SOIL LIQUEFACTION ANALYSIS
BASED ON FIELD OBSERVATIONS

by

M. K. Yegian[I] and R. V. Whitman[II]

SYNOPSIS

Most interpretations of reported data obtained from field observations have invariably been in terms of ground surface accelerations. This paper presents an alternate approach to the interpretation of reported case histories. The use of ground surface accelerations which are subject to great uncertainties has been deliberately avoided. Instead, the reported earthquake magnitudes together with the distances to the nearest source of energy release have been employed to evaluate the shaking at the sites of the case histories. A criterion is recommended herein which relates the possibility of liquefaction and soil density expressed in terms of corrected blow counts. Quantitative measures of uncertainties both in the criterion and the parameters which define the criterion are also presented in this paper.

INTRODUCTION

Current empirical methods of liquefaction analysis make extensive use of case histories where soil liquefaction was or was not observed. Ground surface acceleration values which were either recorded or estimated together with some measure of the durations of the earthquake shakings for selected case histories have been used to develop criteria for soil liquefaction as a function of soil density. These criteria are now used to study the liquefaction potential at other sites subjected to various postulated earthquakes. Such studies are frequently conducted in a deterministic manner employing "conservative" estimates of soil and/or earthquake strength to account for some of the uncertainties present in the analysis and the parameters used.

An alternate approach to liquefaction analysis was developed (Yegian, 1976) which places the problem of soil liquefaction in the context of an overall seismic risk analysis. The proposed liquefaction risk analysis begins with the study of the seismic history of the region surrounding a particular site of interest, making predictions about the future seismic activities in that region together with an investigation of the subsurface conditions at the site and finally yields the probability of foundation "failure" due to liquefaction of foundation soil.

Viewing liquefaction in the form of a risk analysis, the increased benefits of choosing earthquake magnitude, M, and hypocentral distance, R, to describe earthquake intensity is apparent. Liquefaction risk analysis when

---

[I] Assistant Professor of Civil Engineering, Northeastern Univ., Boston, MA, formerly, Graduate Student, Massachusetts Institute of Tech., Cambridge, MA

[II] Professor of Civil Engineering, Massachusetts Institute of Technology, Cambridge, MA

performed in terms of magnitude and hypocentral distance instead of acceleration and duration is much more convenient and avoids the unnecessary use of an acceleration attenuation law which introduces great uncertainties in the analysis. The use of earthquake magnitude to study the likelihood of liquefaction accounts for earthquake duration more directly than an empirically determined parameter such as number of equivalent significant cycles. Moreover, the list of case histories used to develop an empirical criterion for soil liquefaction can be greatly enlarged by choosing to interpret the field data in terms of earthquake magnitude and hypocentral distance. Sites where liquefaction was not observed can now receive equal importance in the field data studies as sites where liquefaction was observed.

M & R APPROACH TO THE INTERPRETATION OF FIELD DATA

Having preferred to perform the liquefaction risk analysis in terms of magnitude and distance, it was necessary to develop a probabilistic model which also employs these seismic parameters to yield the likelihood of liquefaction given a certain level of earthquake shaking. The model developed for this purpose and presented herein is based on the interpretation of case history data.

To interpret the field data a parameter was evolved which describes earthquake intensity or soil strength against liquefaction as a function of earthquake magnitude, M, in Richter scale, hypocentral distance, R, in miles, depth to point of interest, H, in feet and vertical effective stress, $\bar{\sigma}_v$, in psi. Equation (1) presents the parameter, S_c, which is proportional to earthquake induced maximum shear stress.

$$S_c = \frac{e^{aM} H}{(R + 16)^b \bar{\sigma}_v} \qquad (1)$$

Values of the coefficients "a" and "b" were estimated studying the amplification characteristics of various soil profiles using the computer program SHAKE. The results of these studies indicate that the values of "a" and "b" range between 0.32 to 0.8 and 0.64 to 1.6 respectively depending upon the assumptions employed. The values of "a" and "b" under average conditions are 0.5 and 1.0 respectively. Assuming the average values of the coefficients "a" and "b" and incorporating the variances of "a" and "b" into the variance of S_c, the parameter chosen to interpret field observattions takes the form as expressed in Equation (2):

$$S_c = \frac{e^{0.5M} H}{(R + 16) \bar{\sigma}_v} \qquad (2)$$

The parameter S_c described in Eq. (2) was employed to interpret the case histories reported earlier by Seed and Idriss (1971), Castro (1975) and a few Japanese sites where liquefaction was not observed. The results of the study are presented in Fig. (1). The horizontal axis in this figure represents the SPT values corrected for the overburden pressure using Eq. (3).

$$N' = \frac{50N}{\sigma_v + 10} \qquad (3)$$

The vertical axis represents the average strength parameter S_c. In Fig. 1 the solid circles correspond to sites where liquefaction was observed and conversely the open circles signify no liquefaction. To develop a criterion for liquefaction as a function of soil density, N', it is necessary to define a boundary line separating, with some confidence, the solid circles establishing a zone of probable liquefaction from the open circles establishing a zone of no liquefaction. This problem is commonly referred to in statistics as "pattern recognition". In short, the criterion sought should be in the form of a plot which is defined ideally by the lowest possible position that any solid circle would assume or the highest possible position that any open circle would assume.

A number of techniques have been developed in statistics to study patterns of different behaviors very similar to the problem described above. However, a careful study revealed the inapplicability of these techniques to the problem of liquefaction or no liquefaction. Therefore, a rather simplified approach was developed to statistically determine the line separating the solid circles from the open circles of Fig. 1. The method is termed "least square of misclassified points". The method consists of locating the boundary line such that the sum of the squares of the distances of the misclassified points from the boundary line is a minimum. Fig. 2 shows the boundary line which minimizes the sum of the squares of the distances of the misclassified points. The misclassified points are the solid circles which plot below the line in the region of no liquefaction and the open circles which plot above the line in the region of liquefaction.

The process of establishing a criterion such as shown in Fig. 2 involves great sources of uncertainties. An attempt was made to identify the major sources of uncertainties, to quantify them, and finally to incorporate them into the variance of S_c. Some of the sources of uncertainties considered are:

a) form of the parameter S_c due to uncertainties in the values of the coefficients "a" and "b".

b) location of the mean line $\bar{S}_c$ due to variance of misclassified data.

c) parameters M, R and N' for each case history.

Considering these sources of uncertainties the average strength parameter $\bar{S}_c$ and its total variance was estimated. Assuming a lognormal distribution for S_c, the 98% confidence interval of $\bar{S}_c$ was computed and is shown in Fig. 3.

CONDITIONAL PROBABILITY OF LIQUEFACTION

Conventionally, to assess the liquefaction potential of a sand deposit, the shear stresses caused by an earthquake in the deposit are compared with the available cyclic shear strength of the sand. Whenver the earthquake induced stresses exceed the shear strength, liquefaction of the sand is anticipated during that particular earthquake.

For the sake of mathematical simplicity in the probabilistic analysis developed, a new measure of liquefaction potential of a sand is defined, namely, the Liquefaction Potential Index, LPI. By definition, LPI is equal to the ratio of the shear stresses caused by an earthquake shaking to the resistance of the sand to such shaking. Therefore

$$LPI = \frac{\tau \text{ earthquake}}{\tau \text{ strength}} \tag{4}$$

Liquefaction failure of a sand deposit is expected to occur whenever the earthquake induced stresses are greater than the *available shear* strength of the sand,

Therefore:

$LPI > 1.0$	Liquefaction expected
$LPI < 1.0$	Liquefaction not expected

Since the paramter S_c expressed in Eq. (2) is proportional to shear stresses causing liquefaction the plot of S_c shown in Fig. 3 is proportional to the average shear strength of soils against liquefaction. Employing the definition of Liquefaction Potential Index, the expression for average LPI becomes:

$$\overline{LPI} = \frac{S_c \text{ earthquake}}{\bar{S}_c} \tag{5}$$

in which S_c earthquake is given by Eq. (2) and $\bar{S}_c$ is given by the plot of Fig. 3. Substituting for the parameter S_c earthquake:

$$\overline{LPI} = \frac{e^{0.5M} H}{(R + 16)\ \bar{\sigma}_v \bar{S}_c} \tag{6}$$

Eq. (6) expresses the average computed LPI as a function of the earthquake intensity and average soil strength $\bar{S}_c$. However, because of uncertainties in the parameters which are used to compute LPI and especially uncertainty in the strength paramters $\bar{S}_c$, the "true" LPI might be lower or higher than the computed LPI. Thus, accounting for these uncertainties, a statement can be made regarding the probability of liquefaction or no liquefaction at a site.

Liquefaction potential of a sand deposit may therefore be described in a probabilistic manner as the probability of having the "ture" LPI exceeding or equal to 1.0.

$$P[LIQ] = P[LPI \geq 1.0]$$

To compute the probability of liquefaction one is requried to estimate the variance of LPI and then employ a probability density function for LPI. The area under the probability density function for LPI $\geq$ 1.0 will yield the probability of liquefaction. Figure 4 describes schematically the probability of liquefaction computations for different depths assuming a log-normal distribution for LPI.

CONCLUSIONS

A new probabilistic model is developed which can be incorporated into a risk analysis for earthquake induced ground failure by liquefaction. The model is based on case history data interpreted in terms of earthquake magnitude and hypocentral distance. This paper presented the model for liquefaction analysis together with quantitative measures of uncertainties both in the model and the parameters used in the study.

REFERENCES

1. Castro, R. (1975): "Liquefaction and Cyclic Mobility of Saturated Sands", American Society of Civil Engineers GT6, Vol. 101.
2. Seed, H.B. and I.M. Idriss (1971): "Simplified Procedure for Evaluating Soil Liquefaction Potential," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, SM9.
3. Yegian, M.K. (1976): "Risk Analysis for Earthquake-Induced Ground Failure by Liquefaction," Thesis submitted to the Massachusetts Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

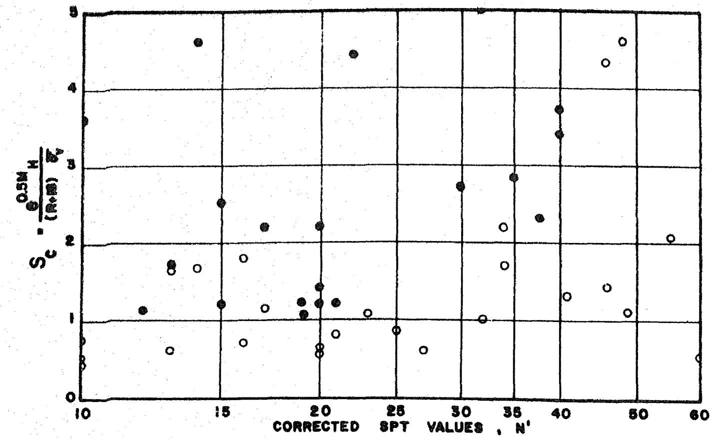

FIG. 1 SHEAR STRESS PARAMETER S_c VS. CORRECTED BLOW COUNTS, N'

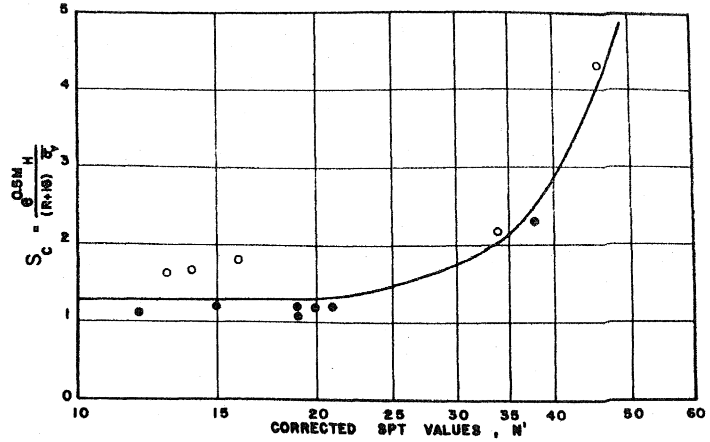

FIG. 2 METHOD OF LEAST SQUARES OF MISCLASSIFIED POINTS

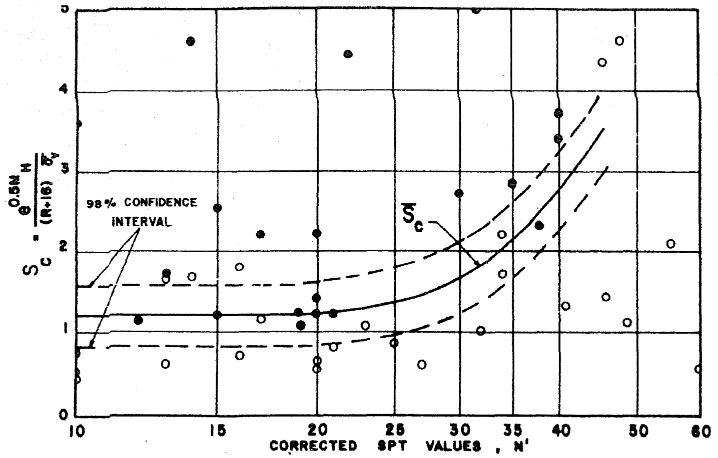

FIG. 3 STRENGTH PARAMETER $\bar{S}_c$ WITH CONFIDENCE INTERVAL

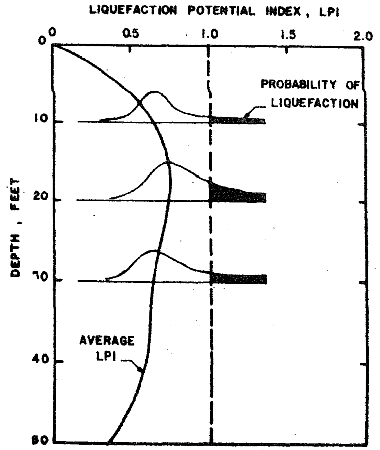

FIG. 4 PROBABILITY OF LIQUEFACTION WITH DEPTH

DISCUSSION

Henry J. Degenkolb (U.S.A.)

The authors are to be congratulated in the presentation of their plotted graphs. They not only show the 'average' curves, but they show the spread of the actual data.

Too often when these studies are combined, simplified and presented to the practicing engineer for use in design, they are shown as simple lines. The engineer consequently has no knowledge of the variability of the data and cannot make suitable design adjustments to his specific project to account for the possibility that the simple rule may not apply to his project.

G.P. Saha (India)

The authors have made a very interesting contribution by presenting formulae for the evaluation of depth of liquefaction of a soil mass. Although the depth of liquefaction can be calculated for a particular case of soil media but since some uncertainties are involved in the evaluation of soil properties, would it be advisable to consider the friction into consideration along the sides of a caisson foundation which is situated in a seismic area.

Author's Closure

Not received.