

## APPROXIMATE VIBRATION ANALYSIS OF A CABLE STAYED BRIDGE

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### SYNOPSIS

An approximate method of vibration analysis of a cable stayed bridge in longitudinal direction has been presented, the results of which have been compared with those of a small scale model test and an exact analysis. It has been found that the fundamental frequency of the structure as determined by approximate method shows good agreement with the results of experimental and exact analysis.

### INTRODUCTION

A cable stayed bridge constitutes a complex structural system particularly when its vibration analysis is carried out. To obtain an insight into the vibration of such a major bridge, it was considered desirable to conduct a small scale model test and check the method of analysis with experimental results. To begin with free vibration in the longitudinal direction of the bridge was considered.

The scaled model of the five span cable stayed bridge (1) is shown in Figs. 1 and 2. In this bridge, fixed bearing is provided only on an end pier and the other piers and towers are provided with vertical links which would permit free translation. This perspex model was tested for free vibrations in the longitudinal direction. For dynamic analysis in longitudinal direction, that is, along the direction of traffic, the deck was regarded as rigid and its mass lumped at the top of pier having fixed bearing. The natural frequencies and modes were computed by treating the pier as a bending shear structure. For checking the results of approximate analysis and that of model tests, the model has further been analyzed (2) using a lumped multimass idealization and stiffness matrix analysis. This work is briefly presented herein.

### MODEL TESTING

A scale factor of 200 was chosen with the consideration of facilities of testing equipment available, cost of model and time required in preparing the model. All the linear dimensions of spans, piers and towers were scaled geometrically. The cross sectional dimensions of the members were fixed with due consideration of their prototype sizes and modulus of elasticity of material. Moreover the deck mass was required to be increased by similitude requirements of the natural frequency. This was done by gluing small metallic cylindrical weights at regular intervals along the length of the deck at both sides of it.

For vibration testing the model was mounted on shake table as shown in Fig. 1. All the wells supporting the piers were clamped at

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their foundation level through steel angles and bolts. Rigid packing plates of appropriate thickness were used under the wells to simulate differences of levels in various foundations the following tests were carried out:

- a) Free vibration test of the first pier after removing the superstructure so that an idea of its base rigidity could be obtained.
- b) Free vibration test of the whole bridge model in the longitudinal direction.

For the above two tests the shake table was clamped and initial displacement condition was used.

- c) Resonance vibration test of the bridge as a whole in longitudinal direction by using a Lazan type mechanical oscillator.

For vibrating the model freely, the table was rigidly attached at one end to a fixed base to arrest its movement. The vibration records were obtained by using an accelerometer mounted at the top of pier 1. The fundamental frequency obtained in each case is shown in Table 1.

#### APPROXIMATE ANALYSIS

As stated earlier except the pier 1, every other pier allows the deck to translate freely in longitudinal direction. In the approximate analysis, the deck is treated as rigid and its mass is lumped at the top of the first pier. For determining the fundamental frequency, the pier itself has been divided into six parts and its mass is lumped at the nodal points. Figure 3 shows the details of this pier.

The pier well system is primarily treated as a cantilever beam fixed at the base. The natural frequency and the mode shapes are computed using modified Holzer's technique (3). In applying the transfer functions from node to node the deformations due to shear and bending and the rotatory inertia have been considered. Satisfying the boundary conditions as fixed against rotation and translation at the base and free to rotate and translate at the top, the natural frequency obtained in the fundamental mode is shown in Table 1.

#### RATIONAL DYNAMIC ANALYSIS

In this analysis, due to symmetry about the longitudinal axis, the whole structure has been lumped as a plane frame as shown in Fig. 4. For this plane frame detailed dynamic analysis has been done for vibrations in longitudinal direction by writing the stiffness matrix of the whole structure and solving for the resulting eigen value by matrix iteration technique. The fundamental frequency obtained is 20.95c/s for the model. This is also shown in Table 1 for comparison.

#### EFFECT OF BASE FLEXIBILITY

Comparison of experimental and analytical frequency values in Table 1 shows that the experimental value is considerably lower than the theoretical frequency. The main reason apparently is that the base support of the pier is not fully fixed as it has been taken in

the theoretical analysis using the concept of generalised mass at top of pier to represent the distributed mass of the pier in the unloaded state (that is without the deck at its top), it is found that the generalised stiffness of the pier would be 107.95 kg/cm for the fixed case and 51.69 kg/cm for the actual experimental pier. Now reducing the overall stiffness of the pier in the ratio of the experimental to the fixed case, the values of the theoretical frequencies obtained by the approximate and rational analysis are reduced from 21.40 c/s to 15.05 c/s and 20.95 c/s to 14.45 c/s. These are also shown in Table 1. In the loaded state experimental frequency is 16.67 c/s which is of the same order the difference being only about 10 per cent.

#### COMPARISON OF FREQUENCIES

Comparing the fundamental frequency values as determined in various ways as given in Table 1, it is seen that the approximate and rational values differ only by 2.5 per cent. These values when corrected for the flexibility of the damping arrangement at the base, compare well with the experimental value, the difference being about 10 per cent.

#### CONCLUSION

The theoretical and experimental results given about clearly indicate that for preliminary seismic analysis of the bridge in the longitudinal direction, the fundamental time period can be determined using the approximate method in which the flexibility of the superstructure is disregarded and the mass of the deck is lumped at the top of the pier with fixed bearing. The error in this approach is seen to be within 2.5 per cent of the rational values for the case studied.

#### REFERENCES

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TABLE 1 - FUNDAMENTAL FREQUENCY OF BRIDGE MODEL

CASE	FREQUENCY IN C/S FOR PIER 1	
	Without deck at top	Without deck at top
1. Experimental		
(a) Free Vibration	178.00	16.66
(b) Resonant	-	16.60
2. Approximate Analysis (treating base fixed)	257.00	21.40
3. Rational Matrix Analysis (treating base fixed)	-	20.85
4. Approximate Analysis using Experimental stiffness)	-	15.05
5. Rational Matrix Analysis using Experimental Stiffness	-	14.45

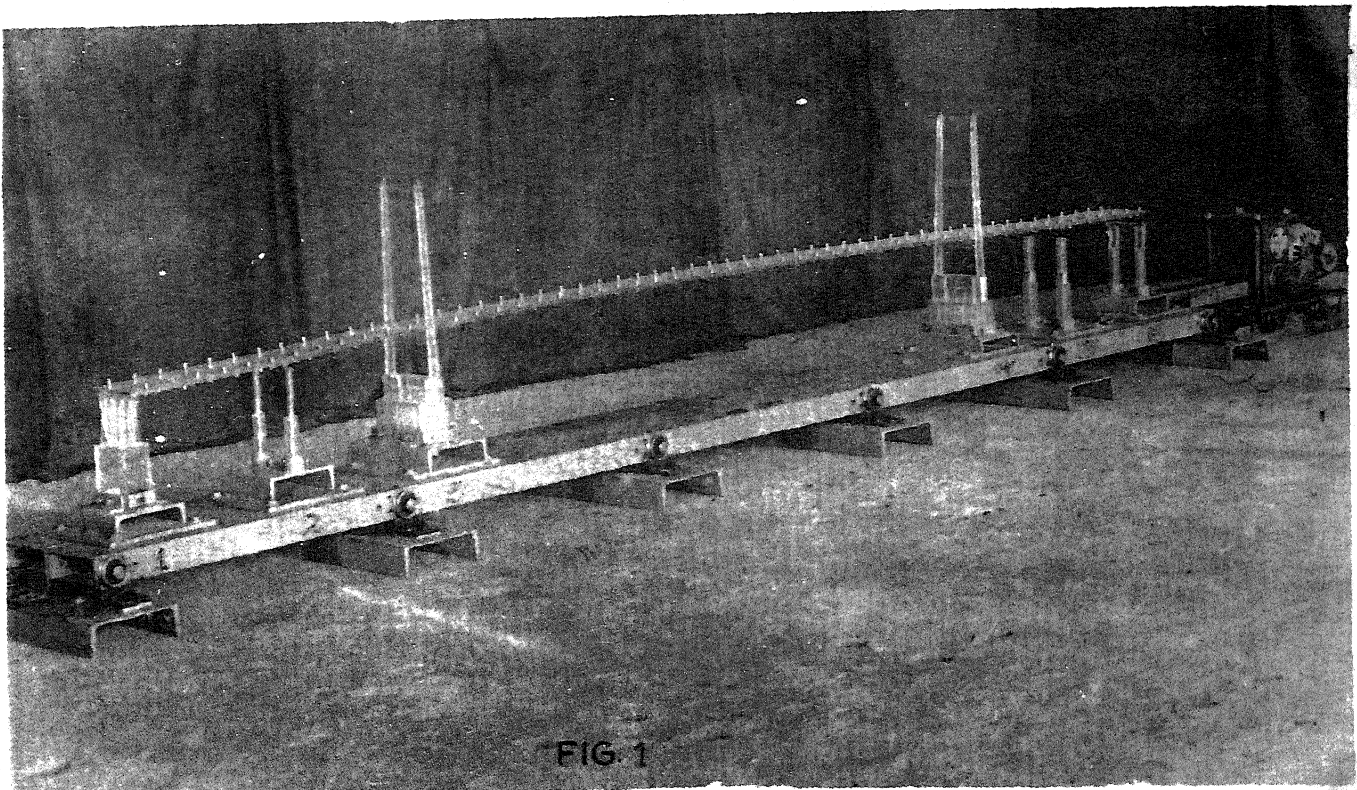
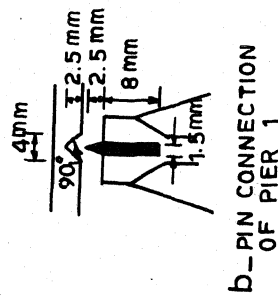
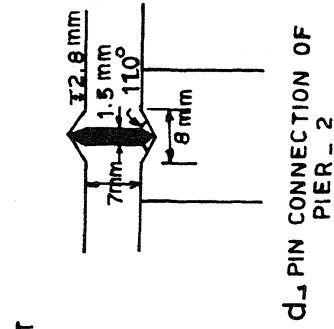
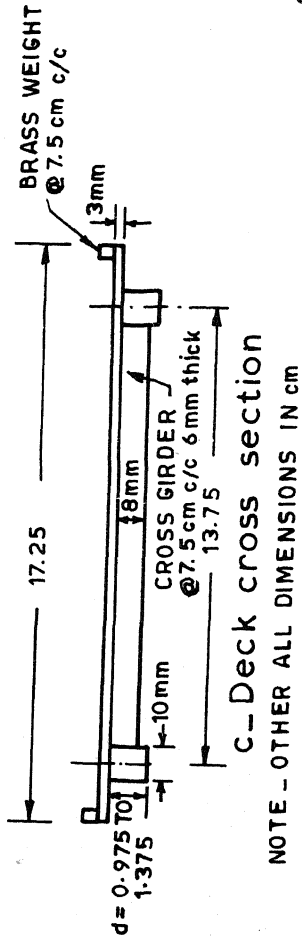
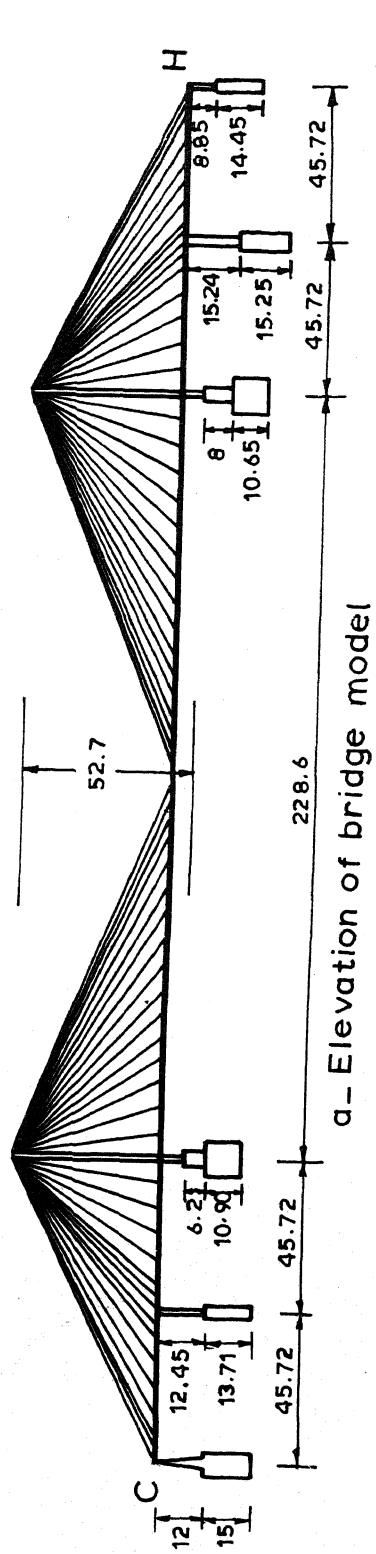


FIG 1



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FIG. 2 - DETAILS OF PERSPEX MODEL OF HOOGHLY BRIDGE

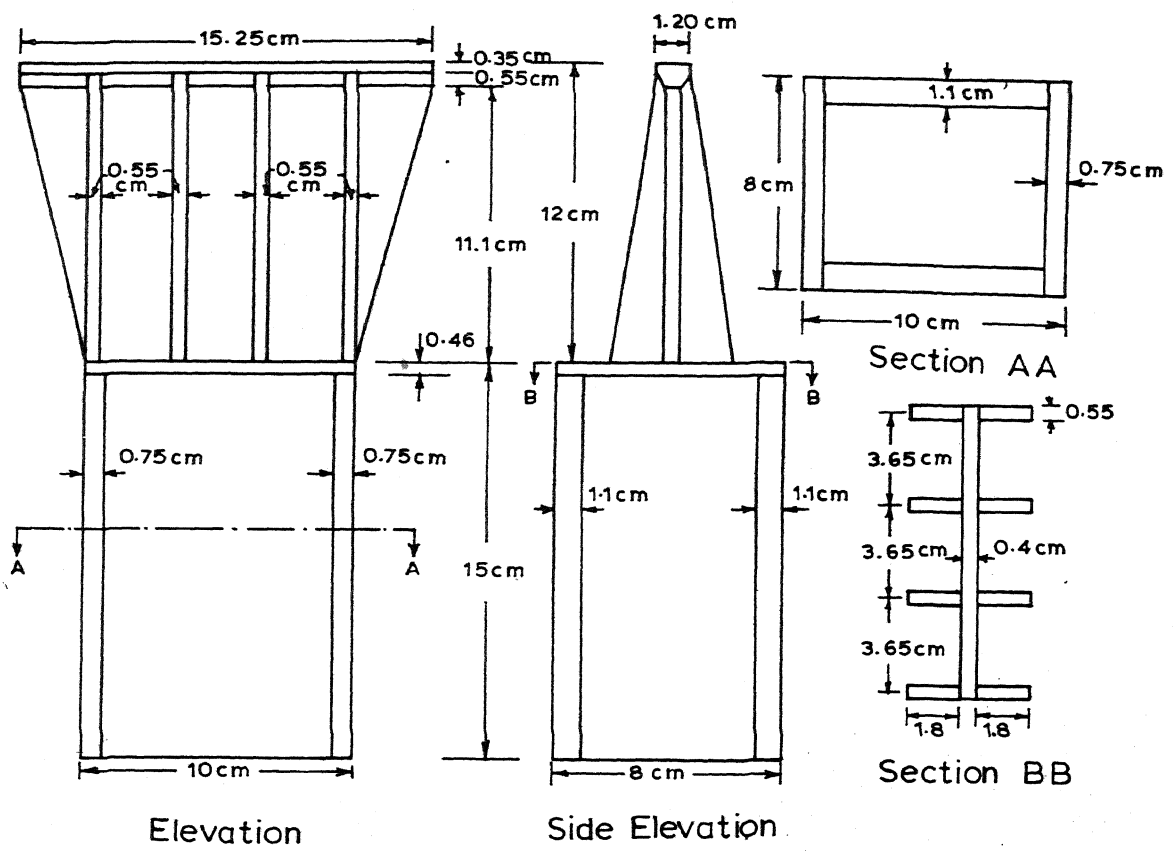


FIG.3 \_DETAILS OF PIER 1

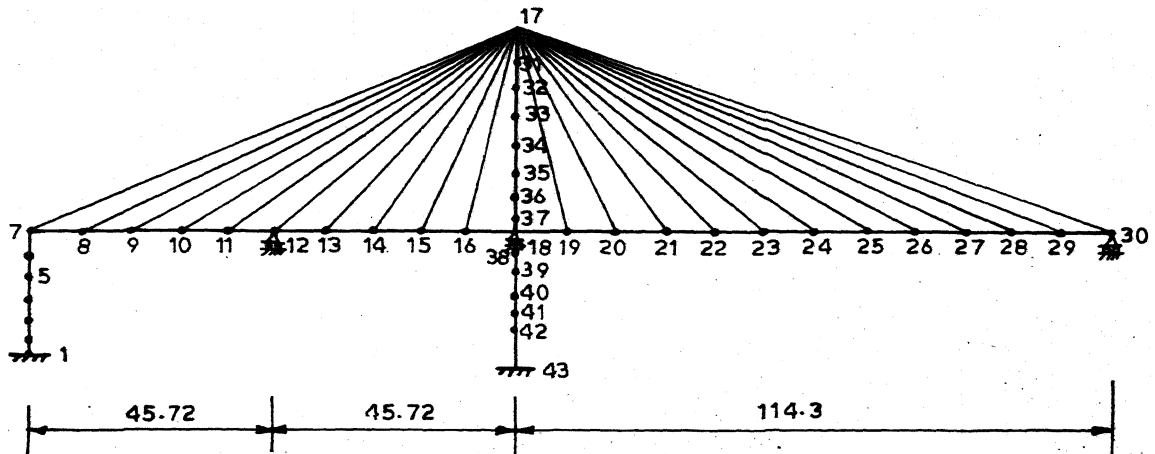


FIG. 4 \_ MATHEMATICAL MODEL FOR LONGITUDINAL VIBRATIONS