DYNAMIC PROPERTIES OF A SUSPENSION BRIDGE

by

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SYNOPSIS

The measurement program is described for determining the natural frequencies, mode shapes, and damping values of the Lions' Gate Suspension Bridge at Vancouver, British Columbia, Canada. The measured natural frequencies are compared with the calculated frequencies using a continuum model and a lumped parameter model.

INTRODUCTION

The Lions' Gate Suspension Bridge at Vancouver, Canada, which opened in 1938, carries three lanes for vehicular traffic and two sidewalks. Because the bridge deck was in need of repair and wider traffic lanes were required, the deck structure is to be rebuilt. The sidewalks will be moved to the outside of the stiffening trusses, and a new orthotropic bridge deck will replace the present steel grid and concrete supported on steel beams and girders. As changes in geometric and structural properties could influence the behaviour of the modified bridge under wind loading, an aerodynamic investigation is being undertaken which includes wind tunnel section tests and a full aeroelastic model. Measurements on the existing structure were made to obtain guidance in establishing the dynamic parameters for the model tests and the design calculations.

DESCRIPTION OF THE BRIDGE

The bridge consists of a main span 473 m long and two side spans, each 187 m. The tower height is 110 m from top of concrete footing to the saddle points; the road deck passes at mid-height. An elevation is shown in Fig. 1, and a typical cross-section in Fig. 2. At the bridge centre and at the north and south ends of the suspension bridge, traction rods connect the cable to the top chord of the stiffening truss. The two stiffening trusses are joined at the lower chords by floor beams that support the road deck and by horizontal cross bracing that decreases in stiffness toward the centre of the bridge. The trusses rest on fixed bearings at the N and S supports of the bridge and have sliding bearings at the towers. The girder has a vertical curve with a rise of 6.8 m from the tower to the centre line.

MEASUREMENT PROGRAM

The motions of the bridge were monitored with battery-powered servo-drive accelerometers of 5 V/g sensitivity placed on the top flange of the cross beam close to the inside of the stiffening truss at the numbered locations shown in Fig. 1. After d-c compensation the signals were passed through 1-Hz low-pass filters and 20-dB amplifiers and recorded on a 7-channel FM tape recorder. Station 1N was used as a reference; the remaining stations were covered by successively moving the transducers to those stations. A special run was made with transducers placed at Stations 2N and 2S.

Ambient Vibrations. - Ambient vibrations were recorded of vehicular traffic only, mainly passenger cars and buses; wind speeds ranged from zero

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to about 8 km/h. Measurements were also made under various wind conditions during early morning hours when traffic was minimal or absent. One limited set of data was obtained at winds gusting to 80 km/h. For each setup the motions were monitored for approximately 45 min. The data were analyzed on a 500-line real-time spectrum analyzer and with 2 and 4 spectrum averages. Phase relations between two stations were obtained by addition and subtraction of signals. Amplitude and phase relations were also confirmed by cross-spectral density calculations using 512 points and four spectrum averages.

Vehicle Impacts. - For torsional and lateral excitation, simulated impacts were applied to the bridge from a truck driven in the outside lane at about 15 km/h, swerved normal to the bridge and brought to a sudden braking stop. Another technique utilized a small car driven normal to the roadway and brought to a sudden braking stop. Vertical impact responses were induced by slowly driving the rear wheels of the truck off a 15-cm ramp; a more successful method, however, was to drive the truck from the main span onto the side span at approximately 65 km/h. The impact response signals were digitized and analyzed on a digital computer using FFT routines. Time decay curves for various modes of vibration were obtained by inverting the filtered spectrum for a particular modal frequency band into the time domain. From this decay, modal damping was computed using the log decrement relationship. A sample record for torsional vibration decay is shown in Fig. 3.

CALCULATIONS OF MODAL PROPERTIES

Continuum model. - Selberg's theory for vibration of suspension bridges (1) was used in calculating the mode shapes and frequencies for the existing bridge. A general computer program was written to solve the series solution of the differential equation by a trial and error method utilizing the first 5 terms for the side spans and 7 terms for the main span. The torsional modal properties were obtained in the same way by replacing the vertical stiffness and mass with the torsional stiffness and mass moment of inertia of the girder. The full torsional stiffness of the roadway was assumed to be activated and the traction rods were assumed to be unstressed.

Lumped mass, linear stiffness model. - The stiffness matrix for the discrete model of some 750 members and 400 joints having over 1000 degrees of freedom was assembled with a space-frame analysis program. For most calculations symmetry properties along the length and the width of the bridge were utilized so that only one quarter of the structure needed to be encompassed. A limited set of calculations was also carried out for the full length of the bridge. The torsional rigidity used for the bridge girder includes full participation of the roadway and stringers and the assumption that the expansion joints have seized. Girder bearings at the towers are taken as fixed or free; traction rods are assumed to be ineffective.

RESULTS

The normal modes identified in this program have been categorized into vertical, horizontal and torsional modes. The usual clear distinction between horizontal and torsional modes becomes obscured, however, because both torsion and horizontal motion are present in most of these modes. For the torsional modes, the horizontal components shown by broken lines in Tables 2 and 3 give the modal amplitude on the top flange of the floor beam, i.e., just below the roadway level. The plotted torsional component shown by solid lines represents the sum of the vertical motions at the inside of the stiffening truss.

The results of the measurements and calculations are summarized in Table 1 for vertical modes, and in Tables 2 and 3 for identified anti-symmetric and symmetric torsional modes.* The damping values from ambient vibrations were determined using the method of half-power bandwidth. Damping calculated from the filtered decay curves varies slightly over the length of the decaying vibration record and consequently the values near the beginning of the impact and those after about 15 cycles are given. Where necessary, a correction was applied to the decaying signal by subtracting the amplitudes of the ambient signal due to wind excitation.

DISCUSSION

Identification of torsional modes proved to be difficult in some instances. For example, multiple frequencies with substantially the same mode shapes were found near 0.32, 0.38, 0.51 and 0.94 Hz. Possible reasons for these might be: (a) the restraints change as the tie rods begin to tighten, (on examination they were found to be rather loose); and (b) the bearings slip owing to changing loading conditions on the bridge. From an examination of various Fourier and cross-spectral calculations, it is suspected that another symmetrical torsional mode exists very close to the symmetric mode at 0.465 Hz and another one near the asymmetric mode at 0.507 Hz. It is thought that the plotted mode at 0.465 Hz is a composite of two modes, a dominant side span mode and a dominant main span mode.

As the results in Table 1 indicate, there is generally excellent agreement between the measured and calculated frequencies for the vertical modes as well as for the lowest horizontal mode for both the continuum model and the lumped parameter model. However, for the torsional modes presented in Tables 2 and 3, the calculated frequencies for the continuum model show substantial differences from measured ones. Reasons for this may be the difficulty in achieving realistic estimates for torsional stiffness of the girder and the assumptions of constant properties in each span.

The damping values from the ambient vibrations are those present at low amplitudes of vibration. There may also be some imprecision in the damping calculation because of changing frequencies with varying constraints on the bridge. These aspects are discussed in more detail in Ref. 2.

ACKNOWLEDGMENT

The Lions' Gate Bridge is owned by the Department of Highways of British Columbia. The cooperation of the Department in the measurement program is gratefully acknowledged, as is their permission to publish the paper. Structural consultants for the bridge modifications are Buckland and Taylor Ltd., of Vancouver, B.C. The results using the lumped mass, linear stiffness model were obtained by Hooley Engineering, Vancouver, B.C., Canada.

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* For the lumped parameter calculations, results for free and fixed bearings are presented, designated in the tables by a) and b), respectively.

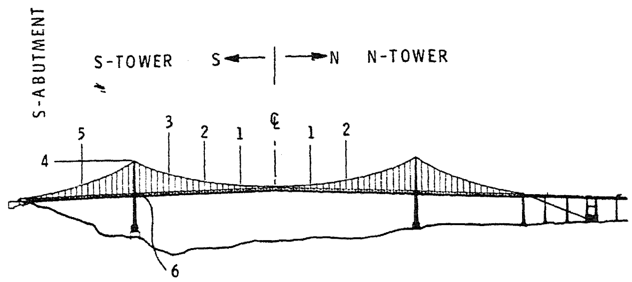


FIGURE 1

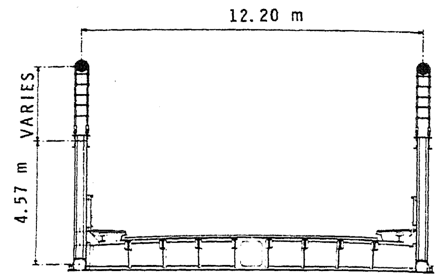
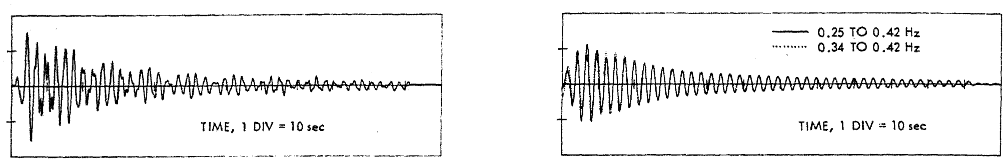


FIGURE 2

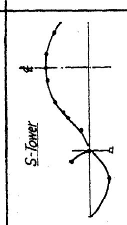
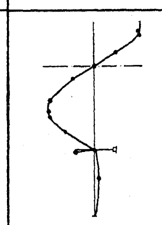
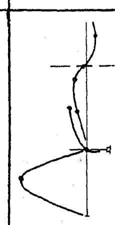
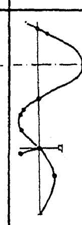
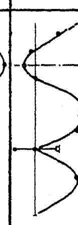
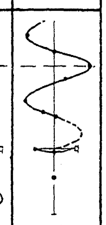


(A) RESPONSE FOR LESS THAN 1 Hz

(B) FILTERED RESPONSE

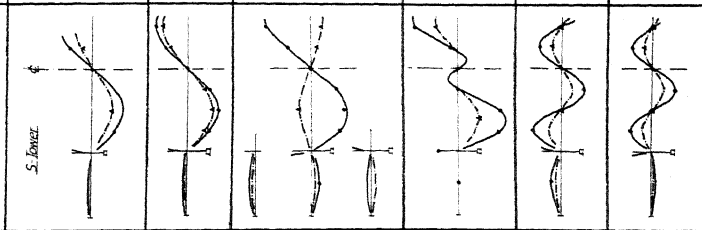
FIGURE 3 TORSIONAL RESPONSE TO TRUCK IMPACT, CENTRE SPAN

Table I: Dynamic Properties of Vertical Modes

Item No.		1	2	3	4	5	6	7	8
Measured Mode Shape									
Measured Frequency, Hz		0.200	0.225	0.30	0.358	0.420	0.590	0.760	0.845
Calculated Frequency Hz	Lumped Parameter a.) b.)	0.198 0.210	0.229 0.216	0.289 0.282	0.344 0.348	0.417 0.424		0.700 0.702	
	Continuum Model	0.185	0.214	0.273	0.345	0.408			
Damping, % critical, from	Fourier Spectra	1.7	1.6	1.7	1.0	0.7	0.6		0.6
	Impulse Decay	0.9*,0.6* 1.1**,0.9**	0.9*,0.8* 1.05**,0.95**						

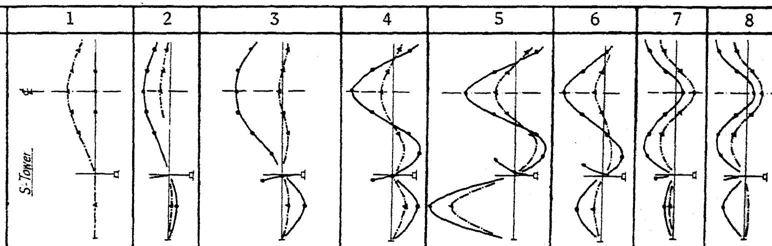
* Uncorrected ** Corrected for ambient vibrations a.) bearings free b.) bearings fixed

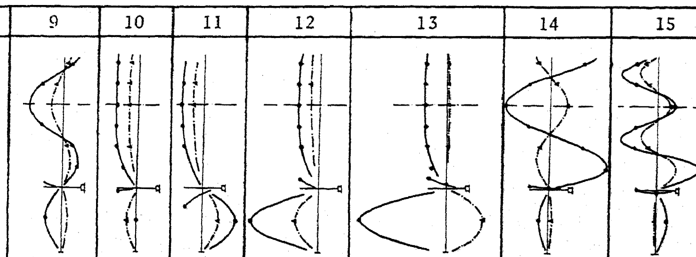
Table 2: Dynamic Properties of Anti-symmetric Torsional Modes

Item No.			1	2	3	4	5	6
Measured Mode Shape ——•—— Difference of Vertical Motion (East + up) —·—▲—·— Horizontal (+ to West)								
Measured Frequency, Hz			0.310	0.32 (0.33)	0.495 0.515 0.507	0.600	0.745	0.765
Calculated Frequency, Hz	Lumped Parameter	a.) b.)		0.311 0.316	0.465 0.468	0.568		
	Continuum Model			0.32 H * 0.411	0.553			
Damping from Fourier Spectra, % critical				1.25	0.79			

* Calculated horizontal mode a.) bearings free b.) bearings fixed

Table 3: Dynamic Properties of Symmetric Torsional and Horizontal Modes

Item No.			1	2	3	4	5	6	7	8
Measured Mode Shape ——•—— Difference of Vertical Motion (East + up) —·—▲—·— Horizontal (+ to West)										
Measured Frequency, Hz			0.12	0.36	0.38 (0.37, 0.39)	0.44	0.465	0.475	0.525	0.535
Calculated Frequency, Hz	Lumped Parameter	a.) b.)	0.118 0.125		0.355 0.364	0.465†	0.441*	0.467 0.477	0.520 0.533	
	Continuum Model		0.12 H**		0.347			0.593	0.646	
Damping, % crit., from	Fourier Spectra		3.3	1.7	0.9	0.34		0.63	0.57	
	Impulse Decay		3.0 and 3.9		1.3, 1.6					

Item No.		9	10	11	12	13	14	15
Measured Mode Shape								
Measured Frequency, Hz		0.555	0.615	0.635	0.65	0.715	0.94	1.00
Calculated Frequency, Lumped Parameter, Hz,	b.)		0.575		0.618	0.654		

* Mainly side span motion; ** Computed horizontal mode; a.) bearings free; b.)bearings fixed;
† Calculated from modeling whole bridge

DISCUSSION

V.K. Gupta (India)

The authors have presented a good piece of measurement programme for dynamic properties of Lion's Gate Suspension Bridge at Van Couver, Canada. The writer would be interested to know how the geometric and dynamic parameter simulation is done between the prototype and the model for wind tunnel testing ? What are the approximations used for doing so.

The authors have presented the systematic procedure for measurement of frequency on the bridge under various vehicular conditions on bridge, for different wind velocities. It would be of interest to know in more detail about the instrumentation which was used for measuring the frequency of bridge. From table 1, it can be seen that for most of the modes, the measured frequency is higher to that of frequency calculated by lumped mass model (free bearing) and continuum model. However, in second mode it is not so. Is there any special reason for this ? Moreover, in mode four the frequency by lumped mass model is less than frequency calculated by continuum model whereas for other modes it is higher. Is it due to special characteristics of fourth mode or for some other reasons?

It will be interesting to know the magnitude and type of damping adopted for the calculation of frequencies and mode shapes.

S.P. Gupta (India)

How was the torsional vibration generated and recorded and what is the effect of torsional vibration on the lateral vibration and vertical vibration of the bridge ?

If the torsional vibration is ignored what could be the effect.

How the torsional vibration was developed and recorded.

Author's Closure

As the paper is concerned only with the results of a full-scale dynamic measurement program and the agreement with calculations, aspects of wind tunnel model testing cannot be discussed here. The aerodynamic model investigations for the Lions' Gate Bridge modifications were carried out by the National Aeronautical Establishment (NAE) of the National Research Council of Canada (NRC), Ottawa. The experiments and results derived therefrom are described in References 1, 2 and 3.

In addition to the description of the measurement program given in the paper, the following elaboration may be of interest.

The random nature of traffic and wind forces acting on the structure produces vibrations predominantly in the natural modes. The amplitudes of the Fourier spectrum peaks of the vibration signals at simultaneous measuring stations give the relative modal amplitudes. This method, sometimes called the ambient vibration survey, has been widely used to determine the dynamic properties of structures (see for example, Refs.4 and 5). Without going into considerable detail, it is not possible to elaborate on the instrumentation beyond the general description already given in the paper. There are many manufacturers who sell suitable components; some provide complete packages to perform such a vibration survey.

The fact that some calculated frequencies agree with the measurements better than others may be due to the fact that the mathematical modelling of mass and stiffness distribution is more realistic for some mode shapes than for others. In an extreme case, the side-span mode (Item 7 in Table 1) would depend overwhelmingly on the modelling of the properties of the side-span, whereas this becomes less important for modes where the main span dominates the dynamic behaviour of the bridge. Similarly, for item 2, table 1, the influence of both inertia and strain energy terms near the centre of the bridge is almost negligible for that mode, whereas for Item 1 these are dominant. Thus, discrepancies in mathematical modelling near the bridge centre would greatly affect the first mode, but not the second. In view of the good over-all agreement between measured and calculated results, however, such additional refinements in mathematical modelling were not carried out.

All frequencies and mode shapes were calculated for zero damping. Because of the low damping values found in the structure, differences between damped and undamped results are negligibly small.

Since the centre of mass and the shear centre for the bridge cross-section do not coincide, the torsional vibrations are coupled with the horizontal vibrations. Thus, in general, lateral translation is accompanied by rotation and vice versa. On the other hand, the vertical modes are completely uncoupled from the horizontal or rotational motions because of symmetry of mass and stiffness in the vertical direction.

The random nature of loads from wind and traffic also excites the torsional modes, just as for the vertical modes. Torsional motion of the bridge was monitored by placing two vertical transducers opposite each other at both stiffening trusses of the bridge. Spectrum amplitudes from a Fourier analysis of the subtraction of the signals (and dividing by the distance between the transducers) gives the rotational modal amplitudes. Addition of the signals gives twice the vertical modal amplitudes, but no rotational components are present because of the uncoupling of the torsional and vertical modes. For a complete modal description, a horizontal transducer was also placed with every pair of vertical ones.

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