NON LINEAR SOIL IDENTIFICATION

by
K.D. Hill[I], V.W. Lee[II] and F.E. Udwadia[III]

SYNOPSIS

Laboratory tests have been performed on free standing soil samples in which deformations in the nonlinear range have been studied. The tests were performed using a shake table upon which the soil samples were placed.

The nonlinear response to white noise inputs has been modeled using methods of nonparametric identification. Wiener kernels have been determined for such tests and these kernels have been further used to model soil responses to sinusoidal excitations. The white noise testing technique for obtaining nonlinear soil characteristization has been evaluated.

INTRODUCTION

Soil experiments using a shake table have been performed in the past using sinusoidal input waves to determine the dynamic properties of the soil (for a review of previous shake table testing see references 1, 2 and 3). This previous testing has been limited to performing each test at a single sinusoidal frequency. To obtain information for a range of frequencies it has been therefore necessary to perform numerous tests. However, soil being a nonlinear material, the individual response to sinusoidal inputs of different frequencies cannot be simply superimposed to obtain the response to earthquake like motions which are typically broad band in nature. Hence, the application of such single sinusoidal testing to study the earthquake response of soils may become suspect at times and may not hence lead to meaningful results.

It was the purpose of this testing program to overcome this basic problem. To accomplish this, analytical modelling of soil properties by white noise testing, which more closely simulates the earthquake environment, has been carried out. Such a characterization would yield soil properties which would need to be interpreted in terms of an average sense.

In its most general interpretation, knowledge of the dynamic characteristics of the soil implies cognizance of the functional relations between the excitations and the soil responses. Such an identification is ultimately aimed at obtaining a mathematical representation, or model. Having obtained by some means such a mathematical model, the investigator can then simulate the soil response to any input. The usefulness of such a model would lie in the feature that soil responses to large ground motions can then be computed.

I Consulting Civil Engineer
II Graduate Student, California Institute of Technology, Pasadena, Ca.
III Assistant Professor of Soil Mechanics, University of Southern California, Los Angeles, Ca.

The problem of identifying the nonlinear properties of a soil may be formulated from two broad viewpoints. In the first, the mathematical structure of the model is given or assumed, but its parameters are not. In the second, no a priori information whatsoever is specified regarding the soil. In the present paper, attention is focused on the second of these viewpoints.

Although soils subjected to strong ground shaking are almost universally nonlinear in nature, there have been no previously reported attempts to characterize the dynamics of soil through the application of a general theory of nonlinear systems. This can be readily understood because the theory developed by Wiener (3) was not available prior to about 1950 and also because it was particularly obscure to the nonmathematician.

This study presents the general soil identification problem and briefly outlines the basic Wiener Theory (3) of nonlinear system characterization, so as to provide the necessary background for an understanding of its extent of applicability to soil systems using earthquake and microtremor ground motion inputs. The effects of possible error sources and the possible effect of the nonlinearities in adversely affecting the linear characterizations of the system are analyzed.

The nonlinear behavior of the same soil as obtained through the determination of the higher order system kernels is determined. Model responses for the same input are determined using first, the linear characterizations and then the nonlinear characterizations.

THEORY

Following Wiener's general theory of nonlinear system identification, there is for a system S, whose input is x(t) and output is y(t),

$$y(t) = F(t, x(\tau), \tau \leq t)$$

where F is a function whose value at time t depends only on the past values of the input. It is assumed that the system is nonlinear with finite memory, M. The memory, M, can be defined as that time t at which the response of the system to an impulse at time zero is negligibly small.

Since soils would in general behave in a time-variant manner, the resulting analytical model would represent an average (in time) characterization (i.e. the nonlinear soil model would exhibit the average characteristics).

Under the above assumptions, Wiener showed that the relationship between the input and the output signal can be written as an infinite series as follows:

$$y(t) = \sum_{n=0}^{\infty} G_n[h_n, x(t)] \tag{2}$$

where $\{G_n\}$ is a complete set of orthogonal functions with respect to Gaussian white noise, and $\{h_n\}$ is a set of kernels that characterizes the system (the soil samples in this case). The orthogonal property of the functions is expressed as

$$E\{G_n[h_n, x(t)]\, G_m[h_m, x(t)]\} = \begin{cases} 0 & \text{for } m \neq n \\ \text{constant} & \text{for } m = n \end{cases} \tag{3}$$

where E { • } denotes the expected value of the quantity in the brackets with x(t) a white-noise input. Moreover, the Wiener G-function of degree n is such that it is orthogonal to all functions of degrees less than n.

Each h_n is a symmetrical function with respect to its arguments. The set of kernels $\{h_n\}$ can be considered to be the generalized "impulse responses" of the nonlinear soil. The first order kernel $h_1(\tau)$, corresponds to the impulse response function of the linear system theory. By considering the system response to inputs of single impulses applied at different times as well as the response to finite trains of impulses, the nonlinear kernels give a quantitative measure of the nonlinear "cross-talk" between different portions of the past input as it affects the present response.

The kernels $\{h_n\}$ can be evaluated directly by means of cross-correlation techniques, utilizing the orthogonal properties of the Wiener series and the fact that the input x(t) is Gaussian white noise with a zero mean. As discussed in Udwadia and Marmarelis (6),

$$h_o = E\{y(t)\} \tag{4}$$

and for $n \geq 1$

$$h_n(\sigma_1, \sigma_2, \ldots, \sigma_n) \tag{5}$$

$$= \frac{1}{n!\,P^n} E\{[y(t) - \sum_{i=0}^{n-1} G_i[h_i, x(t)]]\, x(t-\sigma_1)x(t-\sigma_2) \ldots x(t-\sigma_n)\}$$

where P is the spectral density level of the input white noise, i.e. $P = \phi_{xx}(f)$ is the power spectrum of x(t) at frequency f.

Non-linearities in soil responses depend closely on the amplitude range and frequency content of the exciting source. The characterization of such non-linearities depends on the estimation of the kernels. Such analysis as discussed in Udwadia and Marmarelis (5,6) showed that to obtain a good estimate of the kernels, it is required that the bandwidth of the input signal cover the complete frequency range in which the system's response is of interest. This provides the lower bound for the test noise bandwidth.

To provide its upper bound, the analysis showed that the variance of the estimated first order (linear) kernel is given by

$$\mathrm{Var}\,[\hat{h}_1(\tau)] =$$

$$A(T,M)\left[\frac{\omega_o}{4\pi^3}\int_{-\omega o}^{\omega o} |H(\omega)|^2\, d\omega + \frac{1}{4\pi^2}\left|\int_{-\omega o}^{\omega o} H(\omega)\exp(i\omega\tau)\, d\omega\right|^2\right] \tag{6}$$

where A(T,M) is a function (< 1) of the input duration, T, and system memory, M; ω_o is the bandwidth of the input signal; and $H(\omega)$ is the Fourier transform of $h_1(t)$.

It is seen that Var $[\hat{h}_1(\tau)]$ generally increases with the bandwidth. To minimize error in estimating the kernels, it is concluded from above that the input bandwidth should be larger than the system bandwidth, but should not extend much beyond it.

EXPERIMENT AND DATA ANALYSIS

Free standing cylindrical soil samples (6.35 cms. in diameter and 10.16 cms. in height) consisting of a silty fine sand were used in the testing. The samples were molded to a dry density of 1.3 tons per cubic meter (65 percent relative compaction) and a moisture content of 7 percent by dry weight. The bottom 2.54 cms. of the sample extended into a thin brass ring which was fixed to the shake table leaving 7.62 cms of free standing soil height.

The shake table was driven by a white noise generator. The mechanical impedance of the shake table caused the input motion spectrum to be non-flat. Suitable prefiltering of the generator signal was therefore required prior to amplification before the voltage could be fed to the shake table. The motion of the table was recorded using a Statham accelerometer. A thin metal lid was glued on to the top surface of the soil sample and a light Endevco accelerometer was mounted on the lid to record the response of the free standing soil column. The signals from the accelerometers were amplified, observed on an oscilloscope and recorded on analog tape. The recorded signals on tape were then converted to digital form for further analysis. A schematic flow diagram on Fig. 1 presents the experimental apparatus.

After calibrating the accelerometers, the natural frequencies of two samples were determined by a sinusoidal shake test. Each sample was tested with a different r.m.s. level of input. Having found the fundamental frequencies and the response to sinusoidal tests, two similar samples were subjected to the white noise input. Again each sample was subjected to different r.m.s. levels of white noise. The total recorded time interval for each test was two to three minutes.

The analog records were converted to digital form using an AD converter, sampling 200 points per second of data. Using equation (5), h_1 and h_2 were computed. From these tests, the linear and nonlinear characterization was obtained for each r.m.s. level of signal input.

CONCLUSIONS

Final numerical results are not available, however, preliminary results indicate that nonlinear properties of the soil column can be efficiently analyzed with this procedure and that the nonlinear behavior as characterized by the second order kernel changes in systematic manner with increasing r.m.s. The final results will be presented at the conference.

REFERENCES

1. Kovacs, W.D., Seed, H.B. and Idriss, I.M., "Studies of Seismic Response of Clay Banks" Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 97 SM 2, Proc. Paper 7878, February 1971 pp. 441-455.

2. Shannon & Wilson, Inc. and Agbabian-Jacobson Associates, "Soil Behavior Under Earthquake Loading Conditions" Interim Report No. 1 proposed for U.S. Atomic Energy Commission, December 1971, pp. 128-159.

3. Pyke, R., Seed, H.B., Chan, C.K., "Settlement of Sands under Multi-directional Shaking" Journal of the Geotechnical Division, ASCE, Vol. 10, G.T. 4 Proc. Paper 11251, April 1975, pp. 379-398.

4. Weiner, N., "Nonlinear Problems in Random Theory", MIT Press, 1958.

5. Udwadia, F.E. and Marmarelis, P.Z., "The Identification of Building Structural Systems: I. The Linear Case", Bulletin of the Seismological Society of America, Vol. 66, No. 1, February 1976, pp. 125-151.

6. Marmarelis, P.Z. and Udwadia, F.E., "The Identification of Building Structural Systems: II. The Nonlinear Case" Bulletin of the Seismological Society of America, Vol. 66, No. 1, February 1976, pp. 153-171.

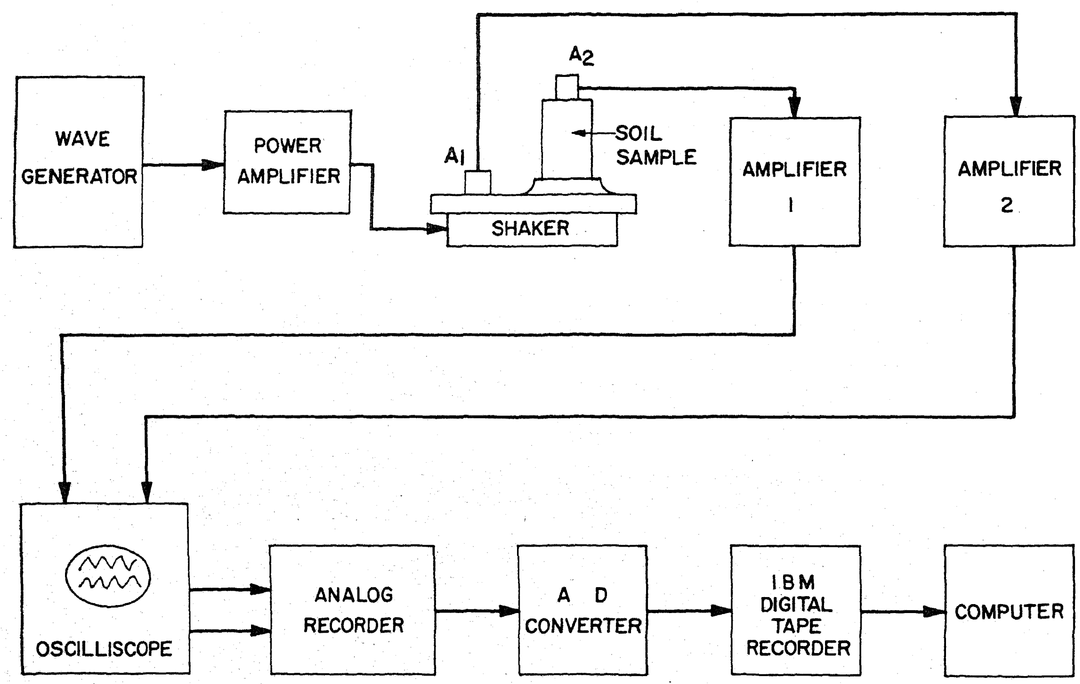

Figure 1