OPTIMAL SENSOR LOCATION FOR IDENTIFICATION OF BUILDING STRUCTURES USING RESPONSE TO EARTHQUAKE GROUND MOTIONS

by

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SYNOPSIS

This paper deals with finding the optimal position along the height of a building structure where a sensor may be located, such that records of strong ground shaking obtained thereat would yield a maximum amount of information about the dynamic characteristics of the structure. After modeling a structure by a shear beam, a general technique for solving this problem has been indicated. Some preliminary results for the estimation of the structural stiffness as a function of height have been presented together with their verification using simulated data.

INTRODUCTION

Numerous tall buildings around the world are being instrumented nowadays with strong motion accelerographs, so that their motion during strong ground shaking can be recorded. One of the prime motivations for this is the construction of improved dynamic models from such data. Often, for economic reasons, two accelerographs are used, one located at the base of the structure and the other at some other floor level in the structure, generally the roof. The record of the base motion is useful for several earthquake related studies, thus justifying the location of the first accelerograph. However, the second accelerograph may be so located as to yield maximal information about the structural model. This becomes all the more important due to the finding [1] that for a structure modeled as a shear beam, identification using base input and roof response may lead to locally nonunique and thus grossly erroneous estimates of the stiffness distribution.

In this paper, we model a building structure by a fixed base undamped shear beam and present a methodology which can be used to determine the optimal location of a sensor such that the noisy strong motion records obtained at that location, when used for identification, would yield the most accurate estimates of the distribution of building stiffness with respect to height. Soil - structure interaction effects have not been considered.

THEORY

Model and Identification. For a structure modeled as a shear beam [2], the horizontal structural motion $r(x, t)$ in response to the known ground motion $w(t)$ is governed by the equations

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$$m(x) \frac{\partial^2 r}{\partial t^2} = \frac{\partial}{\partial x}\left(k(x) \frac{\partial r}{\partial x}\right) , \tag{1}$$

$$r(0, t) = w(t) \ , \ \frac{\partial r}{\partial x}(L, t) = 0 \ ; \ r(x, 0) = 0 \ , \ \frac{\partial r}{\partial t}(x, 0) = 0 \tag{2}$$

where x is the position coordinate along the height of the structure, while $k(x)$ and $m(x)$ are the unknown stiffness and known mass distributions in the structure.

The measurements consist of the record of the displacement at a single location $x = a$ for the time period $(0, T)$. The identification problem consists of determining the stiffness $k(x)$ using the model equations, the base motion $w(t)$, and the observed response, $r^{obs}(a, t)$. A generally used approach consists of finding $k(x)$ such that the corresponding model response $r^{model}(a, t)$ matches the observed response as closely as possible over the period $(0, T)$. A new and highly efficient first-order gradient algorithm for this purpose of history matching is described in [1].

Optimal Sensor Location Problem. For a given structure and a given input, the estimate $\hat{k}(x)$ will depend on the observation site, a. In this paper, we attempt to determine if the estimate $\hat{k}(x)$ will be closer to the true distribution $k(x)$ when the observations are taken at a particular location, as compared to any other location.

For the sake of numerical treatment and ease of description, we utilize a finite dimensional approximation of the problem through the use of suitably fine grids of N and M nodes to cover the intervals $[0, L]$ and $[0, T]$, respectively $(M>N)$. Then the identification problem reduces to determining the vector $\underset{\sim}{k}$ of the N grid point values of $k(x)$, using the M-vectors $\underset{\sim}{r}^{obs}(a)$ and $\underset{\sim}{w}$.

In general, the observations are corrupted by the measurement noise, $\underset{\sim}{e}$. Also, the numerically obtained history match between $\underset{\sim}{r}^{model}(a)$ and $\underset{\sim}{r}^{obs}(a)$ is imperfect. These result in the estimated stiffness $\hat{\underset{\sim}{k}}$ being different from the true stiffness $\underset{\sim}{k}$. We then, due to (1), have the nonlinear implicit relationship,

$$\underset{\sim}{r}^{model}(a) - \underset{\sim}{r}^{obs}(a) = f(\hat{\underset{\sim}{k}}, \underset{\sim}{k} ; \underset{\sim}{e}) .$$

Given the probability densities for $\underset{\sim}{e}$ and the residual history match error, we can obtain the probability density for $\hat{\underset{\sim}{k}}$ for any $\underset{\sim}{k}$ by (say) a Monte Carlo procedure. Then the covariance of the estimate error can be determined. However, this would involve a prohibitive amount of computational effort. As a result, we seek a simple, approximate, linear relation between the history match error and the estimate error, which can be easily inverted to obtain the probabilistic description of the latter. Assuming that the history match error $\delta\underset{\sim}{r}(a)$ and the consequent estimate error $\delta\underset{\sim}{k}$ are small enough to allow the linearization of the above relation, we get

$$\delta\underset{\sim}{r}(a) = A\, \delta\underset{\sim}{k} , \tag{3}$$

where $A(M\times N)$ has the elements $\partial r_i(a)/\partial k_j$. These are the sensitivities of the observations with respect to the parameters $\{k_i\}$.

Using linear algebra, the matrix A can be decomposed as

$$A = U\Lambda V^T \quad , \tag{4}$$

where $U(M\times N) \equiv [\underset{\sim}{u}(1), \underset{\sim}{u}(2), \ldots, \underset{\sim}{u}(N)]$, $V(N\times N) \equiv [\underset{\sim}{v}(1), \underset{\sim}{v}(2), \ldots, \underset{\sim}{v}(N)]$, and $\Lambda \equiv \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ with $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \geq 0$. The sets $\{\underset{\sim}{u}(i)\}$ and $\{\underset{\sim}{v}(i)\}$ each form an orthonormal set of vectors. The vectors $\{\underset{\sim}{v}(i)\}$ form a complete set and span the parameter space E^N, whereas $\{\underset{\sim}{u}(i)\}$ do not span E^M. However, a matrix $U_o(M\times(M-N)) \equiv [\underset{\sim}{u}(N+1), \ldots, \underset{\sim}{u}(M)]$ can be constructed such that the (MxM) matrix $[U|U_o]$ is orthogonal with its columns spanning E^M. Then defining $\underset{\sim}{\alpha} = U^T\delta\underset{\sim}{r}$, $\underset{\sim}{\alpha}_o = U_o^T\delta\underset{\sim}{r}$, and $\underset{\sim}{\beta} = V^T\delta\underset{\sim}{k}$, we obtain

$$\delta\underset{\sim}{r} = U\underset{\sim}{\alpha} + U_o\underset{\sim}{\alpha}_o \quad , \quad \delta\underset{\sim}{k} = V\underset{\sim}{\beta} \quad . \tag{5}$$

Substituting (4) into (3) and premultiplying by U^T,

$$U^T\delta\underset{\sim}{r} \equiv \underset{\sim}{\alpha} = \Lambda V^T\delta\underset{\sim}{k} \equiv \Lambda\underset{\sim}{\beta} \quad \text{or} \quad \alpha_i = \lambda_i\beta_i \quad ; \quad i = 1, 2, \ldots, N \ . \tag{6}$$

This is a linear relation between the components of the history match error $\delta\underset{\sim}{r}$ along $\underset{\sim}{u}(i)$ and the component of the estimate error $\delta\underset{\sim}{k}$ along $\underset{\sim}{v}(i)$.

Assuming that $\lambda_i > 0$ for all i, and that the history match errors δr_i are independent Gaussian random variables with zero means and a uniform variance σ^2, we have $E\{\delta\underset{\sim}{r}\,\delta\underset{\sim}{r}^T\} = \sigma^2 I$, so that

$$P \equiv E\{\delta\underset{\sim}{k}\delta\underset{\sim}{k}^T\} = VE\{\underset{\sim}{\beta}\underset{\sim}{\beta}^T\}V^T = V\Lambda^{-1}U^T E\{\delta\underset{\sim}{r}\delta\underset{\sim}{r}^T\}U\Lambda^{-1}V^T = \sigma^2 V\Lambda^{-2}V^T \ . \tag{7}$$

The last expression gives an approximation, P, to the covariance of the estimate based on the linearization (4). If any λ_i is very small, the variances of the estimates are very large. If $\lambda_i = 0$, for finite $\beta_i \equiv \underset{\sim}{v}(i)^T\delta\underset{\sim}{k}$, $\alpha_i = 0$. Thus, the component of small changes in $\underset{\sim}{k}$ along $\underset{\sim}{v}(i)$ has no influence on the measured data and consequently cannot be estimated from it.

If p of the $\{\lambda_i\}$ are zero, we may restrict the corrections to lie in the (N-p) dimensional subspace spanned by $\underset{\sim}{v}(1), \underset{\sim}{v}(2), \ldots, \underset{\sim}{v}(N-p)$. This corresponds to taking the Lanczos pseudo-inverse [3] of A. Then we obtain finite variances of the estimates. Let $\overline{V} \equiv [\underset{\sim}{v}(1), \underset{\sim}{v}(2), \ldots, \underset{\sim}{v}(N-p)]$, $\overline{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{N-p})$, $\overline{\beta} = \overline{V}^T\delta\underset{\sim}{k}$, and $\overline{\delta k} = \overline{V}\underset{\sim}{\beta}$. Then we obtain the partial covariance,

$$\overline{P} \equiv E\{\overline{\delta\underset{\sim}{k}}\,\overline{\delta\underset{\sim}{k}}^T\} = \overline{V}\,\overline{\Lambda}^{-2}\overline{V}^T \quad . \tag{8}$$

The corrections $\overline{\delta k}$ now contain no components along $\underset{\sim}{v}(N-p+1), \ldots, \underset{\sim}{v}(N)$ and thus the estimates are biased. Without additional information on $\underset{\sim}{k}$, this bias cannot be estimated and the partial covariance $\overline{P}$ cannot be used.

The optimal sensor location for any structure may be obtained by computing the covariance P of $\underset{\sim}{k}$ for given data for all the possible locations and picking the one that yields the smallest value of a scalar measure of P. The scalar measure used here is the trace.

RESULTS AND DISCUSSION

For illustration, the case of a ten-storey building with a uniform mass distribution of 3 units/grid length, whose true stiffness, $k(x)$, linearly decreased with height (fig. 1) was analyzed. The input was the scaled N-S component of the El Centro earthquake of 1940 (fig. 3). The finite differencing of equations (1) and (2) was carried out using a uniform spatial grid of 10 points. For the time integration, the implicit Crank-Nicholson scheme with 400 equal time intervals was used.

In order to keep the computational effort reasonably small, the sensitivities of the measurements, sampled at 66 time instants, uniformly distributed over $(0, T)$, were utilized. It was found that for sufficiently large M and for uniform sampling intervals, the actual sampling times and the number of samples did not affect the results. M and sampling times were kept unchanged during the analyses for the sensor locations at floor levels 2 to 10, level 1 being the basement.

As observed from (7), the larger the values of $\{\lambda_i\}$, the smaller is the covariance of $\underset{\sim}{k}$. The magnitude of the singular values $\{\lambda_i\}$ depends on two factors: (a) the actual values of the sensitivities $\partial r_i/\partial k_i$, which are directly proportional to displacements r_i; and (b) the linear independence of the sensitivities of the observations at different times. The latter determines the rank of A and has a large influence on the smaller of $\{\lambda_i\}$. This factor is determined by the participation of the different modes of motion in constituting the observed displacements. Numerical results indicate that the contribution of the first factor increases as the sensor location moves upwards because the response increases in amplitude, whereas the rank of A decreases. From (7) it is clear that the smaller of $\{\lambda_i\}$ have the greatest influence on the covariance. Figure 2 contains the plots of the normalized, averaged standard deviation of $\{k_i\}$ against the sensor location 'a' for 5 different values of NP, the dimension of the subspace used to compute $\overline{P}$ in (8). For NP = 10, the variances of $\{k_i\}$ are the lowest for sensor location at level 2. For NP = 3 and 5, the partial variances are smallest for the sensor at the top. However, these cases involve unknown bias in the estimates. In the absence of additional information, the second floor level is the optimal sensor location.

An important feature of the sensor location problem is that it can not be exactly solved without the knowledge of $k(x)$ and $w(t)$. Hence, to study the dependence of the optimal location on the actual values of these functions, the foregoing analysis was carried out for cases of uniform and linearly varying stiffness distributions using two different types of input. In addition, buildings with three different heights of 5, 10, and 20 storeys were treated. In all cases, the results about the optimal sensor location were similar. These results indicate that it may be possible to draw conclusions about the optimal sensor location in the absence of *a priori* knowledge about the actual functions $k(x)$ and $w(t)$.

Figure 4 contains the plots of 10 vectors $\{\underset{\sim}{y}(i)\}$ for $a = 2$ and $a = 10$. It is clear that the first few $\underset{\sim}{y}(i)$ do not differ significantly in these cases. The subspace spanned by the first NP of $\{\underset{\sim}{y}(i)\}$ appears to be only weakly dependent on $k(x)$. Thus, the use of a given value of NP implies that the corrections in the estimate lie in approximately the same linear subspace of E^N, irrespective of the values of k and a used.

To verify these conclusions, the identification of the 10-storey building with linear k(x) was carried out using simulated data. Measurements contaminated by noise (zero mean, Gaussian, with standard deviation equal to 12% of the rms roof level displacement) together with the input described in fig. 3 were used for the identification. The history match commensurate with the observation noise level was achieved by the algorithm described in [1] using no constraints on the stiffness distribution, thus implying an absence of any prior knowledge about it. The resulting estimates are plotted in fig. 1. The advantage of the sensor at the second level instead of at the roof is clearly borne out.

CONCLUSIONS

(1) In the absence of any prior knowledge, the sensor located at a floor level immediate to the basement yields the most information about the stiffness distribution when the identification is done using a history match of observed and model responses.

(2) The optimal sensor location problem cannot be exactly solved in the absence of knowledge about the actual stiffness distribution and input motion. However, the foregoing result was found not to strongly depend on the actual stiffness distribution, so far as it is smooth, or on the actual input motion so far as it contains sufficiently many frequency components.

(3) If prior knowledge about the stiffness distribution, either qualitative or probabilistic, is available, the partial variance $\bar{P}$ for an appropriate dimension NP of the estimate correction space can be utilized, making further analysis possible.

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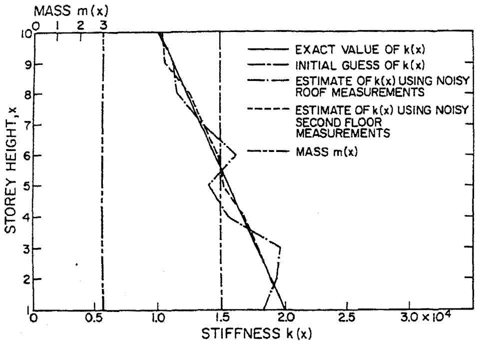
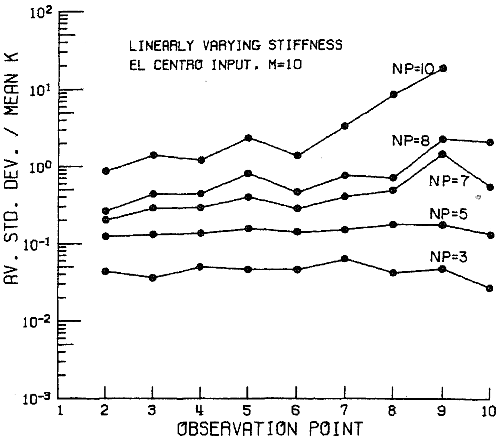
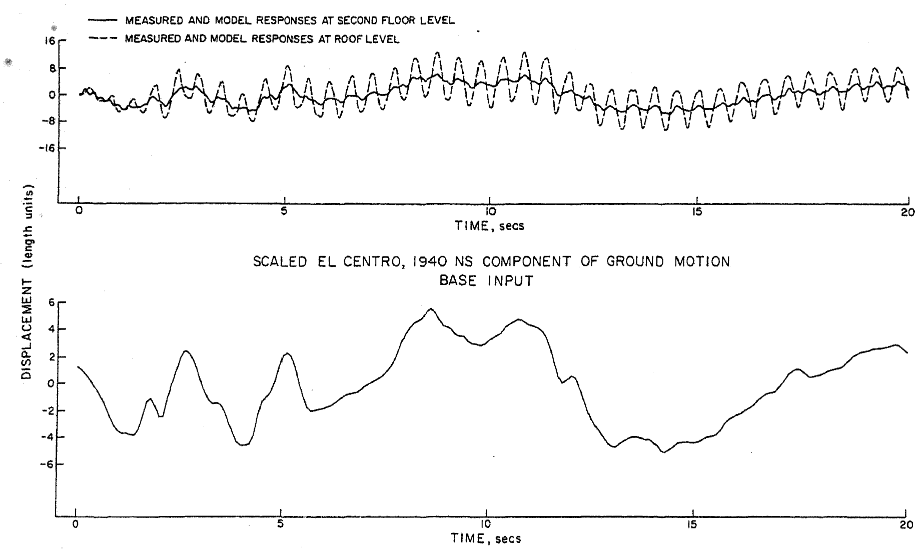

MASS m(x)
EXACT VALUE OF k(x)
INITIAL GUESS OF k(x)
ESTIMATE OF k(x) USING NOISY ROOF MEASUREMENTS
ESTIMATE OF k(x) USING NOISY SECOND FLOOR MEASUREMENTS
MASS m(x)
STOREY HEIGHT, x
STIFFNESS k(x)
3.0 × 10^4

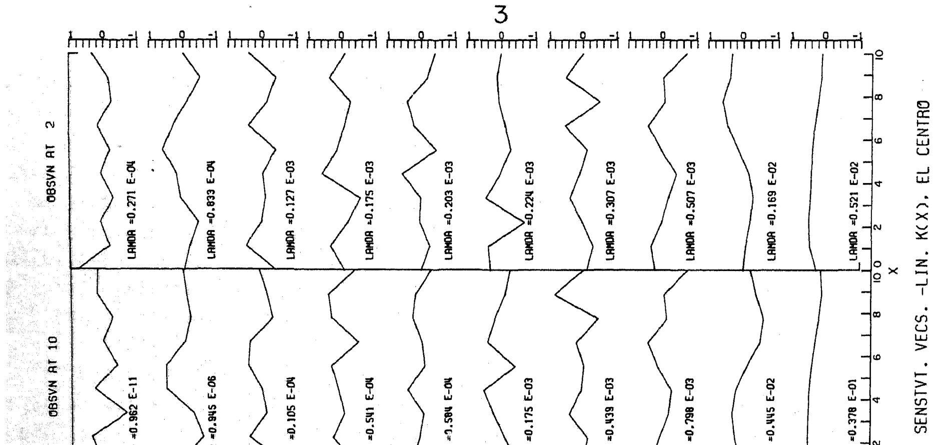
1


LINEARLY VARYING STIFFNESS
EL CENTRO INPUT. M=10
NP=10
NP=8
NP=7
NP=5
NP=3
AV. STD. DEV. / MEAN K
OBSERVATION POINT

2


MEASURED AND MODEL RESPONSES AT SECOND FLOOR LEVEL
MEASURED AND MODEL RESPONSES AT ROOF LEVEL
TIME, secs
SCALED EL CENTRO, 1940 NS COMPONENT OF GROUND MOTION
BASE INPUT
DISPLACEMENT (length units)
TIME, secs

3


OBSVN AT 2
OBSVN AT 10
LAMDA =0.271 E-04
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LAMDA =0.127 E-03
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LAMDA =0.203 E-03
LAMDA =0.224 E-03
LAMDA =0.307 E-03
LAMDA =0.507 E-03
LAMDA =0.169 E-02
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LAMDA =0.962 E-11
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LAMDA =0.584 E-04
LAMDA =0.175 E-03
LAMDA =0.439 E-03
LAMDA =0.798 E-03
LAMDA =0.445 E-02
LAMDA =0.378 E-01
X
SENSTVT. VECS. -LIN. K(X), EL CENTRO

DISCUSSION

P.N. Agrawal (India)

What would be the preferntial location for second sensor in a structure where stiffness is uniform or not varying with height ? What about dams in particular ?

Author's Closure

Not received.