COLLAPSE ANALYSIS OF MULTISTORY BUILDINGS

by

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INTRODUCTION

After a building is noticeably damaged in an earthquake, the earthquake engineer needs to know if the building should be repaired. A method of static incremental analysis suitable for predicting the likelihood of collapse is needed.

The objectives of this paper are to: 1) develop a method for large deflection quasi-static incremental analysis of ductile reinforced concrete frames, and 2) interpret the results of static collapse studies. The frames studied are loaded incrementally with static loads simulating earthquake-like load distributions.

ANALYSIS PROCEDURE

Quasi-static incremental displacement analysis which considers the decrease in story shear resistance after yield can be formulated either with the incremental displacements or incremental forces as the dependent variables. Examine the quasi-static equilibrium equation

$$[K_c(u)]_i\{\Delta u\}_i + [K_g(u)]_i\{\Delta u\}_i = \{\Delta f\}_i \tag{1}$$

in which $\{\Delta u\}_i$, $\{\Delta f\}_i$ = incremental displacement, force vectors; $K_c(u)$, $K_g(u)$ = conventional, geometric stiffness matrices dependent on displacement history, u; i = increment number.

Though arbitrarily choosing the incremental force, $\{\Delta f\}_i$, is mathematically sound, in actuality a meaningful collapse displacement history cannot be found this way. The choice of $\{\Delta f\}_i$ becomes exceedingly difficult when the fundamental eigenvalue of $[K_c(u) + K_g(u)]$ becomes negative, as it normally does after a yield mechanism forms in the structure. If the incremental displacements, $\{\Delta u\}_i$, are chosen, then a straightforward multiplication can be used to find the incrementally applied loads. This approach is unsuitable in the present collapse analysis study because the incremental displacement history is unknown in multidegree of freedom systems.

A systematic calculation procedure which avoids guessing $\{\Delta u\}_i$ or $\{\Delta f\}_i$ can be formulated from principles of incremental work. In the procedure the shape of the load vector $\{\Delta f\}_i$ and the incremental work are input quantities. The magnitude of the incremental load and incremental displacements are output from the analysis.

Consider the following load and displacement relationships

$$\{\Delta f\}_i = \Delta\alpha_i \ \{\Delta\bar{f}\} \tag{2}$$

and $$\{\Delta u\}_i \quad \Delta\alpha_i \ \{\Delta\bar{u}\}_i \tag{3}$$

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in which $\Delta\alpha_i$ = unknown incremental scalar multiplier; $\{\Delta\bar{f}\}$ = specified constant load shape vector; $\{\Delta\bar{u}\}_i$ = unknown scaled displacement vector. The same unknown scalar, $\Delta\alpha_i$, is used in Eqs. 2 and 3 because $\{\Delta u\}_i$ and $\{\Delta f\}_i$ are linearly related in Eq. 1.

The accumulated load is found from the relation

$$\{f\}_{i+1} = \{f\}_i + \{\Delta f\}_i \tag{4}$$

in which $\{f\}_{i+1}$, $\{f\}_i$ = accumulated loads for increments i+1 and i.

The incremental external work done by the accumulated load, $\{f\}_i$, moving through the incremental displacemtnt $\{\Delta u\}_i$ is given by

$$\Delta W_i = \{f\}_i^T \{\Delta u\}_i \tag{5}$$

in which ΔW_i = incremental external work. In the present paper ΔW_i is a specified quantity whereas in usual equilibrium calculations ΔW_i, is sometimes computed, as an afterthought, from Eq. 5.

An incremental procedure can be devised using Eqs. 1 through 5, but some supportive data must be supplied. Before the incremental procedure is carried out the quantities ΔW_i (i=1, ..., number of increments) and $\{\Delta\bar{f}\}$ must be specified. These are input data whose selection will be discussed in the Section, "Input Quantities." At the beginning of each increment the stiffness matrices $K_c(u)$ and $K_g(u)$ must be known. These matrices may be evaluated using known methods for representing uniaxial (1) and biaxial (2,3) behavior. Also known at the start of the increment is the accumulated load vector $\{f\}_i$.

Then the steps followed in each increment are:

1. Substitute Eqs. 2 and 3 into Eq. 1 to obtain

   $$\Big[[K_c(u)]_i + [K_g(u)]_i\Big]\{\Delta\bar{u}\}_i = \{\Delta\bar{f}\}. \tag{6}$$

   Solve for $\{\Delta\bar{u}\}_i$ using a convenient simultaneous equation solution procedure.
2. Combine Eqs. 3 and 5 to obtain

   $$\Delta W_i = \Delta\alpha_i\{f\}_i^T\{\Delta\bar{u}\}_i \tag{7}$$

   ΔW_i is specified, $\{f\}_i$ is known from the previous increment, and $\{\Delta\bar{u}\}_i$ is known from Eq. 6. It follows that Eq. 7 may be modified to solve for the unknown scalar, $\Delta\alpha_i$, i.e.

   $$\Delta\alpha_i = \frac{\Delta W_i}{\{f\}_i^T\{\Delta\bar{u}\}_i} \tag{8}$$

3. Using Eqs. 2 and 3 the unknowns $\{\Delta f\}_i$ and $\{\Delta u\}_i$ are computed.
4. The accumulated load for the subsequent increment is found using Eq. 4. In a similar fashion the accumulated displacement can be found from

   $$\{u\}_{i+1} = \{u\}_i + \{\Delta u\}_i \tag{9}$$

   in which $\{u\}_{i+1}$, $\{u\}_i$ = accumulated displacement for increments i+1 and i.
5. The $K_c(u)$ and $K_g(u)$ are computed for the subsequent increment. Effects of joint coordinate translation on the K_c and K_u are represented in this step.

INPUT QUANTITIES

Some basic understanding of the generalized force-deflection property of the structure is required for choice of ΔW_i. For the present study on collapse of multistory buildings the total work, W, can be found and the increments of work, ΔW_i, can be selected if the distribution of earthquake loads, seismic coefficients, and maximum drifts are estimated in advance. This is not a restrictive requirement since the validity of the estimation can be tested by evaluating the yield shears and drifts obtained in the analysis. The magnitude of the ΔW_i values can be changed to study convergence for the load, seismic coefficient, and drift estimations.

Consider the generalized force-deflection relation shown in Fig. 1. The generalized force and displacement quantities are computed from the square root of the sum of force components squared. Two incremental work quantities ΔW_3 and ΔW_{30} for increments 3 and 30 are depicted as shaded bar areas. The ΔW_3 is positive for loading while the ΔW_{30} is negative for unloading or reversal. The ΔW_3 has a larger magnitude than ΔW_{30}.

Some properties of the present analysis method are demonstrated in Fig. 1. These properties are: 1) when $[\{f\}_i^T\{f\}_i]^{1/2}$ approaches zero then ΔW_i must be selected to approach zero; otherwise the resulting $[\{\Delta u\}_i^T\{\Delta u\}_i]^{1/2}$ will be a large meaningless quantity; 2) $\Delta W_i > 0$ implies $\{f\}_i^T\{\Delta u\}_i > 0$ and $\Delta W_i < 0$ implies $\{f\}_i^T\{\Delta u\}_i < 0$.

In a collapse analysis of a multistory frame the incremental work is assumed to be positive for all increments. The collapse is signified when the generalized force, $[\{f\}_i^T\{f\}_i]^{1/2}$, approaches zero. The ΔW_i is chosen to approach zero as $[\{f\}_i^T\{f\}_i]^{1/2}$ goes to zero.

APPLICATIONS

Single bay portal frame planar studies are conducted in order to understand the characteristics of the present method. The properties and coordinate idealization of the frame are shown in Fig. 2. The reinforced concrete beam and columns are idealized using the moment-curvature characteristics given in Ref. (1). Each of the elements is divided into three zones. An independent moment-curvature relation is represented in each of the zones.

The vertical loads applied initially but don't enter into the incremental work calculation. The magnitude of the vertical loads has a strong effect on the shear-drift relation (Fig. 3) of the frame. Four axial load levels related to the ideal axial short column capacity, P_o, are used in the study.

The incremental work, ΔW_i, is assumed to be linearly increasing from zero until the story shear reaches a maximum. Then it is assumed to decrease linearly with each increment. The rate of decrease of incremental work per increment is found from the relation

$$\frac{d(\Delta W_i)}{d(i)} = \left[\frac{V_i - V_{imax}}{i - imax}\right] \frac{\Delta W_{imax}}{V_{imax}} \tag{10}$$

in which $d(\Delta W_i)/d(i)$ = rate of change of incremental work per increment; imax = increment number when story shear is maximum; V_i, V_{imax} = story shear at increment i, at maximum; ΔW_{imax} = maximum incremental work value. Convergence of shear-drift relations was tested by comparing results of 60 and 120 increment studies. The incremental load shape vector $\{\Delta\bar{f}\}$ is indicated in Fig. 2. The upshot of these assumptions is that $\{\Delta f\}_i$ is positive until the maximum shear is reached. Thereafter it is negative. Meanwhile $\{f\}_i$ and $\{\Delta u\}_i$ are positive for all of the increments.

Several aspects of the shear-drift relations (Fig. 3) are interesting: 1) the initial slope; 2) the maximum value, and 3) the descending slope. The axial load 0.3Po caused maximum initial lateral stiffness and corresponded to the "balanced" axial load. The maximum shear value also occurred for the 0.3Po axial load. The descending slope magnitudes were inversely related to the axial load levels.

The collapse displacements for three axial load levels - 0.3Po, 0.5Po, and 0.8Po - were found to be 0.91 ft, 0.48 ft, and 0.23 ft respectively. The impending collapses were indicated by approach of the generalized force to zero.

In Fig. 4 the critical vertical load is plotted against drift. As with the shear drift relations four axial load levels were used, and three properties of these curves are worthy of discussion.

During the initial stages of loading the frame with column axial loads at balanced condition has the highest critical vertical load. As the lateral load is increased there is a corresponding decrease in the critical vertical load; at this time the critical loads for different vertical load levels merge into a narrow band. When the lateral displacement surpasses yield and the shear-drift relation is descending the critical vertical load is less than the applied vertical load.

Throughout the displacement history the buckling mode shape corresponds to a sway buckling mode.

<u>Multistory planar frame studies</u> are performed to evaluate the adaptability of the present method to large systems. The frame to be studied is shown in Fig. 5. For properties of materials, reinforced concrete element sizes, reinforcement detailing, and structural idealization please refer to Ref. (4).

The incremental load shape vector $\{\Delta\bar{f}\}$ which is intended to represent first mode earthquake loading is assumed to be linearly increasing with height. The vertical load is assumed to come from dead load. The incremental work regime is generated using the same procedure that was used for the portal frame.

Two studies simulating pinned and clamped columns at ground level were performed. The base story shear-drift relation is presented in Fig. 6. For the pinned case the yield mechanism is formed in the first story beams and column pins; in the clamped case yielding occurs at the base of the columns and in the first story beams. The base story shear capacity is approximately three times greater for the clamped case. Collapse displacements were found to be 2.34 ft. for the clamped case and 1.76 ft. for the pinned case.

CONCLUSIONS

The collapse studies performed with the nonlinear incremental work procedure produced a number of interesting results from which the following conclusions are drawn:

1. The shear-drift relation is a curvilinear function starting at zero, going rapidly to a peak, and then declining more gradually back to zero; the shape and magnitude of the relation is influenced by material properties, geometry of the structure, detailing of reinforcement, load configuration, and history.

2. The critical vertical load is a variable quantity. It starts at a high initial value when the undamaged structure is subjected to small lateral loads. The initial value is influenced by the magnitude of the applied vertical load. After yield due to lateral load the critical vertical load is not influenced by the applied vertical load because the lateral loads are dominant.

3. The incremental work procedure can be used to study problems of geometric and material instability. The method is ideally suited for predicting collapse displacements in earthquake damaged structures.

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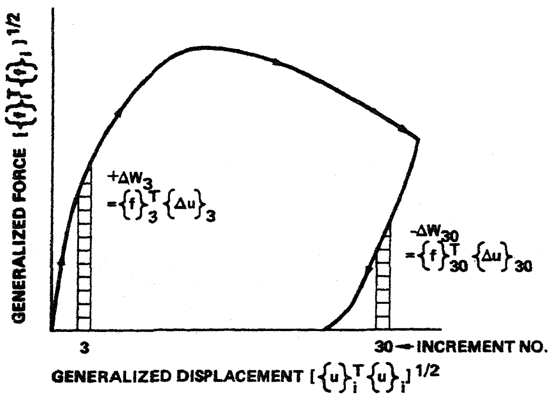

Figure 1. Incremental Work.

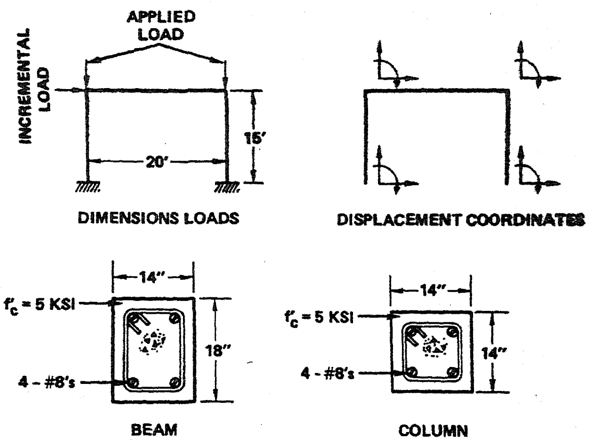

Figure 2. Frame Details.

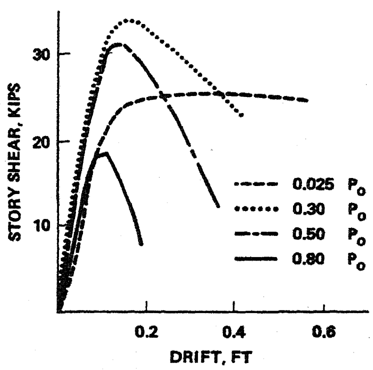

Figure 3. Shear Drift Relations.

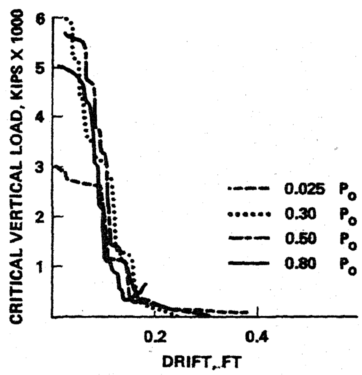

Figure 4. Buckling Load.

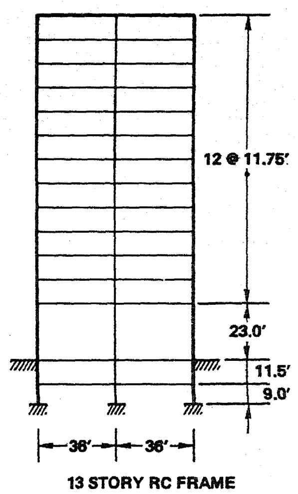

Figure 5. Multistory Frame.

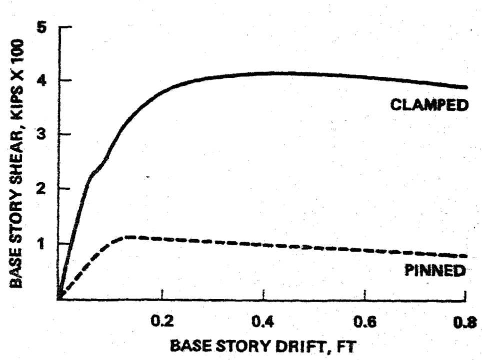

Figure 6. Multistory Frame Shear vs. Drift.