{\varepsilon_x \ \varepsilon_y \ \gamma_{xy}\}^T \tag{1}$$

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{2}$$

where, σ_x and ε_x; σ_y and ε_y are the normal stress and strain components in the horizontal and vertical directions, respectively, τ_{xy} and γ_{xy} denote the shear stress and strain, E and ν are Young's modulus and Poisson's ratio, respectively, and $[D]$ represents the elastic stiffness matrix of an element.

In order to make the structural system anti-seismic, it is important for a wall to be deformable in the plastic range not to cause a brittle failure. Then, the structural element is assumed here to have such a ductile property. The yield condition of the element is supposed to be of von Mises type and to be applicable for the modified condition in the course of plastic flow[1]. So the yield condition is represented as:

$$f = [(\sigma_x - \alpha_x)^2 - (\sigma_x - \alpha_x)(\sigma_y - \alpha_y) + (\sigma_y - \alpha_y)^2 + 3(\tau_{xy} - \alpha_{xy})^2]^{1/2} - K = 0 \tag{3}$$

where, K is a constant, which means the generalized stress corresponding to the yielding set in initially, and α_x, α_y and α_{xy} denote the amount of translation of the yield condition. The relation between stress increment and strain increment in the plastic range according to the flow rule with work-hardening effect is expressed by the next equations[2].

$$d\{\sigma\} = [D]_{ep}\, d\{\varepsilon\} \tag{4}$$

$$[D]_{ep} = [D] - \frac{[D]\{\partial f/\partial \sigma\}\{\partial f/\partial \sigma\}^T[D]}{H' + \{\partial f/\partial \sigma\}^T[D]\{\partial f/\partial \sigma\}} \tag{5}$$

$$\{\partial f/\partial \sigma\} = \{\partial f/\partial \sigma_x \quad \partial f/\partial \sigma_y \quad \partial f/\partial \tau_{xy}\}^T$$

where, H' means the hardening coefficient and $[D]_{ep}$ represents the stiffness matrix of an element under the plastic flow.

EQUATION OF MOTION AND RESPONSE ANALYSIS

According to the dynamic property of an element described in the previous sections, the equation of motion for a wall-frame system is written as follows:

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + \Sigma\int[B]^T\{\sigma\}dv = -[m]\{l\}a\,f(t) \tag{6}$$

where, $[m]$ and $[c]$ are the mass and damping matrices of the system, respectively, $[B]$ represents the matrix to transfer nodal displacements to strain components of an element, Σ means to superpose nodal forces evaluated as $\int[B]^T\{\sigma\}dv$ for each nodal point. And $\{u\}$ is displacement vector, $\{l\}$ is a vector, the component of which is unity for the horizontal displacement and zero for the vertical displacement, and a and $f(t)$ are the intensity and shape function of the earthquake excitation, respectively.

In this paper, the fundamental period of system is fixed to 1/29 of duration time of the excitation function. Elasto-plastic response of the system is computed step-by-step through the Runge-Kutta 4-th order procedure and a time step for numerical integration is chosen as 1/100 of the fundamental period of the system. Moreover, to avoid the accumulation of error, the computation is added for the interpolated time during a step, when one of elements changes the state; elastic to plastic and vice versa.

In the response analysis, it is supposed that the width-hight ratio of wall is 2/3 and the thickness ratios of wall T/D are 1/2, 1/4 and zero for types 1, 2 and 3, respectively. The system of type 3 means an open frame. Poisson's ratio of all elements is fixed to 1/6. The strength of frame elements is considered here two cases; to be equal to wall and far greater than wall. Since the hardening coefficient seems to be adjustable in design, the present study deals relatively wide variation to H'.

The maximum amplitude of excitation is determined equal to the static horizontal load which makes one of elements set in yielding. Response is evaluated for the intensive part, initial one fifth of wave function, of the El Centro earthquake.

NUMERICAL RESULTS AND CONCLUSIONS

Fig. 2 shows displacement response at the top of the system. In these figures, the generalized stress for yielding and hardening coefficient are supposed to be common to both the wall and frame elements. Displacement is remarkably reduced, especially for type 2, by the plastic yielding. And the magnified deformation mode to the maximum displacement is represented in

Fig. 3. A large distortion occurs near the foot of column in type 1, but not in type 2. For more precise illustration of it, principal stress distribution is shown in Fig. 4 with the deformation mode. Though type 2 model seems to have a smoothed distribution of wall stress, the concentration of stress and deformation can be seen for type 1.

Displacement response in Fig. 5 explains that the yielding of wall alone is also effective to the reduction of story displacement. The maximum base shear force summed for the elements attached base is compared for the yielding of wall and / or frame in Table 1. It is apparent that the yielding of wall decreases it considerably. And Table 2 represents the maximum generalized stress of frame elements when the plastic yielding is assumed to occur in wall elements only. The yielding of wall makes the stress of frame element reduce greatly and the smaller hardening in plastic flow is advantegeous for the frame elements.

The relation between displacement at the top of the structural system and base shear force is shown in Fig. 6, and it represents the elasto-plastic restoring force characteristics of the whole structure. The system of type 1 is apt to show the negative slope when the story deflection grows large. And the system of type 2 follows a stable hysteresis loop, though the area enclosed by the loop is relatively small. The system with elastic frame is considered to increase the stability of the elasto-plastic restoring characteristics. Fig. 7 represents the transition of plastic zone during one cycle of response from the initiation of the yielding. In a case of type 1, the plastic zone seems to be bounded to the lower part of the system. On the other hand, the yielding spreads out quickly to another wall elements in type 2 structure and most part of wall is effective to the seismic load.

Though no figure is shown here for open frame of type 3, it can be pointed out as expected deservedly that the stress concentration at both ends of column is remarkable and the story deflection becomes twice or treble as large as that of a system with a wall.

From these response characteristics of wall-frame system dealt in this paper, the following concluding remarks can be pointed out;

It is reaffirmed that a shear wall filled in a frame is effective to reduce the story deflection to the earthquake excitation. And stress distribution or the maximum stress of frame is affected largely by the stiffness and strength of wall. A system with a moderate stiffness and strength ratios of wall represents a stable, elasto-plastic restoring characteristics, though an over-stiffened and -strengthened wall makes an excessive stress arise in a frame. Consequently, it is important to design a wall to have the optimum stiffness and strength to a frame for retaining the ultimate strength and deformability of the whole system.

REFERENCES

1. Ziegler, H., " A Modification of Prager's Hardening Rule ", Q. Appl. Math. Vol. 17, 1959, pp. 55-65.
2. Zienkiewicz, O. C., " The Finite Element Method in Engineering Science ' McGraw-Hill, 1971.

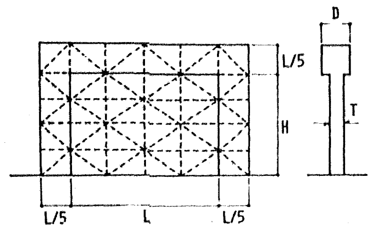

Fig. 1 Structural model

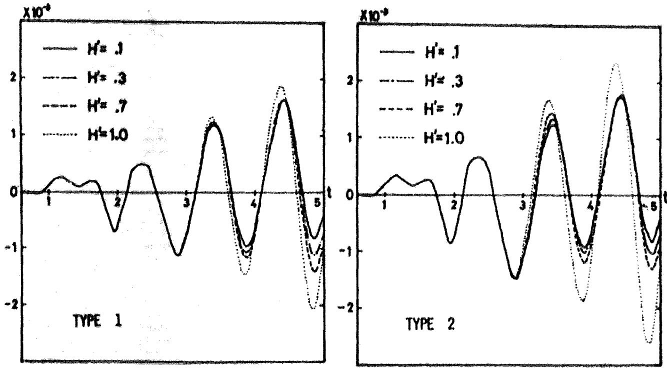

Fig. 2 Horizontal displacement at the top of model

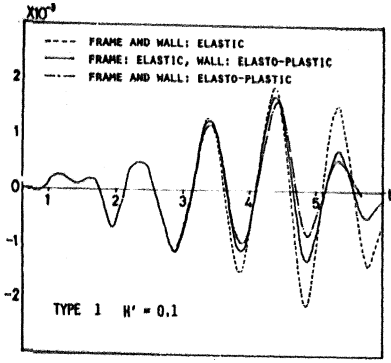

Fig. 5 Horizontal displacement at the top of model

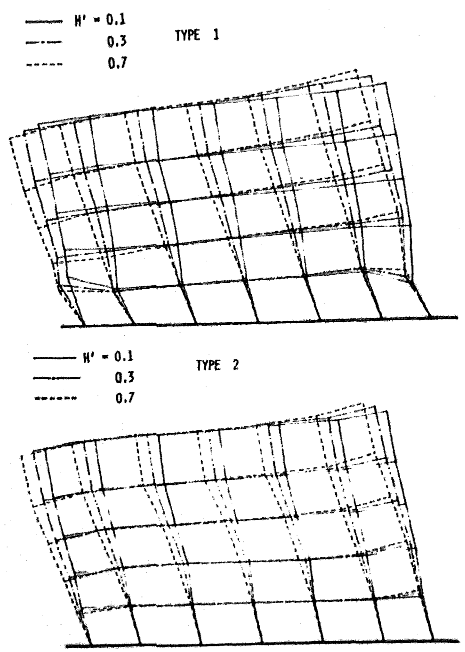

Fig. 3 Deformation mode

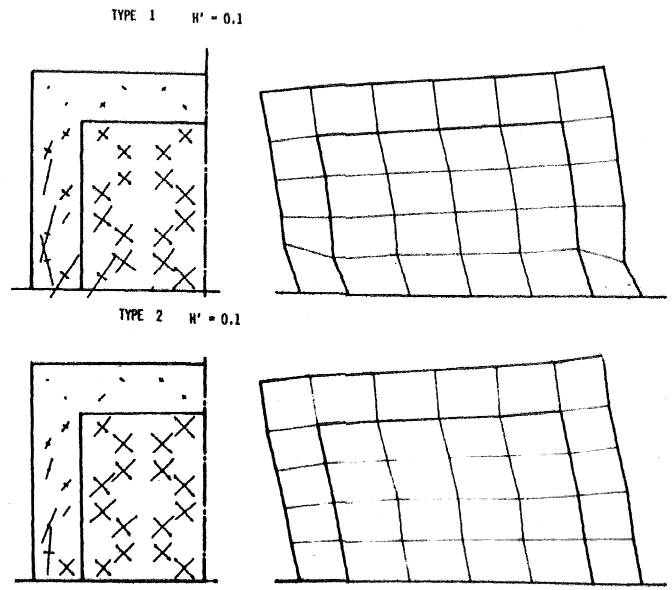

Fig. 4 Principal stress distribution and deformation

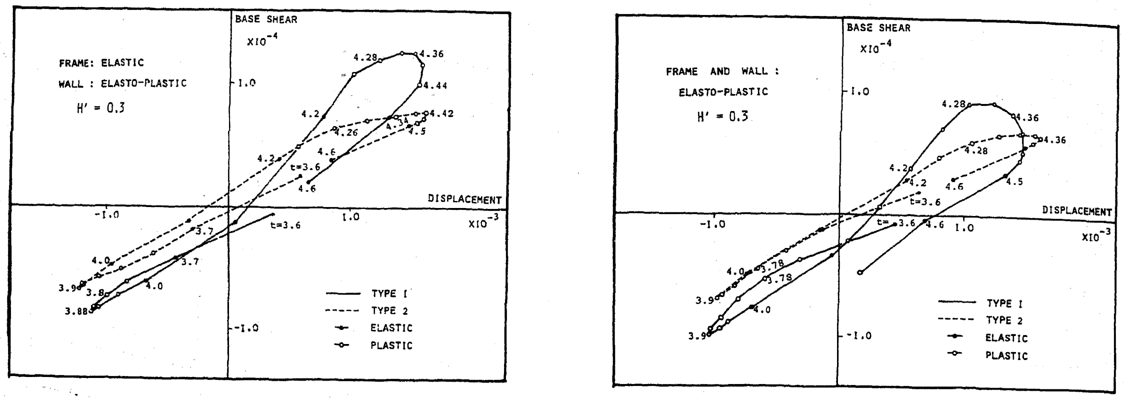

Fig. 6 Relation between top displacement and base shear

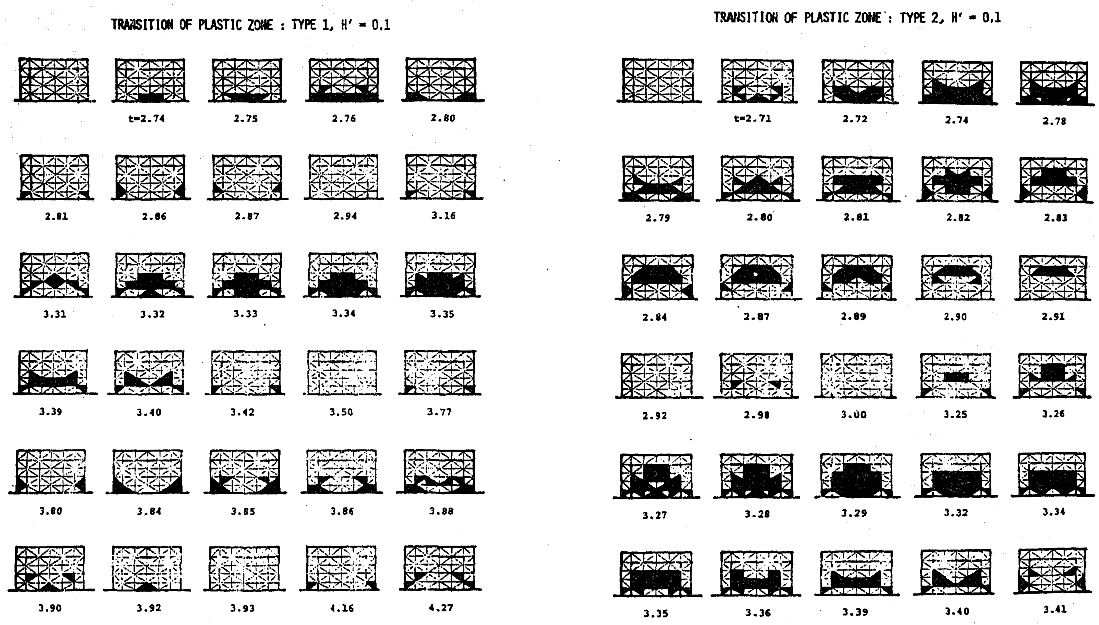

Fig. 7 Transition of plastic zone

Table 1 Maximum base shear (H'=0.1)

	Type 1	Type 2
Frame and wall: elastic	1.516×10^{-4}	1.344×10^{-4}
Frame: elastic Wall: elasto-plastic	1.240	0.787
Frame and wall: elasto-plastic	1.049	0.766

Table 2 Maximum generalized stress of frame elements

	Type 1	Type 2
H' = 1.0	2.409×10^{-3}	2.653×10^{-3}
0.7	1.932	1.661
0.3	1.908	1.578
0.1	1.905	1.521