REGRESSION ANALYSIS ON RESONANCE CURVES OF FORCED VIBRATION TESTS AND ITS ERROR ESTIMATION DUE TO MICROTREMORS

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1. INTRODUCTION

For the examination of vibrational behaviors of structures and their elements, we use a method of forced vibration generators to find natural frequencies and damping ratios. Generous amount of data which includes experimental errors, so to say physical error and sensitive error, should be gathered from vibration tests. The former owes to the properties of measuring instruments. The latter is artificial error in analyzing a resonance curve drawn. Sensitive error will cause larger errors in natural frequencies and damping ratios. On the other side, large error might be introduced into the actual vibration tests influenced by microtremors for smaller force of a vibration generator. The purpose of this paper is twofold:

1) To find a statistical analysis method of the observed data.
2) To search out the error distribution influenced by microtremors in this statistical method.

2. REGRESSION ANALYSIS

The particular solution of the equilibrium equation with one-degree system on forced vibration tests is shown as follows.

$$Qd = (m_or / m)\cdot(f / fo)^2 / \sqrt{[1 - (f / fo)^2]^2 + 4\cdot h_o^2(f / fo)^2}, \quad (1)$$

where f_o= natural frequency, h_o= damping ratio, m = mass, m_or = eccentric moment and f = exiting frequency. Equation(1) will also be transformed as follows.

$$y = c\cdot x / \sqrt{(a - x)^2 + a\cdot b\cdot x}, \quad (2)$$

where $a = f_o^2$, $b = 4\cdot h_o^2$, $c = 1/m$, $x = f^2$ and $y = Qd / m_or$. Equation(2) is useful to decide the most suitable coefficients, a, b and c, for the groups of observed (x,y), i.e. (x_1,y_1), (x_2,y_2), $\cdots(x_n,y_n)$, by regression analysis such as the method of least squares. Finaly the author obtained a set of the non-linear simultaneous equations as follows.

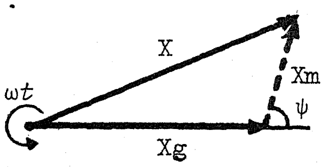
$$\sum_{i=1}^{n}(x_i\cdot y_i\cdot g_i)\cdot\sum_{i=1}^{n}(x_i^3\cdot g_i^4) - \sum_{i=1}^{n}(x_i^2\cdot y_i\cdot g_i^3)\cdot\sum_{i=1}^{n}(x_i^2\cdot g_i^2) = 0,$$
$$\sum_{i=1}^{n}(x_i\cdot y_i\cdot g_i)\cdot\sum_{i=1}^{n}(x_i^2\cdot g_i^4) - \sum_{i=1}^{n}(x_i\cdot y_i\cdot g_i^3)\cdot\sum_{i=1}^{n}(x_i^2\cdot g_i^2) = 0, \quad (3)$$

where $g_i = \{(a-x_i)^2 + a\cdot b\cdot x_i\}^{-\frac{1}{2}}$.

3. ERROR ESTIMATION

Actual response vector X can be obtained using Eq.(4) on Gaussian plane.

$$X = Xg + Xm \quad (4)$$



Gaussian plane

where Xg and Xm show exact and error response vector, respectively. The author considered that $|Xm|e^{i\psi}$ on the stationary random process may be influenced by microtremors, and searched out the amount and distribution of these error taking, as a parameter, s/n ratio, i.e. 20 log(|Xg|/|Xm|) dB, using Monte Carlo Simulation under the conditions that ψ is random value and its distribution is uniform between 0 and 2π radian.

4. CONCLUSION

It is the most important in vibration tests to estimate the damping ratios of the lowest mode which are greatly affected by the soil-structure interaction. For this reason, regression analysis to estimate observed results on vibration tests is very useful.

The error in natural frequencies is much smaller than the error in damping ratios. Standard deviation of comparative error ratio of the former is about one tenth of that of the latter. It is necessary that s/n ratio should be at least 20 dB for a precise vibration test.

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