A STOCHASTIC METHOD FOR SEISMIC STABILITY EVALUATION OF EARTH STRUCTURES WITH STRAIN DEPENDENT PROPERTIES

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SYNOPSIS

A stepwise linear stochastic method is presented for seismic stability analysis of earth structures with strain dependent damping and shear moduli. In this method, the seismic design input defined as response spectra curves can be directly used. The technique of stochastic linearization is used to obtain the best estimates of the soil properties for their use in a linear step of the analysis. Stochastic definition of cumulative damage sustained by soil is used to evaluate safety under seismic excitations. The factors of safety at various locations of an earthen dam are obtained and compared with those obtained by the conventional time history analysis approach.

INTRODUCTION

For stability evaluation of earth structures under seismic loads, currently time history analyses using recorded or synthetic accelerograms are performed in which strain dependent soil properties are used. However, for design the seismic input is more commonly defined in terms of ground response spectra curves rather than a time history accelerogram. Therefore, a method is presented herein whereby design response spectra curves can be directly used with due consideration of strain dependent soil properties.

Hysteretic behavior of soil is commonly defined by equivalent strain dependent damping and shear modulus curves, Seed and Idriss (1), such as shown in Fig. 1. As in the currently used approach, Ref. (2), an iterative procedure is proposed herein for making use of these nonlinear soil property curves. The best estimates of soil properties for use in an iteration are, however, obtained through stochastic linearization technique, instead of obtaining them corresponding to a certain fraction of the peak strain. The formulation, with stochastic linearization, is developed for its use with the finite element method of analysis which is commonly used with soil structures. Stochastic concepts are used to obtain the damage sustained by the soil finite elements and to obtain their factors of safety.

ANALYSIS TECHNIQUE

The equations of motion for a finite element discretization can be written as:

$$M\ddot{X} + D(\varepsilon)\dot{X} + S(\varepsilon)X = -M\{1\}\ddot{X}_g(t) \tag{1}$$

where M, is mass matrix and D & S are damping and stiffness matrices which depend upon the elemental shear strains. X is the vector of nodal displacement relative to fixed base. A dot over a vector represents its time derivative. $\ddot{X}_g(t)$ is the base acceleration. Depending upon the level

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of strain in an element, damping and shear modulus values are obtained and used to form element stiffness and damping matrices (2). These element matrices are assembled to obtain the total structural matrices. Eq. 1 is a nonlinear set of equations as ε, the elemental strain, depends on the nodal displacements. It is intended to replace Eq. 1 by a linear equation, i.e.:

$$M\ddot{X} + C\dot{X} + KX = -M\{1\}\ddot{X}_g(t) \tag{2}$$

This introduces an error, the magnitude of which can be written as:

$$e = [D(\varepsilon) - C]\,\dot{X} + [S(\varepsilon) - K]\,X \tag{3}$$

The most commonly used procedure for selection of C and K is to minimize a norm of e. This norm is usually taken as the mean square value, i.e., minimize $E\{e'e\}$. Here prime over a vector indicates its transpose. As the only variables which define C and K in Eq. 2 are the elemental damping and shear modulus values, the conditions for minimization are:

$$\frac{\partial}{\partial \lambda_q}[E(e'e)] = 0, \text{ and} \tag{4a}$$

$$\frac{\partial}{\partial G_q}[E(e'e)] = 0 \tag{4b}$$

where λ_q and G_q are the damping and shear modulus for the qth element of the model. It is assumed that the excitation is a Gaussian stationary random process. This assumption is not a serious handicap. Stationary response due to such excitation is examined. At a node, the stationary response and its time derivative are uncorrelated at the same time instant. It has also been observed that the correlation between them for any two different points on a structure excited by ground motion is not significant either. With this it can be shown that:

$$E[\dot{X}'C\frac{\partial C}{\partial \lambda_q}\dot{X}] = E[\dot{X}'D(\varepsilon)\frac{\partial C}{\partial \lambda_q}\dot{X}] \tag{5a}$$

$$E[X'K\frac{\partial K}{\partial G_q}X] = E[X'S(\varepsilon)\frac{\partial K}{\partial G_q}X] \tag{5b}$$

Eqs. 5 represent two sets of simultaneous equations in λ_q's and G_q's which can be solved to obtain the best linearized values of these parameters. In the paper, however, only the results with the error being minimized in a local sense, i.e., for each finite element of the model, are obtained. Error equation analogous to Eq. 3 can be written separately for each element, and minimization of the mean square value defines λ_q and G_q as follows:

$$\lambda_q = E[\bar{\lambda}_q(\varepsilon)\,\dot{X}_q'\,D_q\,\dot{X}_q] \,/\, E[\dot{X}_q'\,D_q\,\dot{X}_q] \tag{6a}$$

$$G_q = E[\bar{G}_q(\varepsilon)\,X_q'\,S_q\,X_q] \,/\, E[X_q'\,S_q\,X_q] \tag{6b}$$

where X_q is the nodal displacement vector for the element nodes, D_q and S_q are element damping and stiffness matrices with maximum values of damping coefficient and shear modulus, and $\bar{\lambda}_q(\varepsilon)$ and $\bar{G}_q(\varepsilon)$ are damping and shear modulus variables. As Eq. 6 corresponds to minimization of error for each element, it does not represent a true minimum of e for the whole system.

However, as minimization of the norm of e was arbitrarily selected (mainly for mathematical convenience), these values of λ_q and G_q are, probably, as good as the ones obtained from Eq. 5. In Eq. 6a strain in an element depends only on the nodal displacements and not the velocities. Making use of the fact that a response and its derivative are uncorrelated at the same node and approximately uncorrelated for different nodes, Eq. 6a reduces to:

$$\lambda_q = E[\bar{\lambda}_q(\varepsilon)] \tag{7}$$

To obtain λ_q and G_q from Eq. 6 and 7, quantities $E[x_i^2]$, $E[\varepsilon^2]$, $E[x_i x_j]$ and $E[\bar{G}_q(\varepsilon)x_i x_j]$ are required. These quantities in terms of ground spectra curves can be defined as follows:

$$E[x_i^2] = (\sum_{j=1}^{N} \psi_j^2(i)\gamma_j^2 R_j^2/w_j^4)/F^2 + \text{Second Order Terms}$$

$$E[\varepsilon^2] = (\sum_{j=1}^{N} \xi_j^2 \gamma_j^2 R_j^2/w_j^4)/F^2 + \text{Second Order Terms}$$

$$E[x_i x_j] = \sum_{k=1}^{N} \gamma_k^2 \psi_k(i)\psi_k(j)R_k^2/w_k^4/F^2 + \text{Second Order Terms}$$

$$E[\bar{G}_q(\varepsilon)x_i x_j] = a\ E\{\bar{G}_q(\varepsilon)\varepsilon^2\} + b\ E\{\bar{G}_q(\varepsilon)\} \tag{8}$$

where

$$a = E[x_i\varepsilon]\ E[x_j\varepsilon]/\{E(\varepsilon^2)\}^2$$

$$b = E[x_i x_j] - E(x_i\varepsilon)E(x_j\varepsilon)/E(\varepsilon^2)$$

$$E(\varepsilon x_i) = (\sum_{j=1}^{N} \xi_j\psi_j(i)\gamma_j^2 R_j^2/w_j^4)/F^2 + \text{Second Order Terms}$$

where it is assumed that ε, x_i and x_j are jointly normal. The second order terms can generally be neglected. Here w_j, γ_j and $\psi_j(i)$ are the modal frequency, participation factor and displacement for node i, respectively; N= number of degrees of freedom; R_j=the acceleration response spectrum value for mode j; ξ_k=element strain mode shape. Modal damping is defined as $\beta_j = \psi_j^T C\psi_j/(2\beta_j)$; F=the peak factor by which the root mean square response is multiplied to obtain the design response. These equations are similar to those in the method of square root of the sum of the squares (SRSS) commonly used in seismic analyses.

Procedure is essentially iterative. For initially assumed values of λ_q and G_q for each element total damping and stiffness matrices are obtained. These matrices are used to obtain modal quantities to define a better estimate of λ_q and G_q. In a few iterations a good convergence in the values of λ_q and G_q can be obtained. The modal quantities in the final iteration are used for evaluation of damage potential and factor of safety as follows.

DAMAGE POTENTIAL: To evaluate the safety of earth structures Palmgren-Miner's linear cumulative damage concept is commonly used in practice. For safety of a structure this cumulative damage should be less than 1.0. The relationship between the amplitude of stress response, s, and number of cycle to fatigue failure, N, for most soils can be represented by the following relationship:

$$N\ s^b = C \tag{9}$$

where b and c are constant which depend upon the type of soil and its state of stress at site. This curve if plotted on a log-log scale represents a straight line. For such strength curves it can be shown (3) that the expected value of damage sustained by the soil due to a stationary response of duration T is:

$$E[D(T)] = T\ M\ C^{-1} \int_0^\infty s^b p_s(s) d_s \tag{10}$$

where M = expected number of peaks per unit time, s = shear stress and $p_s(s)$= the probability density of stress peaks, defined as (3),

$$p_s(s) = \sqrt{1-\alpha^2}\ \exp\ (-s^2/[2\sigma^2(1-\alpha^2)])/\sqrt{2\pi}\sigma$$

$$+ \frac{\alpha s}{2\sigma^2}\{\ 1+erf[s\alpha/\{\ \sigma\sqrt{2(1-\alpha^2)}\}]\}\ \exp(-s^2/2\sigma^2) \tag{11}$$

where $\alpha = \nu_0/M$ is a measure of band width of the response, ν_0=zero crossing rate, and σ = standard deviation of stress response. For stochastic response, the expression for the variance of damage is not available at this time. However, through approximate analysis Crandall and Mark (4) have shown that for high damping values the variance of the damage is small. For soils the damping values are high, and therefore, expected damage is a good measure of the actual damage potential of soil structures.

FACTOR OF SAFETY: A more conventional measure of safety of a structure is the factor of safety. For soils under dynamic stresses it can be defined, for a predecided number of cycles, as the ratio of the cyclic strength to the equivalent stress to which the soil is subjected. For the strength relation of Eq. 9 which is a straight line on log-log scale, the factor of safety does not depend on the chosen number of cycles, and can be related to damage D by the following equation:

$$F.S. = D^{-1/b} \tag{12}$$

NUMERICAL RESULTS: The above approach has been used to obtain the factor of safety of a small earth dam shown in Fig. 2. This dam represents a dike used in the ultimate heat sink of a nuclear power plant. The seismic stability of such dikes is required to be evaluated to ensure the availability of the cooling water for at least 30 days in case the main source of cooling water is lost. The cross section of the dike was discretized into finite elements; for horizontal excitation, only half the cross section of the dam was considered due to symmetry of the structure. Strain dependent shear modulus and damping curves are shown in Fig. 1. The maximum values were taken as 1080 ksf and 0.350. Different strength curves were used for different elements. These are shown in Fig. 3. A peak horizontal ground acceleration of 0.12g was considered. Input was defined in terms of response spectra curves as described by Newmark, Blume and Kapur (5). The factor of safety also depends upon the peak factor used for obtaining the mean square response from the SRSS response. This factor may vary between 2.0 to 3.5. The guidelines provided by Hou (5) have been used to estimate the peak factor and equivalent stationary duration of the earthquake. For a 10 secs. duration of earthquake with 2 secs. of strong motion phase, the peak factor was estimated to be 2.874 and equivalent stationary duration as 4 secs. corresponding to the fundamental frequency of 8.0 cps.

The factor of safety values for a few important core elements of the model obtained by the method proposed in this paper and by the commonly used time history analysis are given in Table 1. The values for the peripheral elements which are not of much significance have been omitted from the Table. In the time history analysis, a synthetic time history whose response spectra enveloped the prescribed spectra was used. It is seen that the values obtained by the two method agree reasonably well with each other.

SUMMARY AND CONCLUSIONS

For seismic stability evaluation of earth structures, a simple approach which can be directly used with prescribed response spectra and which considers strain dependent soil properties is presented. Stochastic principles are invoked, but the computations involved are only a trifle more complicated than the usual SRSS approach. No synthetic time history consistent with given spectra are required for analysis. The approach can also be applied for evaluation of liquefaction potential of soil strata.

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Table 1

Factors of Safety by Time History Analysis and Proposed Approach

Elem. No.	Str. Curve No.	Factor of Safety		Elem. No.	Str. Curve No.	Factor of Safety	
		Time Hist.	Proposed Method			Time Hist.	Proposed Method
3	2	3.63	3.74	10	1	3.07	2.75
4	1	3.30	3.22	11	1	2.86	2.55
5	1	2.61	2.52	14	2	3.11	2.44
6	1	2.41	2.31	15	2	3.10	2.44
9	2	3.03	2.55	18	3	4.73	3.31

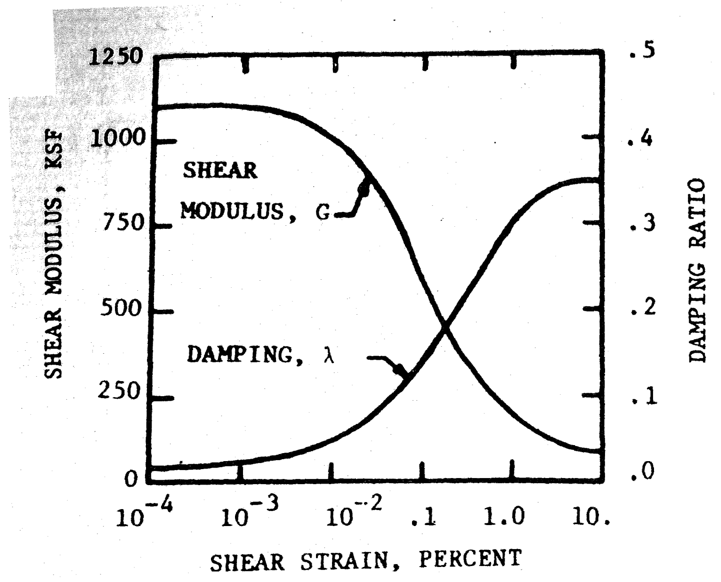

FIG. 1: STRAIN DEPENDENT G & λ CURVES

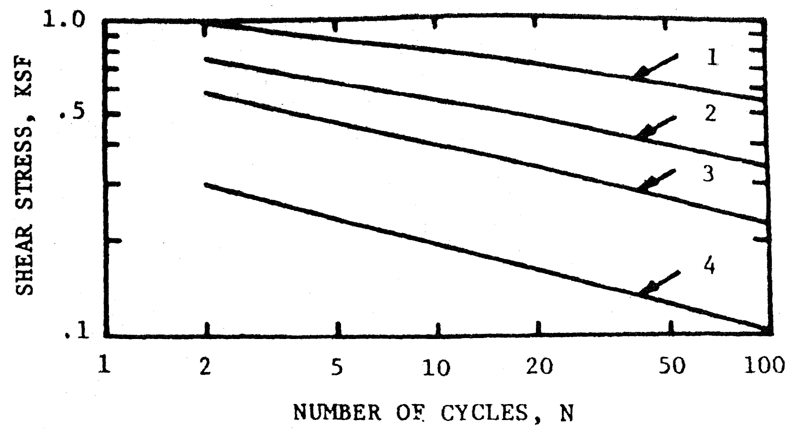

FIG. 3: SHEAR STRESS V/S NUMBER OF CYCLES CAUSING 5% STRAIN

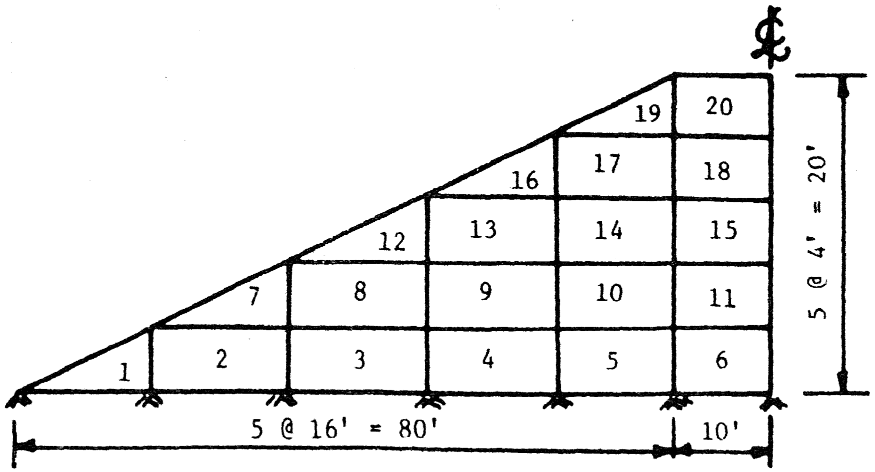

FIG. 2: HALF CROSS SECTION OF AN EARTH DAM