# DYNAMIC PROPERTIES OF A STRATIFIED FLUID SATURATED SOIL LAYER

BY

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## ABSTRACT

Dynamic properties of the soil layer with a microstructure are investigated by idealizing the soil composed of soil skeletons and pore-fluid to the mixture model. A wave propagation theory in multi-phase continuum mechanics is developed briefly. Its method and FEM are applied to two-phase medium and then, free vibration and frequency transfer properties in a fluid saturated soil layers are presented. The wave dispersion based on the microstructure of the ground is recognized and shows one of the damping mechanism of multi-phase soil ground.

## INTRODUCTION

Soils as constituents of the ground are idealized generally to a monophase medium such as elastic, elastoplastic, and/or viscoelastic one by the way of expression in microscopic behavior in soils. However, soils are really composed of soil skeletons and fluid saturated in pore. This internal structures remarkably influence on material properties and dynamic behavior. In further understanding the soil behavior, the microstructure of the ground should be estimated in the governing relation. But the classic rigorous approach is analytically unrealizable. Then, in this paper, the fundamental properties in a fluid saturated porous soil layer where the saturated layer is replaced by mixture, is presented. The wave propagation method and FEM in two-phase mixture are used as an analytical approach. By applying the bi-mixture theory, free vibration and steady state response are obtained. The influence of variable water level in soil layers are discussed.

## FUNDAMENTAL THEORY

*GOVERNING EQUATION* [1]

The governing equations of saturated soil layer are written in matrix form;

$$-\partial_{tt}\begin{bmatrix} \beta_s\rho_s\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \beta_f\rho_f\mathbf{I} \end{bmatrix}\begin{bmatrix} U_s \\ U_f \end{bmatrix} + \begin{bmatrix} \beta_s(\lambda+\mu)\nabla\nabla\cdot + \beta_s\mu\nabla\cdot\nabla & \mathbf{0} \\ \mathbf{0} & \beta_f r\nabla\nabla\cdot \end{bmatrix}\begin{bmatrix} U_s \\ U_f \end{bmatrix}$$
$$= -\rho_{sf}\partial_{tt}\begin{bmatrix} \mathbf{I} & -\mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{bmatrix}\begin{bmatrix} U_s \\ U_f \end{bmatrix} + B\partial_t\begin{bmatrix} \mathbf{I} & -\mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{bmatrix}\begin{bmatrix} U_s \\ U_f \end{bmatrix} + Q\nabla\nabla\cdot\begin{bmatrix} \mathbf{I} & -\mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{bmatrix}\begin{bmatrix} U_s \\ U_f \end{bmatrix} \quad (1)$$

where $(U_s, U_f)$ denote the solid and fluid displacement, $\lambda$, $\mu$ and $r$ are Lamé constants of solid and bulk modulus of fluid, $\rho_{sf}$, B and Q are the coupling constants between two-phase, $\beta_\alpha$ is volume ratio in $\alpha$-phase to the whole, $\mathbf{I}$ and $\mathbf{0}$ are the unit and zero matrices.

*FREQUENCY RESPONSE ANALYSIS*

We extend the theory of stratified soil layers [3] to the compound ground composed of soil skeleton and water in pore sketched in Fig. 1 and Fig. 2. This bi-phased ground is replaced by the equivalent mono-phase ground by introducing the dispersion relation as the dynamic constitutive

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equation in frequency domain. The interface between mono-phase layer and bi-phase one is expressed by another replacement of boundary condition. This means that the bi-phase is reduced to apparent equivalent mono-phase in the neighborhood of the interface by applying the phase occupation ratio to individual phase quantities.

The solution of Eq.(1) in the saturated ground, taking the x-axis to the direction of the wave propagation, are

$$U = \begin{bmatrix} U_s \\ U_f \end{bmatrix} = \begin{bmatrix} \mathbb{I}\cos KX & \mathbb{I}\sin KX & 0 & 0 \\ 0 & 0 & \mathbb{I}\cos KX & \mathbb{I}\sin KX \end{bmatrix} \begin{bmatrix} a^s_1 \\ a^s_2 \\ a^f_1 \\ a^f_2 \end{bmatrix} e^{i\omega t} = \mathbb{C}\, a\, e^{i\omega t} \tag{2}$$

where K denotes wave number, ω circuler frequency, and $a$ integral constants.

The constitutive equations of this media are in terms of strain and stress as follows;

$$e = \begin{bmatrix} e_s \\ e_f \end{bmatrix} = \begin{bmatrix} \mathbb{B}_s & 0 \\ 0 & \mathbb{B}_f \end{bmatrix} U = \mathbb{B} U \ , \quad \sigma = \begin{bmatrix} \sigma_s \\ \sigma_f \end{bmatrix} = \begin{bmatrix} \mathbb{D}_{ss} & \mathbb{D}_{sf} \\ \mathbb{D}_{fs} & \mathbb{D}_{ff} \end{bmatrix} e = \mathbb{D} e = \mathbb{D}\mathbb{B}\mathbb{C} a = \mathbb{E} a \ .$$

The state vector consisting of displacements and stresses in the layer are

$$S = \begin{bmatrix} S_s \\ S_f \end{bmatrix} = \begin{bmatrix} U_s \\ \sigma_s \\ U_f \\ \sigma_f \end{bmatrix} = \begin{bmatrix} \mathbb{C}_{s1} & \mathbb{C}_{s2} & 0 & 0 \\ \mathbb{E}_{s1} & \mathbb{E}_{s2} & \mathbb{E}_{s3} & \mathbb{E}_{s4} \\ 0 & 0 & \mathbb{C}_{f1} & \mathbb{C}_{f2} \\ \mathbb{E}_{f3} & \mathbb{E}_{f4} & \mathbb{E}_{f1} & \mathbb{E}_{f2} \end{bmatrix} a = \begin{bmatrix} \Theta_{ss} & \Theta_{sf} \\ \Theta_{fs} & \Theta_{ff} \end{bmatrix} a = \Theta a \ . \tag{3}$$

By the above relations the state vectors and integral constants in equivalent mono-phase in the interface are defined by the spatial averaging of individual quantities, as follows;

$$S^* = \begin{bmatrix} \mathbb{I}\beta_s & 0 \\ 0 & \mathbb{I} \end{bmatrix} S_s + \begin{bmatrix} \mathbb{I}\beta_f & 0 \\ 0 & \mathbb{I} \end{bmatrix} S_f \ , \quad a^* = \begin{bmatrix} \mathbb{I}\beta_s & 0 \\ 0 & \mathbb{I} \end{bmatrix} a_s + \begin{bmatrix} \mathbb{I}\beta_f & 0 \\ 0 & \mathbb{I} \end{bmatrix} a_f \tag{4}$$

or

$$S^* = \mathbb{B} S \ , \quad a^* = \mathbb{B} a \tag{4}$$

in abbreviated form. Assuming the inverse relation to Eq.(4)' , and denoting the inversal matrix, corresponding the inverse matrix of $\mathbb{B}$ by $\mathbb{B}'$ , the following relation is obtained.

$$S = \mathbb{B}' S^* \ , \qquad a = \mathbb{B}' a^* \ . \tag{5}$$

The matrices $\mathbb{Q}$ combining the state vectors with integral constants are spatially averaged in the form

$$\mathbb{Q}^* = \mathbb{B}\ \mathbb{Q}\ \mathbb{B}' \ . \tag{6}$$

The boundary conditions in free surface and base with respect to $a$ are conventionally given by the following relation;

$$\begin{aligned} (a_1 \ . \ a_2) &= (1, 0) a_1 \\ a_1 + i a_2 &= 2i \end{aligned} \ . \tag{7}$$

The magnification factor M(ω) on surface response in frequency to the base incident wave are obtained,

$$M(\omega) = 1/|q_1 - i q_2| \tag{8}$$

where

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \sum_{\substack{\text{layer} \\ i}} T_i \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \sum_i Q^{-1}_{(x_{i-1}=0)} Q_{(x_i=H_i)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \prod_i \begin{bmatrix} \cos K_i H_i & \sin K_i H_i \\ \alpha_i \sin K_i H_i & \alpha_i \cos K_i H_i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} . \tag{9}$$

The frequency response by FEM is carried out by discretizing the governing equation (1) and considering the radiation damping in the base boundary. The detailed discussion are given elsewhere [1] .

INTERACTION PARAMETER B

The numerical example is shown in Fig. 2. The water inclusion in soil layer and the change of the water level are examined. The important parameter B in analysis, which is the function of permeable constant $k$ , porosity $\beta_f$ , density $\rho_f$ and gravity acceleration $g$, expressed by $B = \rho_f \beta_f^2 g/k$ This interaction parameter is assumed to be 0 ~ 100 ton $*$ sec/m$^4$ Other material constants in ground are shown in table 1.

RESULTS AND DISCUSSIONS

*FREE VIBRATION*

By substituting the time harmonic solution $U = Ue^{i\omega t}$ into the fundamental FEM equation without external forces, the complex eigen frequency $\omega = Re(\omega) + iIm(\omega)$ as an index of dynamic properties of the saturated soil layer is obtained. The real part $Re(\omega)$ is dependent on the rigidity of the ground and the imaginary part $Im(\omega)$ on the damping properties. The vibrational characteristics are discussed by two part of eigen values.

*Transverse wave motion (S wave)* The relation between $Re(\omega)$ and $Im(\omega)/Re(\omega)$ in the first mode of shear wave motion is shown in Fig. 3a. The existence of water in the ground exhibits the decrease of stiffness and the damping action. The uppering of the water level in the ground i.e. the increase of saturated soil region promotes these properties. The change of permeability in a certain water level shows the mutual action of deterioration and damping. The right point of $Re(\omega)$ axis (B = 0) is the natural frequency of dry soil layer with no damping. As the soil layer contains the water, the relative motion of the soil skeletons and water in pore, results in the decrease of stiffness and damping in the ground. The interaction by relative motion of two phases is dependent on the permeability and relative velocity and then the damping action is remarkable in a certain value of B , and the relative motion is decreased until the soil and water behave simultaneously. This motion is shown in the left side point on $Re(\omega)$ axis.

*Longitudinal wave motion (P wave)* In two types of P wave motion in saturated soil layer, the result of the P2 branch propagated through soil skeleton is shown in Fig. 3b. The saturated soil layer in P wave motion also shows the deterioration. Increasing the coupling parameter B, the damping action is considerable. The interaction between two-phases is more complex than that of the S wave as later.

*DISPERSION RELATION*

The frequency equation is obtained by substituting the time harmonic function into the governing equation (1). The result of frequency equation gives the relation of phase velocities and attenuation coefficient in wave motion depending on frequency. The phase velocity in mixture ground is dependent on frequency as shown in Fig. 4a. There are upper and lower limit of phase velocities of each P and S wave motion. The attenuation coefficient $Im(\omega)$ defined by the imaginary part of solution of frequency equation is show- in Fig. 4b.

*FREQUENCY RESPONSE BY THEORY OF WAVE PROPAGATION*

The wave impedance ratio of saturated soil layer to elastic base is obtained by relation; $\alpha^{*}=\beta_s\mu K/(\mu K)_{base}=(\beta_s\mu/V)/(\rho V)_{base}$ in S wave, $\alpha^{*}=(\beta_s(\lambda+2\mu)+\beta_f r/V)/(\rho V)_{base}$ in P1 wave and $\alpha^{*}=(\beta_s(\lambda+2\mu)/V)/(\rho V)_{base}$ in P2 wave, respectively. These $\alpha^{*}$'s are frequency dependent. Frequency response analysis needs the imedance ratio.

*S wave* The frequency response on surface motion in shear wave propagation, is shown in Fig. 5 in variable water level and three typical interaction parameter B = 1, 10 and 100. Uppering the ground water level in small coupling parameter B gives the monotonous decrease of amplitude response and gradual decrease of resonant frequency. This effect is remarkable in large B. The water in soil layer decreases the stiffness of the ground and gives the damping action itself.

*P wave* The magnification factor in various ground water level in a moderate coupling parameter B are shown in Fig. 6. The effect of water inclusion exhibits the increase of stiffness in the ground in P1 branch response, and the remarkable damping action in P2 branch. The response of P1 branch does not decrease the magnification.

*FREQUENCY RESPONSE BY FEM*

Similiar response analysis are carried out by FEM. The S wave response good agreement with the results of wave propagation method shown in Fig. 5, as the solid phase branch in former case one-to-one-corresponds that of FEM. The frequency response of P wave by FEM has no branch. The low frequency response corresponds the P2 branch by wave propagation method. Results in variable water level and three interaction parameter B are shown in Fig. 7. In P wave motion, the stiffness and damping by water inclusion, is increased. The increasing stiffness action in P wave is contrary to results in S wave. In large parameter B, this effect is remarkable.

## CONCLUDING REMARKS

The purpose of this paper is to discuss the influence of the microstructure of soil layer to its dynamic properties and especially the influence of water level of soil layer to frequency response and eigen properties. By applying the bi-mixture theory to fluid saturated elastic soil layer, basic properties of a layer, are obtained and discussed. The wave propagation method in stratified soils is extended to complex layer with multi-phase medium and FEM in two-phase medium are simultaneously used. Some dynamic properties in mixture soil layers are clarified. Results obtained are summarized as follows.

(1) Saturated soil layer exhibits the damping and attracting action between two-phases under transverse wave motion, whereas predominant in the damping properties under longitudinal motion.

(2) In permeable soil ground, the uppering of the ground water level in the soil layer indicates the monotonous decrease in amplitude characteristics and the variable change of ground rigidity. The impermeable layer does not exhibit the monotonous deterioration and damping, and cancels the dispersive properties by unsaturated soil layer. This phenomenon is remarkable in longitudinal motion.

(3) The proposed method of wave propagation, in the stratified layer with bi-phased media, is fairly good agreement with frequency response analysis by FEM. Therefore the theory is valid in the application of two-phase medium.

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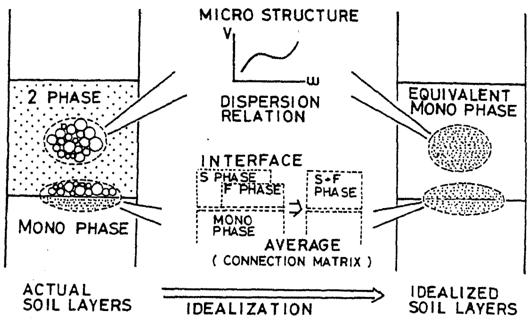


Fig.1 Idealization

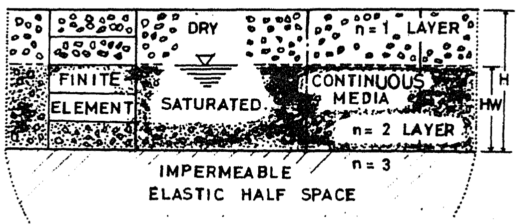


Fig.2 Analytical Model

Table 1 Material Constants

| | | DIMENSION | 2-PHASE MEDIA SOLID | 2-PHASE MEDIA FLUID | 2-PHASE MEDIA COUPLE | ROCK ROCK |
|---|---|---|---|---|---|---|
| WEIGHT DENSITY | | ton/m$^3$ | 2.7 | 1.0 | 0.0 | 2.7 |
| YOUNG MODULUS | E | ton/m$^2$ | 1500.0 | | | |
| POISSON-RATIO | | | 0.4 | | | |
| MODULUS OF VOLUME EXPANSION | R | ton/m$^2$ | | 2.137x10$^5$ | | |
| MODULUS OF RESTORING COUPLE | Q | ton/m$^2$ | | | 0.0 | |
| COEFFICIENT OF DISSIPATION | B | ton.sec/m$^4$ | | | 0~1000 | |
| WAVE VELOCITY S-WAVE P-WAVE | V | m/sec | 44.1 108.0 | | | 362 628 |
| LENGTH OF LAYER | H | m | 5 | | | half space |
| $\beta_s = \beta_f = 1$ | | | | | | |

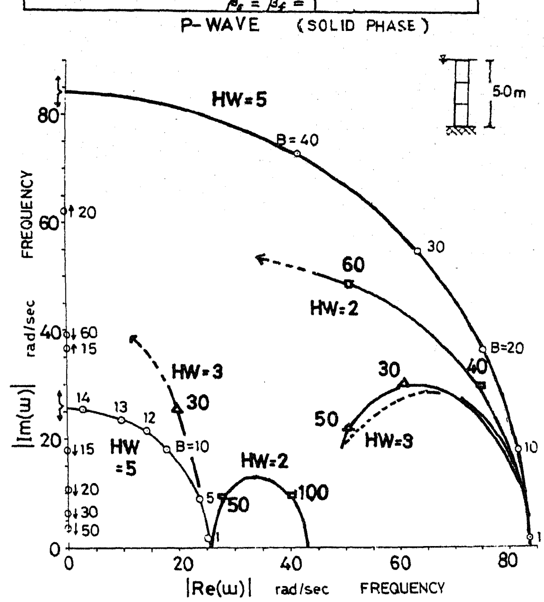


Fig.3b Natural Frequency (P-wave)

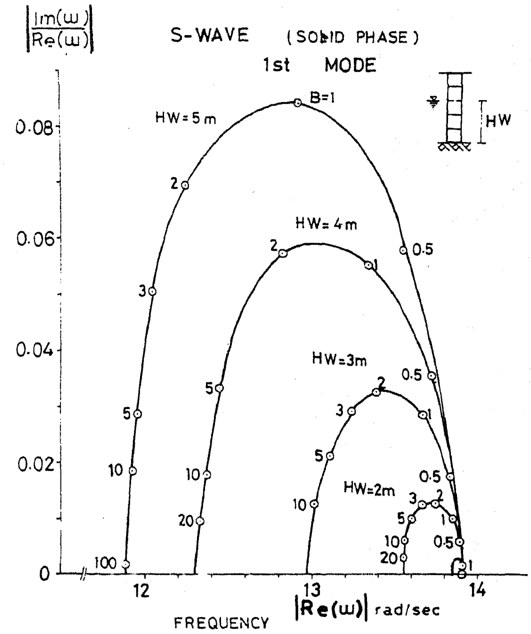


Fig.3a Natural Frequency (S-wave)

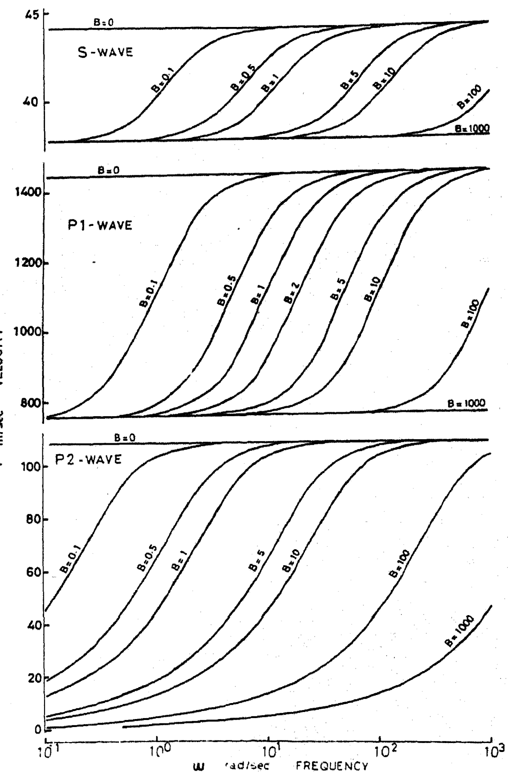


Fig.4a Phase Velocities

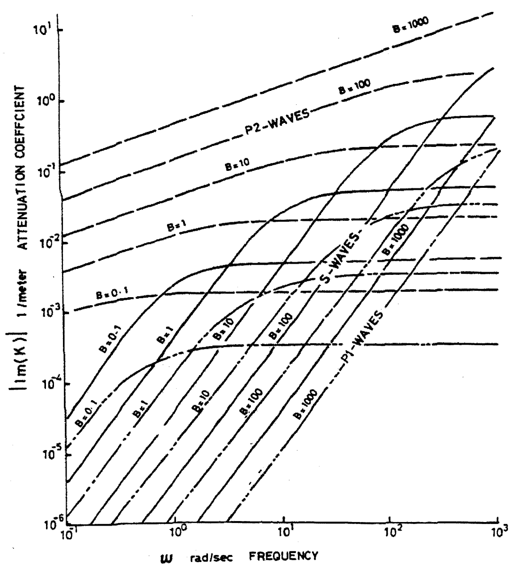


Fig.4b Attenuation Coefficients

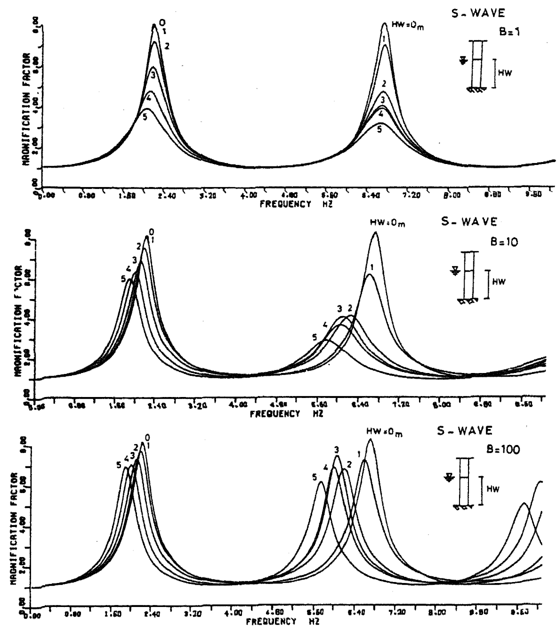


Fig.5 Magnification Factor (S-wave)

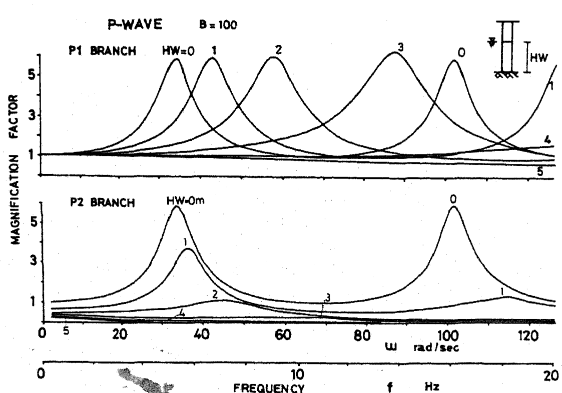


Fig.6 Magnification Factor (P-wave)

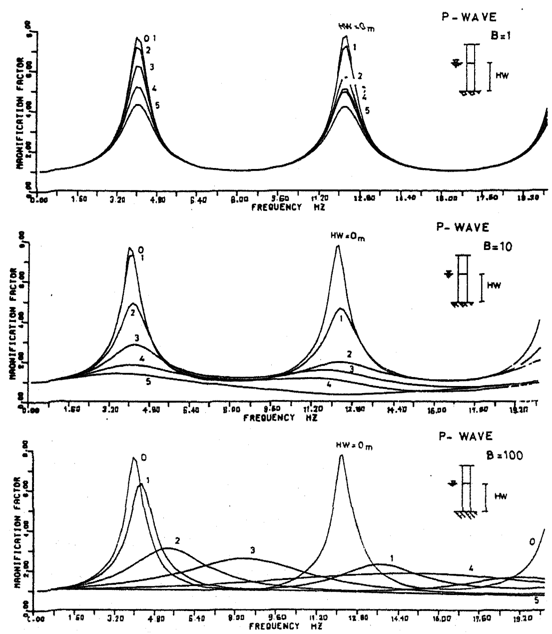


Fig.7 Magnification Factor (P-wave) by FEM