VARIABILITY IN ENGINEERING ASPECTS OF STRUCTURAL MODELING

by

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SYNOPSIS

The state-of-the-art in modeling of structures has not kept pace with the advances made in computational techniques. The modeling of structures is still an exacting art. The engineering judgment that goes into the modeling process is of extreme importance as regards to the outcome of the calculations. This human aspect of structural modeling is explored by two separate experiments involving an idealized structure and a seven story steel frame structure. The results of these experiments are statistically evaluated and a probabilistic model is suggested to deal with the effect that engineering judgment has on frequency calculations.

INTRODUCTION

The need for the proper prediction of the frequencies and mode shapes of nuclear power plant structures is due to the fact that significant response amplification occurs at the structure frequencies (1). Thus, the safe and economical design of safety-related equipment and systems requires either experimental confirmation of theoretical values, or the incorporation of an acceptable probability variation of the calculated structural frequencies.

There are three main sources of uncertainty in the prediction of structural frequencies. The first is the problem of predicting the "as-built" material properties of the structure-soil system. This problem has been studied in sufficient detail (2). The second source of uncertainty is due to the choice of the appropriate differential equation to describe the response phenomenon. The commonly used linear modeling method is adequate to describe the response of a structure only for small displacements, whereas earthquakes produce large displacements. The third source of uncertainty in the calculation of structural frequencies results from the subjectivity of the process by which an engineer translates a set of drawings into a set of matrices describing the structure. This paper is an investigation into the nature of this uncertainty.

Although the impetus of the earlier work (2) and the present effort at defining variabilities of structural frequencies comes from a designer's concern for predicting a realistic response of equipment in nuclear power plants, the earlier and present work could well be used in the interpretation of the results of dynamic testing of structures.

DESCRIPTION OF THE EXPERIMENTS

Two experiments were conducted with two different groups of students who had just completed a course in earthquake engineering. Because of the lack of any significant experience in both groups relating to mathematical modeling of structures, the results of this study are not typical of experienced engineers, who are expected to narrow, but not to eliminate,

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the range of variabilities reported herein. Linear lumped parameter modeling was considered appropriate for these structures.

The first experiment involved obtaining the eigenparameters of the idealized structure shown in Figure 1. The structure was simplified on purpose so that inferences can be made when compared to the second experiment. All pertinent design data are given in Figure 1. Being a shear wall box type structure, shear deformations, proper shear area coefficients and shear lag effects were incorporated into the analysis (3). Soil-structure interaction effects were also considered. The site was assumed to be a deep alluvial deposit with a mean shear wave velocity of 2000 ft/sec. and a Poisson's ratio of 0.40.

The second experiment involved a seven-story structure occupied by the group (Figure 2). As-built drawings were provided the group together with all pertinent material characteristics. Space does not allow for a detailed description of the structure. It could be characterized though as a modern seven story steel rigid frame structure with two significant features: a) the exterior columns are encased at the first floor level in concrete columns that in turn rest on the basement walls; and b) the interior vertical load carrying system is capable of resisting a limited lateral load in the N-S direction as a result of an uncommon column-beam joint detail. The effects of this detail will be discussed subsequently. Soil-structure interaction effects were not considered. After the eigenparameters were calculated, the structure was dynamically tested (4). This experiment, unlike the first one, affords the opportunity of determining the source of uncertainties by a comparison of the calculated structure properties with those obtained by a structure parameter identification based on the results of the dynamic tests (5).

RESULTS

Tables 1a and 1b display the pertinent results of the first experiment. Although a statistical review of these data would be useful, a more meaningful endeavor would be to generalize these results in the form of a probability density function such that general inferences could be made on future frequency calculations. The intent therefore is the development of tractable smooth mathematical functions such that subsequent dealings with this variable will be simplified.

The usual procedure in such situations is to select a model that best represents the randomness of the data. However a more direct approach is suggested by the present data. The cumulative probabilities (m/N+1) are plotted in Figure 3a. The study of this plot in conjunction with similar plots (Figures 4 and 5) suggests that the only reasonable fit (in the middle portion of the cumulative density function - CDF) is a straight line. Thus, a least squares fit is made to the data points. This straight line translates into the horizontal segment of the probability density function (PDF) plotted in Figure 3b. The inference from this segment of the PDF is simply that engineers will predict the frequency of a structure, within a given range, with equal likehood. The probability outside that range is expected to diminish and, as a first approximation, the sloping segments of the PDF as shown in Figure 3b seem to be most appropriate. Thus a trapezoidal PDF is deemed to be a meaningful function to describe the judgment engineers apply in translating a set of drawings to mathematical models. The sloping tail ends of the PDF translate into parabolas on the CDF plot. These parabolas are then used to determine, by eye, the cut-off frequencies in the PDF plot. Standard deviation, coefficient of variation and the ten percentile value are also shown in Figure 3b. These parameters refer to the idealized curves.

Table 2 summarizes theoretical and experimental results obtained during the seven-story building research project. The range of experimental values results from the nonlinearities of the structure. All groups except two used lumped mass models, uncoupled in the N-S (transverse) and E-W (longitudinal) directions. One group used the finite element method. The floors were modeled as rigid diaphragms and soil-structure interaction effects were not considered. The same procedure as for the first experiment is used to cast the results in a more tractable form. Again, the CDF of the raw data suggests a straight line and a least square fit is adopted. Both the CDF and the PDF for the fundamental modes in the N-S and E-W directions are shown in Figures 4 and 5.

All three PDF's were arbitrarily taken as being symmetric. Before any non-symmetric model can be adopted more data must be generated and studied.

DISCUSSION OF RESULTS

Although the higher modes can be similarly treated, the fundamental mode was emphasized since it is usually the important mode. It is instructive to compare the PDF of Figures 3 through 5. It is noticed that the coefficient of variation increases sharply from the first to the second experiment. This is as it should be since the structure depicted in Figure 1 is extremely simple compared to that shown in Figure 2. The much larger coefficient of variation of Figures 4 and 5 underscores the difficulty of modeling real structures. Although the increase in the coefficient of variation from the E-W to N-S frequencies is small, it could explain the more complex nature of the N-S model. As mentioned earlier, the N-S interior frames of the seven story building have a special beam-column connection detail. The stiffness calculation of that connection was a subject of discussion during the analysis phase of this project. The actual cumulative distribution plot in Figure 5a also points to this difficulty.

The modeling of the seven story structure was specifically done for "low" strain levels (so that meaningful comparisons could be made with test results), and so the difficulties associated with evaluating the stiffness contribution of exterior walls, window panes and interior partitions must be appreciated. Additionally the evaluation of the actual mass on floors would be overestimated simply because some fraction of design live loads is used. A third reason why test results and the mean of the idealized distribution differ could be due to the fact that elastic moduli used in design are lower than in-situ properties. Thus, all three considerations would tend to give a mean calculated frequency below the test results, even though this difference is not that large. The study of the "best choice" mass and stiffness characteristics identified from these test data will be the subject of a subsequent paper.

CONCLUSIONS AND RECOMMENDATIONS

Admittedly the data base on which any conclusions will be based is limited. Nevertheless, the results discussed here suggest the tentative adoption of a trapezoidal probability model where two parameters are required to define the PDF. The width of the flat top segment of the model and the coefficient of variation are selected as having some direct relationship to the complexity of the model and the expertise of the engineer. Although these two parameters are interactive, some trends can be discerned from the available data. As the complexity of the structure increases, both the width of the flat top of the PDF and the coefficient of variation increase. It is expected that as the expertise of the engineer increases both the flat top and the coefficient of variation will decrease. Based on these and similar observations and the numerical data presented, the probability distributions of Figure 6 are suggested. The mean of 10 is assumed for purposes of illustration.

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Table 1a. N-S Lateral Frequencies (Hz)-First Experiment

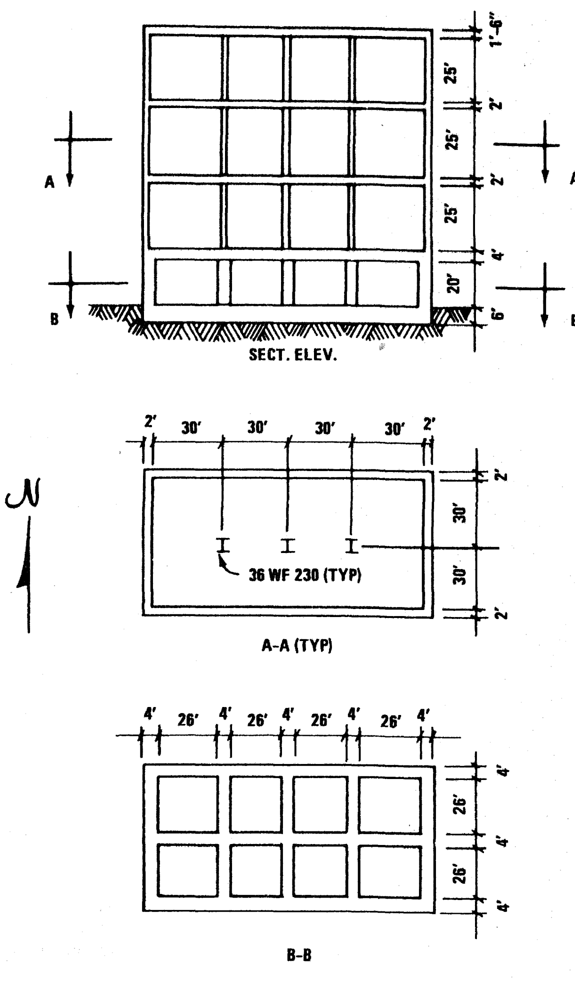
Group # / Mode #	1	2	3	4	5	Sample Average
1	4.41	4.30	4.50	4.24	4.32	4.35
2	12.38	11.96	12.37	12.41	11.97	12.22
3	19.98	19.99	21.37	19.14	20.09	20.11
4	27.96	29.26	32.08	-	29.44	29.67

Table 1b. Vertical Frequencies (Hz)-First Experiment

Group # / Mode #	1	2	3	4	5	Sample Average
1	10.12	10.18	10.39	10.37	10.18	10.25
2	30.24	30.22	30.84	30.85	30.22	30.47
3	64.61	64.51	65.87	65.89	64.51	65.08
4	88.25	88.11	-	90.00	88.12	88.62

Table 2. Frequencies (Hz) - Seven Story Building

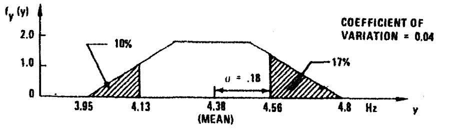
Mode #	Description	Group #									Sample Average	Experiment	
		1	2	3	4	5	6	7	8	9		Ambient Vibration	Forced Vibration
1	N-S Translation	0.99	0.49	1.03	1.11	1.36	0.67	0.64	0.63	0.69	0.85	1.03	0.92
2	E-W Translation	0.92	0.50	0.78	0.99	1.05	0.73	0.77	0.51	0.58	0.76	1.07	0.93
3	Torsion				Values not calculated							1.35	1.20
4	N-S Translation	2.51	1.46	2.70	2.95	3.55	1.83	1.83	1.80	1.80	2.27	3.20	3.10
5	E-W Translation	2.15	1.49	2.16	2.64	2.89	2.02	2.12	1.51	1.63	2.07	3.20	3.00
6	Torsion				Values not calculated							4.00	?
7	N-S Translation	3.59	2.64	4.45	4.73	5.72	3.00	3.16	2.96	2.92	3.69	5.10	4.60
8	E-W Translation	3.16	2.63	3.56	4.22	4.63	3.23	3.51	2.55	2.63	3.35	5.22	5.00
9	Torsion				Values not calculated							7.18	6.50



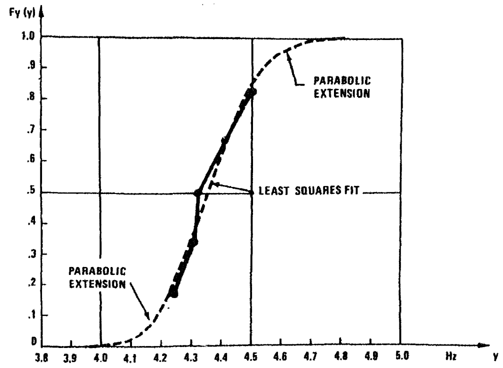
f'_c = 4000 psi
A36 steel
γ_{soil} = 125 lbs/ft^3

Figure 1. DETAILS OF THE STRUCTURE FOR THE FIRST EXPERIMENT

Figure 2. NORTHEAST CORNER VIEW OF THE SEVEN STORY STRUCTURE USED IN THE SECOND EXPERIMENT

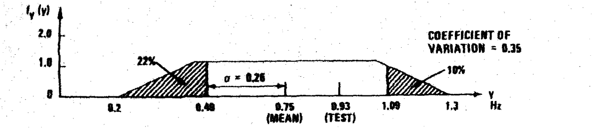


(B) PROBABILITY DENSITY FUNCTION

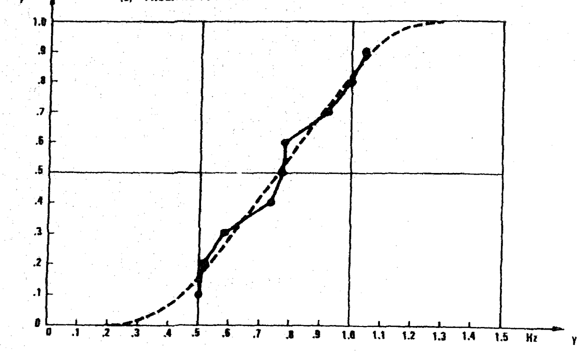


(A) CUMULATIVE DISTRIBUTION FUNCTION

Figure 3. FREQUENCY DISTRIBUTION FOR THE FUNDAMENTAL MODE DERIVED FROM THE FIRST EXPERIMENT

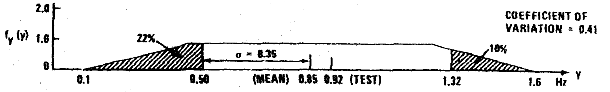


(B) PROBABILITY DENSITY FUNCTION

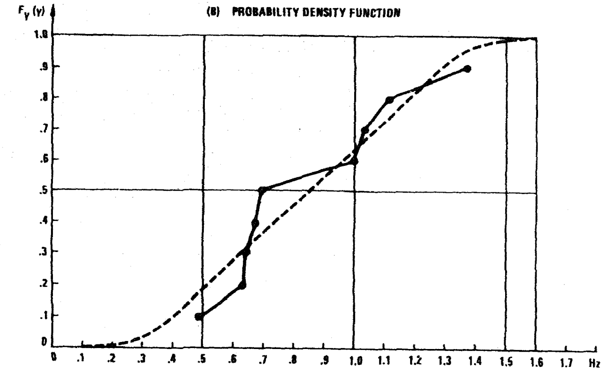


(A) CUMULATIVE DISTRIBUTION FUNCTION

Figure 4. FREQUENCY DISTRIBUTION FOR THE FUNDAMENTAL E-W MODE OF THE SEVEN STORY BUILDING

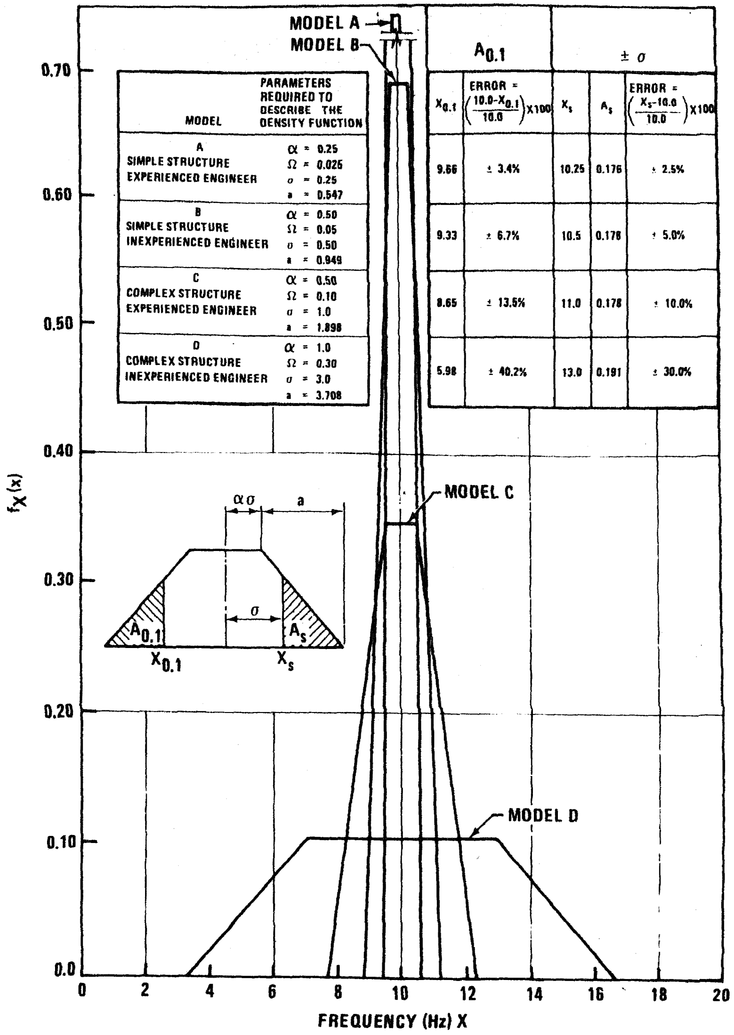


(B) PROBABILITY DENSITY FUNCTION



(A) CUMULATIVE DISTRIBUTION FUNCTION

Figure 5. FREQUENCY DISTRIBUTION FOR THE FUNDAMENTAL N-S MODE OF THE SEVEN STORY BUILDING



MODEL	PARAMETERS REQUIRED TO DESCRIBE THE DENSITY FUNCTION
A SIMPLE STRUCTURE EXPERIENCED ENGINEER	α = 0.25 Ω = 0.025 σ = 0.25 a = 0.547
B SIMPLE STRUCTURE INEXPERIENCED ENGINEER	α = 0.50 Ω = 0.05 σ = 0.50 a = 0.949
C COMPLEX STRUCTURE EXPERIENCED ENGINEER	α = 0.50 Ω = 0.10 σ = 1.0 a = 1.898
D COMPLEX STRUCTURE INEXPERIENCED ENGINEER	α = 1.0 Ω = 0.30 σ = 3.0 a = 3.708

$A_{0.1}$		$\pm\sigma$		
$X_{0.1}$	ERROR = $\left(\frac{10.0-X_{0.1}}{10.0}\right)$X100	X_s	A_s	ERROR = $\left(\frac{X_s-10.0}{10.0}\right)$X100
9.66	± 3.4%	10.25	0.176	± 2.5%
9.33	± 6.7%	10.5	0.178	± 5.0%
8.65	± 13.5%	11.0	0.178	± 10.0%
5.98	± 40.2%	13.0	0.191	± 30.0%

Figure 6. TENTATIVELY RECOMMENDED PROBABILISTIC MODEL FOR ENGINEERING JUDGMENT IN STRUCTURAL MODELING