PRINCIPLES OF PHYSICAL MODELLING OF STRUCTURES RESISTANT TO EARTHQUAKES

by

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SYNOPSIS

Presented are basic principles and an algorithm of physical modelling theory for the linear boundary value problems. Obtained are the necessary and sufficient conditions of modelling. These conditions are generalization of the known conditions similarity. Given are examples of the application of the theory for solving structural seismic stability problems by model tests. The modelling conditions which allow to simplify methods and technique of modelling of seismic impacts, material properties and geometry of structure are described.

INTRODUCTION

The substitution of the prototype tests by investigations into its physical model is founded on the modelling conditions usually obtained by the equation or dimensional analysis. The starting point of the similarity theory is the approximation of deformation processes of a structure and its model by mathematical models /1, 2/. Similarity is the simplest form of a linear correspondence between a prototype "p" and a model "m", at which modelling scales are constants

$$m_f = \frac{f^p}{f^m} = \text{const.} \qquad (1)$$

The dimensional analysis is still a more simplified correspondence with modelling scales of dimensionless values equal to unity. Transformation of type (1) limits the scope of a model test as the similarity conditions require the identity of the idealizations (mathematical models) "p" and "m". For instance, the similarity of a seismic stress-strain states of a structure and its model necessitate the observance of the following conditions:

- similarity of laws of variation of the external effects "p" and "m" with time, which requires to test a model for an impact similar to that of an accelerogram;
- similarity of the equations of state for the materials of "p" and "m", which requires in practice to restrict the idealization to that of a linear-elastic body;
- geometric similarity of a structure and its model, which presents serious difficulties for model testing of thin-walled structures, etc.

To simplify the requirements for the models and to widen the scope of modelling problems developed is a linear theory of physical modelling based on linear operator transformations. An algorithm of this theory and its several applications for modelling structural seismic stability problems are presented below.

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1. LINEAR THEORY OF PHYSICAL MODELLING

Let us consider a linear boundary problem "p" in an operator form:

$$
\begin{aligned}
& a_i^{jp} L_i^{jp} y_j^p = f_i^p \quad \text{in } V^p \times T^p , \\
& b_k^{jp} K_k^{jp} y_j^p = g_k^p \quad \text{on } \Sigma^p \times T^p , \\
& c_s^{jp} N_s^{jp} y_j^p = q_s^p \quad \text{in } V^p + \Sigma^p ,\ t^p = 0 ,
\end{aligned} \tag{2}
$$

$$(i, j = 1,2,\ldots, n;\ k = 1,2,\ldots, m;\ s = 1,2,\ldots, r),$$

where a, b, c, f, g, q are the preassigned functions of coordinates and time; y - unknown functions, L - operators of a set of equations, K - boundary condition operators, N - initial condition operators, V - a volume with a surface Σ , T - one-dimensional time space, summed over j .

Linear operator transformations of the independant variables and functions are introduced by the following relations:

$$F^p = m_F F^m \tag{3}$$

and the linear transformations of the operators:

$$L^p = m_L L^m ,\ K^p = m_K K^m ,\quad N^p = m_N N^m \tag{4}$$

where m - linear operators.

Substituting (4) and transformations (3) formulated for the preassigned and for unknown functions into (2), assuming all the operators to be commuting and taking into account that the transformations of the independent variables are centred affinity, obtain the following:

$$
\begin{aligned}
& m_{a_i^j} m_{L_i^j} m_{y_j} a_i^{jm} L_i^{jm} y_j^m = m_{f_i} f_i^m , \\
& m_{b_k^j} m_{K_k^j} m_{y_j} b_k^{jm} K_k^{jm} y_j^m = m_{g_k} g_k^m , \\
& m_{c_s^j} m_{N_s^j} m_{y_j} c_s^{jm} N_s^{jm} y_j^m = m_{q_s} q_s^m
\end{aligned} \tag{5}
$$

(summation over j).

To make (5) describe the boundary problem "m", it is necessary and sufficient that the scales-operators obey the conditions:

$$
\begin{aligned}
& m_{a_i^1} m_{L_i^1} m_{y_1} m_{a_i^n}^{-1} m_{L_i^n}^{-1} m_{y_n}^{-1} = I , \\
& \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
& m_{q_s} m_{c_s^n}^{-1} m_{N_s^n}^{-1} m_{y_n}^{-1} = I ,
\end{aligned} \tag{6}
$$

where I is a unit operator, m^{-1} are inverse operators. By analogy to the similarity conditions, the equations (6) establishing the ties between scales are named as the linear modelling conditions and can be written in a criterion form. As a number of scale-operators N always surpasses a number of the equations N^* , $N - N^*$ of scales are specified on the basis of investigation method employed, test conditions, available testing equipment, etc. If scale-operators are constant and $m_{f_i} \equiv m_{f_j}$ (for any i, j) set (6) transforms into the usual similarity conditions. Substantiation of the linear modelling theory in terms of the functional analysis and group theory can be found in /3/.

2. LINEAR MODELLING OF SEISMIC STABILITY PROBLEMS

Modelling of seismic impacts /4, 5/. The idealization of the deformation process of a structure under seismic impacts is made with the following assumptions: the material is supposed uniform, isotropoic and ideally linear-elastic; volumetric forces are ignored, over the interface of a structure with an ideally rigid foundation Σ_1 a displacement vector $u_{oi}(t)$ is given, the structural surface $\Sigma_2 = \Sigma - \Sigma_1$ is free, the initial conditions are zero. A corresponding boundary value problem in terms of the images of Fourier is written down as follows:

$$\begin{gathered} \bar{\sigma}_{ij,j} = -\rho\omega^2 \bar{u}_i , \quad 2\bar{\varepsilon}_{ij} = \bar{u}_{i,j} + \bar{u}_{j,i} , \\ E\bar{\varepsilon}_{ij} = \bar{\sigma}_{ij} + \nu(\bar{\sigma}_{ij} - \bar{\sigma}_{kk}\delta_{ij}), \\ \bar{u}_i \big|_{\Sigma_1} = \bar{u}_{oi} , \quad \bar{\sigma}_{ij} n_j \big|_{\Sigma_2} = 0 , \quad (i,j,k = 1,2,3), \end{gathered} \tag{7}$$

where δ_{ij} - Kroneker symbol, the summation is made over the repeating indices, the rest of the notation is that generally adopted for the theory of elasticity. In (7) the given and the unknown functions are the complex functions of parameter ω , therefore the scales of the transformation images are introduced in the following form:

$$\begin{gathered} \bar{m}_{\bar{f}} = \frac{\bar{f}^p}{\bar{f}^m} = m_f \exp(i\varphi_{\bar{f}}) , \\ m_f = \frac{f^p}{f^m} , \quad \varphi_{\bar{f}} = \arg \bar{f}^p - \arg f^m , \end{gathered} \tag{8}$$

where m_f - scale-operator of the moduli, $\varphi_{\bar{f}}$ - scale-operator of the arguments. Note that the argument transformations form the Abelian additive group with unity $\varphi = 0$ and the properties identical to multiplicative groups.

Applying an algorithm described in p. 1 to (7), obtained is a system of modelling conditions comprising the conditions for the modulus scales identical in form to the similarity conditions:

$$\begin{gathered} m_\ell^2 m_\omega^2 m_\rho m_E^{-1} = 1 , \quad \tilde{m}_\varepsilon m_\ell \tilde{m}_u^{-1} = 1 , \\ m_\nu = 1 , \quad m_E \tilde{m}_\varepsilon \tilde{m}_\sigma^{-1} = 1 , \quad \tilde{m}_u \tilde{m}_{u_0}^{-1} = 1 , \end{gathered} \tag{9}$$

and conditions for argument scales:

$$\tilde{\varphi}_{\bar{\sigma}} - \tilde{\varphi}_{\bar{u}} = 0 , \quad \tilde{\varphi}_{\bar{\varepsilon}} - \tilde{\varphi}_{\bar{u}} = 0 , \quad \tilde{\varphi}_{\bar{u}} - \tilde{\varphi}_{\bar{u}_0} = 0 , \tag{10}$$

where scale-functions of ω are marked by $\sim$, other scales are constants. Relations (9, 10) do not impose restrictions on the choice of scales-functions $\tilde{m}_{u_0}, \tilde{\varphi}_{\bar{u}_0}$ and scale-constants m_ℓ, m_ρ, m_E . Hence, model may be tested under "any" easily simulated dynamic impact. Responses of the prototype under seismic impacts described by a seismogram or a accelerogram are calculated from model reactions by a linear modelling algorithm. For instance, stress tensors of the prototype and the model are related by

$$\sigma_{ij}^{p}(x^{p},t)=\widetilde{m}_{\sigma}\,\sigma_{ij}^{m}(x^{m},t),$$

in which the scale-operator is of the form

$$\widetilde{m}_{\sigma}=m_{E}\,m_{\ell}^{-1}\,\Phi^{-1}\,\widetilde{m}_{u_0}\exp(i\,\widetilde{\varphi}_{\bar{u}_0})\,\Phi,$$

where Φ, Φ^{-1} are direct and inverse Fourier transformation symbols.

Modelling of linear visco-elastic material properties /6/. Building materials possess dissipative properties which can be approximated by models of visco-elastic bodies. The appropriate boundary value problem is represented in Laplace images:

$$\begin{aligned}
&\sigma_{ij,j}^{*}=\rho\,\bar{p}^{2}u_{i}^{*}, \quad 2\varepsilon_{ij}^{*}=u_{i;j}^{*}+u_{j,i}^{*},\\
&E^{*}\varepsilon_{ij}^{*}=\sigma_{ij}^{*}+\nu^{*}(\sigma_{ij}^{*}-\sigma_{kk}\delta_{ij}),\\
&u_{i}^{*}\big|_{\Sigma_1}=u_{0i}^{*}, \quad \sigma_{ij}^{*}n_{j}\big|_{\Sigma_2}=0,
\end{aligned} \tag{11}$$

where E^{*}, ν^{*} - complex functions of parameter $\bar{p}$, characterizing material dynamic visco-elastic properties. Inserting complex scales-functions of the image transformations and applying a linear modelling algorithm to (11), the following relationship is obtained:

$$\begin{aligned}
&m_{\ell}^{2}\,m_{\rho}\,\widetilde{m}_{\bar{p}}^{2}\,\widetilde{m}_{E}^{-1}=1, \quad \widetilde{m}_{\varepsilon}\,m_{\ell}\,\widetilde{m}_{u}^{-1}=1, \quad \widetilde{m}_{\nu}=1, \quad \widetilde{m}_{E}\,\widetilde{m}_{\varepsilon}\,\widetilde{m}_{\sigma}^{-1}=1,\\
&\widetilde{m}_{u}\,\widetilde{m}_{u_0}^{-1}=1, \quad \widetilde{\varphi}_{E^*}-2\,\widetilde{\varphi}_{\bar{p}}=0, \quad \widetilde{\varphi}_{\varepsilon^*}-\widetilde{\varphi}_{u^*}=0, \quad \widetilde{\varphi}_{\nu^*}=0,\\
&\widetilde{\varphi}_{E^*}+\widetilde{\varphi}_{\varepsilon^*}-\widetilde{\varphi}_{\sigma^*}=0, \quad \widetilde{\varphi}_{u^*}-\widetilde{\varphi}_{u_0^*}=0.
\end{aligned} \tag{12}$$

Here $N-N^{*}=6$, the following scales may be assigned: the scales-constants m_{ℓ}, m_{ρ}, scales-functions $\widetilde{m}_{E}$, $\widetilde{\varphi}_{E^*}$, characterizing the longitudinal properties of materials "p" and "m", scales-functions $\widetilde{m}_{u_0}$, $\widetilde{\varphi}_{u_0^*}$, characterizing spectral composition of the impacts "p" and "m". Modelling conditions (12) permit to perform recalculations of the response under "any" dynamic linear longitudinal material properties of "p" and "m". In particular, one of the materials can be ideally linear-elastic or linear-elastic with a frequency independent decay decrement, both materially may be linear-elastic with different decrements and so on.

Model investigations of soil hydraulic structure seismic stability admit material property idealization by a model of an non-compressive linear visco-elastic body. In this case conditions (12) allow to conduct a test on both elastic and visco-elastic (polymer) models.

Modelling of thin-walled structures /7, 8/. As is well known, in thin-walled elements stresses and strains vary linearly with thickness, therefore the equilibrium equations or the equations of motion of the following shape are formed for a middle surface

$$f(\delta)\,D(\Phi,w)=q, \tag{13}$$

where $f(\delta)$ - thickness function, Φ-stress function, w - flexure function, D - linear or non-linear differential operator. The structure of the equation (13) allows to select a thickness scale different from a dimension scale in plan, which means that in the bounds of the condition of wall thickness, the affine modelling of the geometry is possible. The latter is of substantial value in the cases when the manufacture of a geometrically similar model is technologically impossible.

CONCLUSIONS

When compared to similarity, linear modelling theory permits to widen the range of structural seismic stability problems solved by physical model tests. The complication of the algorithm for rescaling data from a model to a prototype is no barrier if computers are made use of. The application of a linear theory for modelling quasi-static problems is presented in /9/ and some other publications of the same author.

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