A THREE-SPHERICAL MODEL : AN APPROACH TO THE DYNAMIC STUDY OF REGULAR COHERENT SOIL.

by

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SYNOPSIS

A coherent soil constituted by elements whose dimensions aren't very variable is considered; idealizing this soil in a set of spherical elements, joined by elasto-plastic braces, a very acceptable Mohr's curve is drawn. The model permits to obtain the disgregation bond between the frequency and the amplitude of a sinusoidal given motion, i. e. the conditions under which the cohesion disappears.

1) Let us consider a soil constituted by elements whose dimensions aren't very variable, bed of gravel, or sand, or lapillus. This soil can be idealized in a set of spherical elements having the same radius R and the same unit weight, and placed touching one another in horizontal strata. The center C of the generical sphere (fig.1), and the centers A and B of the spheres on which it rests, must determine a vertical plane. Therefore the model becomes the set of spheres contained between two parallel vertical planes, whose distance is 2 R; the cartesian orthogonal axes x y z are fixed so that y is vertical and downward directed, and the centers of the spheres are contained in the plane y z. The static range can be examined fixing the spheres A and B, and charging the sphere C by a radial vertical load N and by a radial horizontal load T parallel to the z axis. The system is plane, and the C point is obliged on two circular lines, whose radius is R, and centers are B if $\varphi > 0$, A if $\psi > 0$; we have a one freedom's degree system, but the line on which C is obliged isn't regular in $\varphi = \psi = 0$. Let N be fixed, and T increasing from zero in R; the surfaces are smooth, the sphere C doesn't move until

$$-TR \cos 30° + NR \operatorname{sen} 30° \geq 0 \tag{1}$$

that is until $T \leq N$ tg 30°. If T = N th 30°, the position $\varphi = 0$ is a position of equilibrium, but it is an unstable one; if a little displacement $d\varphi$ is induced, the sphere C steps over the sphere B and all the spheres of the same stratum. According to that, we can write $\sigma = \frac{N}{4R^2}$, $\tau = \frac{T}{4R^2}$ the (1) leads to the well known Coulomb's bilatera (fig.2); the diagram τ, γ, for a fixed value of σ , ahows a classic rigid-plastic behaviour (fig.3).

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2) The cohesion can be simulated by two elastic braces joining the points A C and B C; the behaviour of the braces is linearly elastic if $\varepsilon \in [0, \varepsilon_\ell[$ $(F = k\,\Delta\ell)$, and fully plastic if $\varepsilon \in [\varepsilon_\ell, \rightarrow[$. For $\varphi \in [0, 60°]$ we can write referring to the point C,

$$
\begin{aligned}
v &= -2R\,[0{,}5\,\mathrm{sen}\,\varphi - 0{,}866(1-\cos\varphi)] \\
w &= 2R\,[0{,}5(1-\cos\varphi) + 0{,}866\,\mathrm{sen}\,\varphi] \\
\Delta\ell &= AC' - AC = R\left\{[8(1-0{,}5\cos\varphi + 0.866\,\mathrm{sen}\,\varphi)]^{1/2} - 2\right\}
\end{aligned}
\tag{2}
$$

the potential total energy is

$$E = \frac{k}{2}(\Delta\ell)^2 - Nv - Tw\,;$$

the equilibrium equation $\frac{dE}{d\varphi} = 0$ gives

$$N = \frac{0{,}5\cos\varphi - 0.866\,\mathrm{sen}\,\varphi}{0{,}5\,\mathrm{sen}\,\varphi + 0{,}866\cos\varphi} + 2kR\left(1 - \frac{1}{[2(1-0{,}5\cos\varphi + 0{,}866\,\mathrm{sen}\,\varphi)]^{1/2}}\right) \tag{3}$$

The mapping $\varphi \rightarrow T$ is defined in $]\,0\,,\,\varphi_\ell\,[$, when φ_ℓ is the value of φ that we can calculate from the (2) for $\varepsilon = \varepsilon_\ell$

$$0{,}5\cos\varphi_\ell - 0{,}866\,\mathrm{sen}\,\varphi_\ell = 1 - 0{,}5\,(\varepsilon_\ell + 1)^2 \tag{4}$$

For exemple, we have $\varepsilon_\ell = 0.2 \rightarrow \varphi_\ell = 13°.5$.

When $\frac{dT}{d\varphi} < 0$, the equilibrium is unstable; so the limit value T_0 is T (o) if the mapping is strictly decreasing, T (φ_ℓ) if the mapping is strictly increasing, T (φ_m) if $\left(\frac{dT}{d\varphi}\right)_{\varphi=\varphi_m} = 0$. In fig.4 two diagrams N T are given, for two values of ε_ℓ ; we can observe that these diagrams have the well known look of the Mohr's curves for coherent soils; while proceeding along the σ axis, the diagrams tend asymptotically to the Mohr's curve in absence of cohesion, and that looks natural enough. The (3) gives, for N = 0,

$$T_0 = 4R^2c = 2kR\left(1 - \frac{1}{[2(1 - 0.5\cos\varphi_\ell + 0.866\,\varphi_\ell)]^{1/2}}\right)$$

that is the bond between C and k; for ε_ℓ = 0,2, we have k = 12,17 C R. Generally, we can write

$$k = \alpha\, c\, R \tag{5}$$

3) In order to simulate the earthquake and to value its effects on the cohesion, let us study the sketch of fig. 5, where the three-spherical model lies on a floating table. As all the mass is involved by the earthquake, we must suppose that if $\varphi \neq 0$ the load transmitted to sphere C by the upper strata acts along the straight line AC' ($\varphi < 0$) or BC' ($\varphi > 0$). Therefore, the potential energy of N doesn't vary during the motion. Let the motion of the table be given by

$$w_p = \eta\,\mathrm{sen}\,\omega_p t\,; \tag{6}$$

the motion of the point C is given by

$$v = R(-\varphi + 0.866\,\varphi^2)$$

$$w = R(1.732\,\varphi + 0.5\,\varphi^2) + \eta \,\mathrm{sen}\, \omega_p t$$

from which

$$\dot v = -R\dot\varphi + 1.732\,\varphi\dot\varphi$$

$$\dot w = 1.732\,R\dot\varphi + \varphi\dot\varphi + \eta\,\omega_p \cos\omega_p t.$$

Neglecting the terms of higher order, we can write

$$\mathcal{L} = C - E = m/2\,(4R^2\dot\varphi^2 + \eta^2\omega_p^2\cos^2\omega_p t + 1.732\,R\eta\omega_p\varphi\cos\omega_p t) - 1,5K\varphi^2R^2$$

Hamilton's equation gives

(7) $$4R^2m\ddot\varphi + 3KR^2\varphi = 0.866\,Rm\eta\,\omega_p^2\,\mathrm{sen}\,\omega_p t,$$

that is

(8) $$\ddot\varphi + \omega^2\varphi = a\,\mathrm{sen}\,\omega_p t$$

where

(9) $$\omega^2 = 0.75\;K/m$$

(10) $$a = 0.2165\;\eta\,\frac{\omega_p^2}{R}$$

The general integer of the (8) is

$$\varphi = A\,\mathrm{sen}\,\omega t + B\cos\omega t + \frac{a}{\omega^2-\omega_p^2}\,\mathrm{sen}\,\omega_p t;$$

the initial conditions $t = 0 \rightarrow \varphi = \dot\varphi = 0$ allow to write

(11) $$\varphi = -\frac{a}{\omega^2-\omega_p^2}\left(\frac{\omega_p}{\omega}\,\mathrm{sen}\,\omega t - \mathrm{sen}\,\omega_p t\right).$$

From the (5), and

$$m = \frac{\gamma}{g}\,4/3\,\pi R^3$$

where γ is the unit weight, one derives

(12) $$\omega^2 = 0.179\,\frac{\alpha c g}{\gamma R^2}$$

Since φ can reach the value $\frac{a}{\omega^2-\omega_p^2}$, imposing

(13) $$\frac{a}{\omega^2-\omega_p^2} = \varphi_e$$

one obtains the <u>disgregation bond</u> between η and ω_p ; this bond, from the

(10) and the (12), is

$$\varphi_e = 0.2165 \frac{\eta \omega_p^2}{R} \frac{1}{0.179 \frac{\alpha c g}{\gamma R^2} - \omega_p^2}$$

from which

(14) $$\eta = \varphi_e \frac{0.179 \alpha c g - \gamma R^2 \omega_p^2}{0.2165 \gamma R \omega_p^2}$$

For ε_e = 0,2, we have φ_e = 0,236 rad, α = 12,17; in the table the graph of the mapping $\omega_p \rightarrow f$, for the following values of the soil's physical parameters

c = 0,1 Kg cm^{-2}
γ = 0,0016 Kg cm^{-3}
R = 1 cm

and for g = 981 cm sec^{-2}, is related.

f_p (sec^{-1})	ω_p (sec^{-1})	η (cm
1	6,28	3686,88
2	12,57	920,90
5	31,41	146,42
10	62,83	37,78
50	314,15	0,38
	365,46	0

We observe immediately that the principal wave of the earthquake cannat influence the cohesion, because its amplitude is too small to satisfy the (14). On the contrary, the danger rises from the micro earthquake. It should be advisable, if one ought to be afraid of this phenomenon, don't rely on all the cohesion.

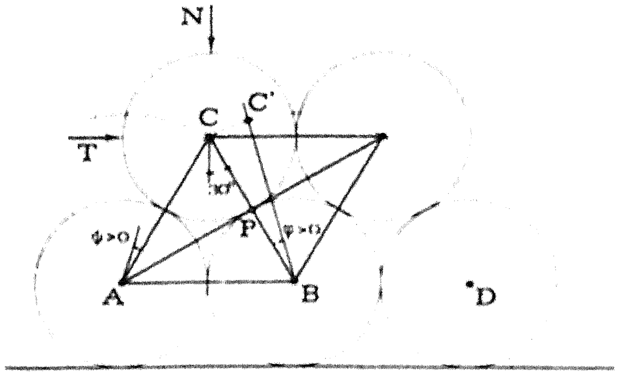

fig 1

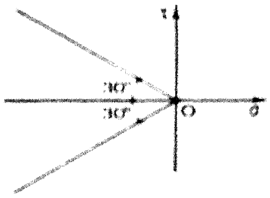

fig 2

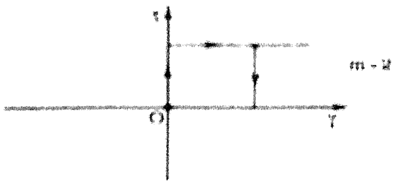

fig 3

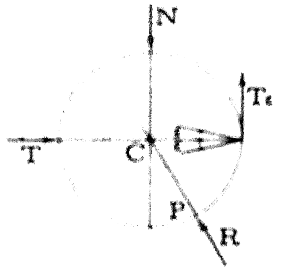

fig 4

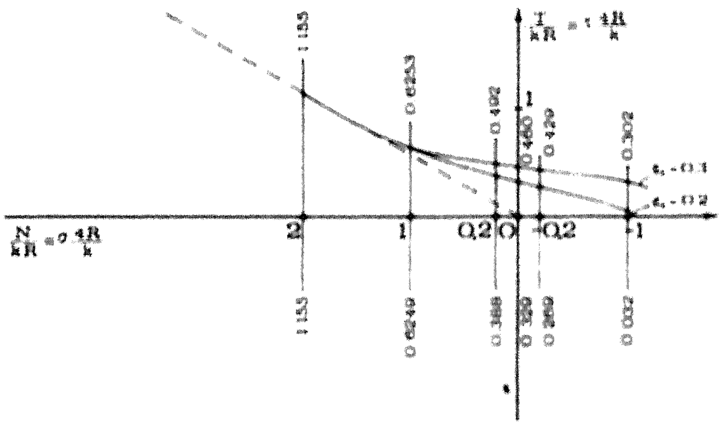

fig 5

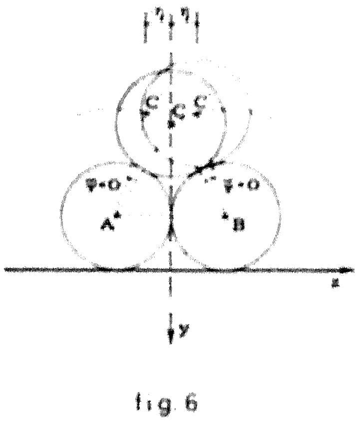

fig 6