EVALUATION OF CONTRIBUTION OF FLOOR SYSTEM TO DYNAMIC CHARACTERISTICS OF MOMENT-RESISTING SPACE FRAMES

by

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SYNOPSIS

Parametric, finite element analyses are used to study the contribution of beam-slab floor systems to the overall stiffness of moment-resisting frames, and to establish graphs of the influence of different floor parameters on the floor stiffness. The graphs are the basis of a practical and sufficiently accurate method for computing the contribution of the floor system to the lateral stiffness of a moment-resisting space frame. The proposed method was found to be more accurate than other methods currently used.

INTRODUCTION

The overall lateral stiffness of a moment-resisting frame is governed by the lateral and rotational stiffness of the columns, and the rotational stiffness of the floor system at its supports. The floor system contributes to the stiffness primarily by restraining the rotation of the columns at the floor levels. This type of restraint has a considerable influence on the overall frame stiffness which can usually be increased more efficiently by increasing the stiffness of the floor system rather than that of the columns. In general, a floor system of a moment-resisting frame building consists of a floor slab that is cast monolithically with the floor beams and thus becomes an integral part of the resisting frames. Studies have shown(1) that the lateral stiffness of such frames, which is an essential parameter in the determination of the dynamic characteristics of the structure, is very sensitive to the assumed participation of the floor slabs.

Several methods for estimating the floor slab's contribution to the lateral stiffness of buildings are presently used in the U.S. The ACI 318-71 Code(2) recommends the "equivalent frame method" whereby a three-dimensional slab-column system is represented by a series of two-dimensional frames which are then analyzed for vertical or horizontal loads acting in the plane of the frames. The frames consist of equivalent columns and beam-slab strips, bound laterally by the centerline of the panel on each side of the column centerlines. This method is difficult to use with most available computer programs, and of questionable accuracy when used to compute the lateral stiffness of buildings. Another approach is the effective slab width method whereby a building is modeled as a series of plane frames and the floor as equivalent beams. The floor beams and the effective width of the slab determine the stiffness of the uniaxial, prismatic equivalent beams. Several effective slab width ratios have been suggested, ranging from 0.50 of the half panel width on each side of the column centerline to values greater than unity. This method greatly simplifies the analysis but is open to serious questions as to its underlying assumptions and its accuracy in predicting building responses.

<u>Objectives and Scope</u>. - The objectives of the studies reported herein are: (1) to evaluate accurately the contribution of the main parameters affecting

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the stiffness of a floor system composed of a two-way slab supported on beams between the columns; (2) to develop a practical and reliable method to compute the contribution of such floors to the lateral stiffness of moment-resisting frames; and (3) to evaluate the adequacy and practicability of the method to compute the lateral stiffness of space frame structures. The floor systems consist of a slab of constant thickness, and beams with a 2:1 depth-to-width ratio (i.e., a constant relationship between the flexural and torsional stiffnesses of a beam is maintained). Homogeneous, isotropic, and elastic materials are also assumed. Different slab thicknesses, beam depths, bay aspect ratios and boundary conditions are considered in a parametric study of numerous floors to evaluate their influence on floor stiffness. These results are then used to develop a Stiffness Matrix Method (SMM) to model the stiffness of a composite beam-slab floor by a set of uniaxial members. The SMM is evaluated by using it to compute the lateral stiffness of several simple space frame structures and by comparing these results with those obtained using a finite element procedure and other currently used methods.

SUMMARY OF STUDIES CONDUCTED AND OF THEIR RESULTS

The investigation consisted of establishing an 8 x 8 stiffness matrix for each panel in a floor, based on two rotational degrees-of-freedom (DOF) at each support (Fig. 2). This was accomplished by calculating the moments needed to produce a unit rotation at one DOF while restraining the other DOF at the supports. The slab was modeled (Fig. 1) as a series of rectangular finite elements, where an LCCT9 element formed from four compatible triangles, is used to represent the bending behavior of the slab and a constant strain element with plane stress properties is used to represent its membrane behavior. The beams were modeled as uniaxial, prismatic members, connected by rigid links to the finite element nodes along the beams' centerline. The rigid links impose at the nodes the condition of plane sections in the beam remaining plane. Analytical tests confirmed this method's accuracy when used for slender, flexible floors, but it was found to lose accuracy as the beams' depth-to-span ratio increases and shear begins to dominate the behavior.

The influence of different combinations of the following parameters (Fig. 2) on the stiffness of a two-way slab floor were investigated: (1) the aspect ratio, L_1/L_2, where L_1 and L_2 are the panel span dimensions as indicated in Fig. 2; (2) the ratio of the slab thickness, d_s, to L_1; (3) the ratio of the flexural stiffness of the beam to that of a slab width bounded laterally by the centerline of the adjacent panel, if any, on each side of the beam, α; (4) the ratio of the torsional stiffness of the transverse (torsional) beam to the flexural stiffness of the flexural beam, β; (5) partial and full composite beam-slab action. Partial composite action is where the beams are symmetrical around the slab neutral axis and hence only vertical shears are transmitted between them. Full composite action was studied by introducing a beam-slab eccentricity due to having the tops of the beams and the slab coincide; (6) the different boundary conditions of the panel occurring in single-panel floors, and in corner, exterior and interior panels in multi-panel floors.

A total of 122 panels were analyzed, of which 46 were single-panel floors with partial composite action. The rest had full composite action of which 33 were single-panel floors, 14 were corner panels, 28 were interior panels, and 1 was an exterior panel. Table 1 gives some typical results describing the first line of the 8 x 8 stiffness matrix of a single-panel floor. The position of the neutral axis in floors with eccentric beams varies throughout the floor and is a function of the slab stiffness relative to that of the beams and the coordinates of the point being considered in the floor. The neutral axis of the composite beam-slab floor is closer to the neutral axis of the beams at the edges of a panel and moves closer to that of the slab as it moves toward the center of the panel. The term, γ, defines the position of the neutral axis of the composite beam-slab section at the supports (Fig. 5). $(K_{11})_s$ is the first term of the floor's 8 x 8 stiffness matrix and $(K_{11})^{\xi}_{B}$, is the stiffness of the bare beams around a fictitious neutral axis defined by ξ (Fig. 5), so that:

$$(K_{11})^{r}_{B} = 4EI^{\xi}_{B}/L_1 + GJ_{CB}/L_2 \qquad (1)$$

and

$$I^{r}_{B} = \frac{BD^3}{12} + A\xi^2 \qquad (2)$$

The carry-over factors, CF_{ij}, shown in Table 1 are the off-diagonal terms of the floor stiffness matrix normalized with respect to $(K_{11})_s$. Figures 4 to 6 show typical trends of the important terms of the floor stiffnesses. It was found that the terms, γ, $(K_{11})_s/(K_{11})^{\xi}_{B}$, and CF_{13}, are primarily dependent on the values of L_1/L_2, α, and the boundary conditions, while CF_{15} is dependent on these three terms plus β. Furthermore, it was found that corner panel stiffnesses could be adequately estimated from results of single-panel floors, and exterior panel stiffnesses from results of interior panels. The slab participation, as reflected by $(K_{11})_s/(K_{11})^{r}_{B}$ (Fig. 4), is highest with shallow beams, but drops off quickly and the floor stiffness approaches that of the bare beams, $(K_{11})^{\xi}_{B}$, as α increases. The neutral axis of the floor (Fig. 5) also approaches that of the bare beams as α increases, but at a slower rate than the stiffness. The results show that while the coupling between DOF 1 and 3 is the most substantial, the coupling between DOF 1 and 5 should not be neglected, especially for cases with $\beta > 0.1$ and $L_1/L_2 > 1.0$. It was also found that coupling between panel support DOF along a diagonal or more than one panel length away is weak. Notice that the two-way action of the slab redistributes the moments between the floor beams. This is reflected in Fig. 6 where the values of CF_{13} are substantially smaller than 0.5 which is the value of the carry-over factor for prismatic members.

Stiffness Matrix Method and Its Applications. - The SMM was developed from the results of this investigation. In this method, the elastic stiffness of a beam-slab floor, Fig. 3, is estimated as the stiffness of equivalent uniaxial members, each with three DOF. The SMM does not identify a physical shape for these members; rather, it establishes a procedure by which the position of the neutral axis of the equivalent member in relation to the top of the slab and its member stiffness matrix can be computed directly from a set of graphs. The 3 x 3 member stiffness matrix has the form of:

$$[K]\ \text{member} = (K_{11})_s \begin{bmatrix} S_{11} & 0 & 0 \\ 0 & S_{22} & k_{23} \\ 0 & k_{23} & S_{22} \end{bmatrix} \qquad (3)$$

Each member stiffness matrix is added directly to obtain the structure's stiffness matrix which can then be used to analyze the building response. The effects of shear on the floor stiffness are already included in the terms of Eq. 3; hence, no other terms to model shear effects are necessary. The results obtained show that in the practical application of the SMM, it is sufficient to consider only edge members and interior members (Fig. 3). Figures 4 to 7 give the graphs necessary for computing the stiffness matrix of an edge member. The procedure is as follows: (1) With L_1 as the span length of the member and L_2 the transverse span, enter Fig. 5 to determine

γ, which identifies the position of the neutral axis with respect to the top of the slab, as well as ξ. This allows the determination of I_B^r and $(K_{11})_B^r$ based on Eqs. 1 and 2. (2) Enter Fig. 4 to determine the value of $(K_{11})_s/(K_{11})_B^r$ which is used to compute $(K_{11})_s$. (3) Enter Figs. 6 and 7 to determine k_{23} and S, where S is a term developed from the results of the investigation and used to determine the value of S_{11}, so that:

$$S_{11} = \frac{[(K_{11})_s] \text{ transverse member}}{[(K_{11})_s] \text{ member}} (S) \tag{4}$$

And, finally

$$S_{22} = 1.0 - \frac{[(K_{11})_s S_{11}] \text{ transverse member}}{[(K_{11})_s] \text{ member}} \tag{5}$$

It should be noted that in the calculations of edge members not framing into a corner support (e.g. member B'C') only half of the transverse beam (i.e. Beam BE in the example) is assumed to act in torsion with the flexural beam in the calculations of $(K_{11})_B^r$, β, S_{11} and S_{22}. The other half is assumed to act with the beam in the adjacent span. The stiffness matrix of the interior members is determined in a similar fashion. The member stiffness matrices are symmetric but differ from those for prismatic beams.

The accuracy and applicability of the SMM were evaluated by applying it to calculate the lateral stiffness, P/Δ, of 27 single-panel, single-story structures (Fig. 8) and of one single-story, 3 x 3 panel structure. The results were compared with those from analyses using a finite element method, the ACI equivalent frame method and an effective slab width method where the slab width was based on the requirements of section 8.7 of the ACI 318-71 Code. In each case, the lateral stiffness was defined as the force, P, necessary to produce a unit lateral displacement, Δ, of the top of the slab in the direction of P. The results, a sample of which is given in Table 2, show the equivalent frame method registering the largest deviation, 169%, from the finite element analysis, and especially, as the relative column-to-floor stiffness increases. This corresponds to a 39% deviation in the structure's period, which can lead to substantially different estimates of its dynamic responses. The equivalent frame method was also found to be inconsistent in that it overestimated or underestimated the lateral stiffness, depending on the individual case considered. The effective slab width method generally fared better than the equivalent frame, with a 13.8% maximum deviation from the results of the finite element analysis. However, this method is not based on reliable experimental or analytical studies, and the correlation of results from the effective slab width method with the finite element analysis was inconsistent and did not establish well defined trends. This suggests that the equivalent slab width could lead to much higher errors in estimating the lateral stiffness than the 13.8% deviation registered in this study. The SMM consistently estimated the lateral stiffness closer to the finite element analysis than the other two methods, with a maximum deviation of 6.1%, and maintained its accuracy even in cases where interpolation between the graphs was necessary. The graphs and the procedures of the SMM were found to be simple to apply in practice.

A trial-and-error procedure was used to compute an effective slab width, b_f, that would yield the same lateral stiffness of the single-panel, single-story structures analyzed, as estimated by the finite element method. The results, a sample of which is given in Table 3, show a wide range of values of b_f which vary with each of the parameters considered. This points out the difficulty of attempting to model the stiffness of a two-way floor system with an effective slab width method.

ACKNOWLEDGEMENT

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REFERENCES

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TABLE 1 TYPICAL RESULTS OF THE FIRST LINE OF THE 8 X 8 STIFFNESS MATRIX OF A FLOOR PANEL

L_1/L_2	α	β	d_s(in)	γ	$(K_{11})_S$ $\frac{K\text{-in}}{rad}$	$(K_{11})_S/(K_{11})^r_B$	CF_{12}	CF_{13}	CF_{14}	CF_{15}	CF_{16}	CF_{17}	CF_{18}
1.0 L_1=240"	3.0	.064	6.5	.39	684715	1.17	-.06	.35	.01	-.03	-.04	.02	-.01
			9.0	.35	1709489	1.15	-.06	.34	.01	-.03	-.03	.02	-.01
		.160	6.5	.40	736792	1.16	-.08	.34	.02	-.07	-.04	.03	-.01
	8.0	.064	6.5	.31	1625482	1.09	-.05	.35	.01	-.03	-.03	.02	--
0.5 L_1=120"	8.0	.064	6.5	.20	2750200	1.00	-.04	.33	.01	-.02	-.01	.01	-.02
2.0 L_1=240"	8.0	.064	6.5	.30	887192	1.22	-.06	.36	--	-.07	-.05	.04	--

TABLE 2 TYPICAL VALUES OF THE LATERAL STIFFNESS OF A SINGLE-PANEL, SINGLE-STORY STRUCTURE*

L_1/L_2	C(in)	α_{AB}	β_{AC}	d_s(in)	$\frac{P}{\Delta}$ (Fin. El)	Equivalent Fr.		Effective Slab Width		Stiff. Matrix Meth.	
						P/Δ	Diff %	P/Δ	Diff %	P/Δ	Diff %
1.0 L_1-240"	20	8.0	.064	6.5	349.6	393.3	-12.3	382.4	-9.4	357.3	-2.2
				9.0	488.8	485.0	0.8	520.5	-6.5	505.8	-3.5
			.160	6.5	353.0	390.6	-10.7	382.4	-8.3	359.5	-1.9
0.50 L_1=120"	25	0.4	.064	6.5	561.5	1512.8	-169.4	556.8	0.8	564.4	-0.5
0.75 L_1=180"	25	8.0	.064	6.5	1517.2	1967.3	-25.2	1748.4	-11.3	1656.0	-5.4
2.0 L_1=240"	20	8.0	.064	6.5	488.0	600.1	-23.0	555.3	-13.8	512.1	-4.9

$$* \left(\frac{P}{\Delta}\right) \text{in k/in., Diff \%} = \frac{\left(\frac{P}{\Delta}\right)_{\text{Finite El.}} - \left(\frac{P}{\Delta}\right)_{\text{model}}}{(P/\Delta)_{\text{Finite El.}}} \times 100$$

TABLE 3 EFFECTIVE WIDTH OF EQUIVALENT BEAMS

L_1/L_2	α_{AB}	β_{AC}	ds(in)	b_f(in)	$\frac{b_f}{0.5L_2}$
1.0 L_1=240"	0.80	.064	6.5	25.0	.21
			9.0	29.8	.25
		.160	6.5	29.3	.24
	3.0	.064	6.5	17.6	.15
0.5 L_1=120"	3.0	.064	6.5	12.5	.10
2.0 L_1=240"	3.0	.064	6.5	17.1	.29

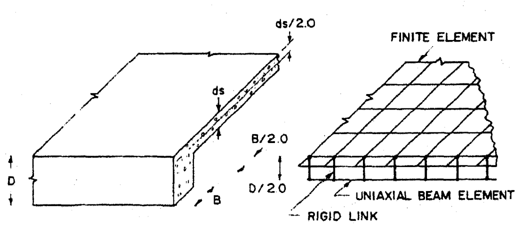

FIG. 1 FINITE ELEMENT MODEL OF A COMPOSITE BEAM-SLAB FLOOR

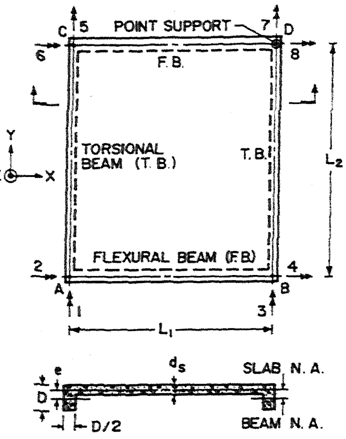

FIG. 2 DEGREES-OF-FREEDOM FOR A SINGLE-PANEL FLOOR

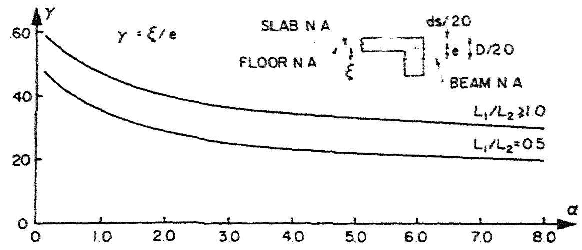

FIG. 5 γ VS. α FOR EDGE MEMBERS

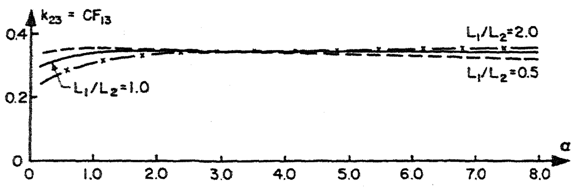

FIG. 6 k_{23} VS. α FOR EDGE MEMBERS

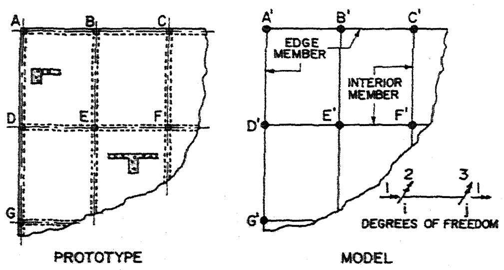

FIG. 3 MODELING OF STIFFNESS OF A BEAM-SLAB FLOOR BY SMM

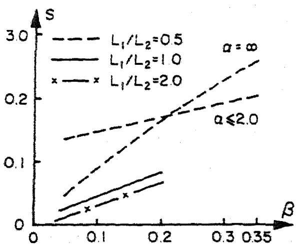

FIG. 7 S VS. β FOR EDGE MEMBERS

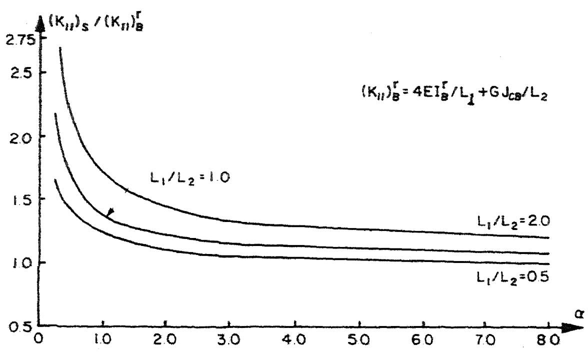

FIG. 4 $(K_{11})_s/(K_{11})_B^r$ VS. α FOR EDGE MEMBERS

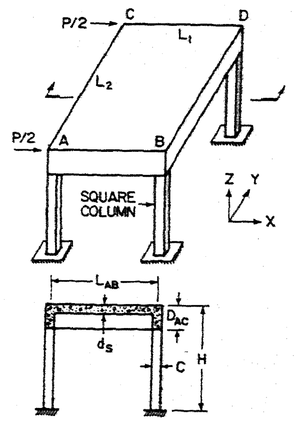

FIG. 8 SINGLE-STORY, SINGLE-PANEL FRAMED STRUCTURE

DISCUSSION

M.K. Agrawal (India)

What is going to be effect if the slab is not solid and may be ribbed slab in one direction or in both the directions we can say it may be waffle slab as this type of contruction is becoming very common in multi-storeyed concrete frames.

T. Paulay (New Zealand)

What would be the torsional effect, induced by two way slabs, on the beams that span at right angles to the direction of earthquake excitation ? Would excessive twisting and torsional cracking reduce the potential flexural capacity of these beams ?

What would be the contribution of slab bars to the flexural strength of beams (T beams), particularly at beam column junctions ? How should we estimate the increased beam flexural capacity due to slab contribution if we wish to protect the beam against shear failure.

Author's Closure

Not received.