BOUNDARY CONDITIONS IN SOIL AMPLIFICATION STUDIES

by

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SYNOPSIS

Usefulness and limitations of the different boundary conditions frequently used in numerical wave propagation formulations are discussed. New approximate boundary conditions for numerical determination of the response of earth masses and structures to earthquakes are proposed. Their formulation is general, applicable in one, two or three dimensional problems. Model requires definition of excitation in terms of a front of plane seismic waves. A modified earthquake simulation algorithm, which includes source parameters, is recommended for the purpose. Computer applications of the theory are presented using a two dimensional finite element programme.

INTRODUCTION

Determination of the earthquake response of soil deposits using discrete methods of solution requires the definition of fictitious boundaries with conditions that minimize or prevent spurious reflections. This problem has been studied for several years[2, 3, 5, 6]. Analytical techniques used have generally been restricted to the absorption of outgoing waves generated within the discrete model, applying the excitation at an infinitely rigid bedrock and assuming that the seismic motion at this level is produced by shear waves propagating upwards.

The purpose of this paper is to define conditions at boundaries, here called active, which allow free transmission of waves across the boundaries as it would occur if the discrete domain were continuous (fig 1). In the formulation the restrictions of rigid bedrock and shear wave excitation are automatically removed. The general procedure of including these boundary conditions in a finite element model is introduced using the principle of virtual displacements which is valid for both linear and nonlinear soil models and requires the assumption of linear elasticity only in the vicinity of the active boundaries. Then follows a discussion of the definition of the seismic waves required by the model. The effectiveness of the boundary conditions is illustrated by numerical examples. Finally recommendations are given for the use of the model proposed.

ACTIVE BOUNDARIES FORMULATION

Consider for simplicity the one dimensional wave equation

$$\frac{\partial^2 u_2}{\partial x_1^2} = \frac{1}{C_s^2} \frac{\partial^2 u_2}{\partial t^2} \tag{1}$$

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where u_2 = the vertical component of the displacement field u_i*, x_1 = horizontal coordinate, t = time and C_s = shear wave velocity.

The general solution of eq 1 may be written as

$$u_2 = f_2 (x_1 - C_s t) + F_2 (x_1 + C_s t) \quad (2)$$

where f_2 and F_2 are any differentiable functions of their arguments.

The physical interpretation of eq 2 is that f_2 is a disturbance travelling with speed C_s in the positive direction of the x_1 axis (fig 2) and F_2 a disturbance moving in the negative direction. This suggests that if u_2 represents the displacement at point $x_1 = a$ produced by an earthquake, then f_2 is the part of u_2 transmitted into the region of $x_1 > a$ – seismic waves, and F_2 is the part that leaves it – reflected waves.

Thus considering a fictitious boundary at point $x_1 = a$, it is always possible to find boundary conditions that allow the free passage of waves. This fact leads to the definition of a semi-infinite domain, $x_1 > a$ which models the behaviour of the original infinite one.

To derive these boundary conditions, consider the time derivative of displacement $\dot{u}_2$ as

$$\dot{u}_2 = -C_s f_2' + C_s F_2' \quad (3)$$

where the prime represents the derivative of a function with respect to its argument.

According to the theory of linear elasticity of shear stress σ_{12} is given by

$$\sigma_{12} = G \frac{\partial u_2}{\partial x_1} \quad (4)$$

where G = shear modulus of the medium and

$$\frac{\partial u_2}{\partial x_1} = f_2' + F_2' \quad (5)$$

Combining eqs 3 and 4 it follows that the shear stress in a medium with mass density ρ is

$$\sigma_{12} = -\rho C_s (2 \dot{f}_2 - \dot{u}_2) \quad (6)$$

which is equivalent to the vertical external traction

$$t_2 = \rho C_s (2 \dot{f}_2 - \dot{u}_2) \quad (7)$$

which in turn defines the active boundary condition for a one dimensional shear wave propagation problem.

The foregoing formulation can be easily extended to body waves of the compressional type, so that the horizontal traction associated with a volumetric disturbance travelling along the horizontal axis is

$$t_1 = \rho C_p (2 \dot{f}_1 - \dot{u}_1) \quad (8)$$

* Index notation is used with variation one to three unless otherwise stated.

Restricting the domain of definition of eq 13 to a finite element, the displacement vector may be approximated as

$$u_i = \phi_{ik} \; U_k \qquad \text{where} \qquad k = 1, 2, \ldots, N \tag{14}$$

and the seismic excitation as

$$f_i = \phi_{ik} \; U_k^r \qquad \text{where} \qquad k = 1, 2, \ldots, N \tag{15}$$

where $\phi_{ik} = \phi_{ik}(x_i)$ are the modal or interpolation functions, $U_k = U_k(t)$ the N generalized coordinates – unknown nodal displacements, and U_k^r is the seismic excitation vector whose non-zero components correspond to nodal points contained in an active boundary.

From eqs 13-15 follows a matrix equation of the form[1]

$$k_{lm} \; U_l + m_{lm} \; \ddot{U}_l = p_m \tag{16}$$

where k_{lm} and m_{lm} are the stiffness and mass matrices of the element respectively and p_m is the load vector.

The load vector for the particular case of surface tractions given by eqs 11 and 12 is

$$p_m = \int_S \rho \, [C_p \, (2\dot{U}_\ell^r - \dot{U}_\ell) \, \phi_{1\ell} \, \phi_{1m} + C_s \, (2\dot{U}_\ell^r - \dot{U}_\ell) \, \phi_{i\ell} \, \phi_{im}] \, dS + \int_V B_j \phi_{jm} \, dV$$

$$\text{where} \quad i = 2, 3 \qquad \ell, m = 1, \ldots, N \tag{17}$$

As for nonlinear problems the use of active boundaries is also direct inasmuch as the domain is linear elastic in the vicinity of these boundaries, but the finite element formulation has to be made incremental with linear properties in each increment.

NUMERICAL EXAMPLES

To show the effect of boundaries in the numerical solution of wave propagation problems two examples are presented. First, a one dimensional model whose shear wave excitation is a horizontal pulse (fig 3) applied at base level. Second, a two dimensional soil structure subjected to a similar excitation.

In fig 4 a comparison is made between the response of a model with active boundary and another with rigid bedrock. The result of the comparison is conclusive. Infinitely rigid bedrock supplies an ever increasing amount of energy into the system whereas active boundaries transmit approximately the correct amount of the energy back to the source.

A comparison between the responses at different points of the two dimensional model (fig 5) with rigid and active boundaries is shown in fig 6. Once again the effect of the restrained energy in the system is present.

CONCLUSIONS

The feasibility of modelling a semi-infinite soil deposit by a finite one with active boundaries was illustrated in this paper. The formulation is approximate inasmuch as it is based on the assumption of plane waves for which the propagation direction is known. The hypothesis of plane waves is only valid at points sufficiently distant from the source[2]. Furthermore, although it is always possible to evaluate the direction of the waves inside the active boundaries, very little is known about the actual arrival of seismic waves; hence the assumption of normal incidence at active boundaries may lead to erroneous results, i.e., for incident angles less than 30°.[3]

where C_p is the velocity of compressional wave.

Generalising the previous concepts for a front of plane body waves of shear and/or compression type advancing along a line defined by its direction cosines n_i, the boundary forces are

$$\bar{t}_1 = \rho\, C_p\, (2\dot{\bar{f}}_1 - \dot{\bar{u}}_1) \tag{9}$$

$$\bar{t}_i = \rho\, C_s\, (2\dot{\bar{f}}_i - \dot{\bar{u}}_i) \qquad \text{where } i = 2, 3 \tag{10}$$

equivalent to eqs 7 and 8 with all the variables written in bar form referred to a system of axes in which the x_1 axis coincides with the direction of propagation.

In general the direction of a front of body waves is not known so it is not possible to develop exact active boundary conditions. An approximation, which generally allows the transmission of waves well within the required engineering accuracy, is to consider that the wave front arrives normal to the boundary,[3] e.g.,

$$t_1 = \rho\, C_p\, (2\dot{f}_1 - \dot{u}_1) \tag{11}$$

$$t_i = \rho\, C_s\, (2\dot{f}_i - \dot{u}_i) \qquad \text{where } i = 2, 3 \tag{12}$$

are the forces at a boundary normal to the x_1 axis.

SEISMIC WAVES SIMULATION

The problem of defining the input motion in soil amplification studies is indeed an important one. The assumption that seismic waves arriving at a bedrock surface are produced essentially by shear waves propagating upwards covers only a limited range of applications. Earthquake motions are in general the result of a very complex wave transmission process dependent on local geology and on source parameters. The ideal way of solving this problem would be to use strong motion records made upon rock at sites of interest. Unfortunately most of the recorded earthquake motions have been made upon soil, i.e. records affected by local site conditions.

An earthquake simulation model has been devised, however, which includes in its formulation source mechanism effects[4] and can be used to generate seismic waves at a fictitious boundary of a soil deposit. Three of the components of an earthquake –two horizontal and one vertical, can be simulated at two different sites simultaneously. The excitation includes the contribution of body and surface waves as well as the effect of fault inclination.

FINITE ELEMENT MODEL

Inclusion of active boundary conditions in a general finite element formulation may be achieved using the principle of virtual displacements, which for a linear elastic body with elastic constants λ and G,can be stated as

$$\int_V [\lambda \frac{\partial u_k}{\partial x_k} \frac{\partial \delta u_i}{\partial x_i} + G\left(\frac{\partial u_i}{\partial x_k} \frac{\partial \delta u_i}{\partial x_k} + \frac{\partial u_i}{\partial x_k} \frac{\partial \delta u_k}{\partial x_i}\right) + \rho \ddot{u}_i \delta u_i]\, dV =$$

$$= \int_S t_i \delta u_i dS + \int_V B_i \delta u_i dV \tag{13}$$

where V and S are the volume and the boundary of the body.respectively, B_i = body force vector and δ = first variation operator.

A way of improving the effectiveness of the active boundaries is to use in their vicinity a cylindrical wave approximation in terms of potentials. This formulation requires the evaluation of the curvatures and the direction of the wave front at each instant, which substantially increases the solution time of a finite element programme. An easier way of achieving a good approximation without increasing the computing effort is to estimate the general trend of the plane wave front, orienting the active boundaries normal to them. With few exceptions, the consideration of the free passage of waves in soil amplification studies is essential since it affects not only the magnitude of the ground response, but also its frequency content and duration.

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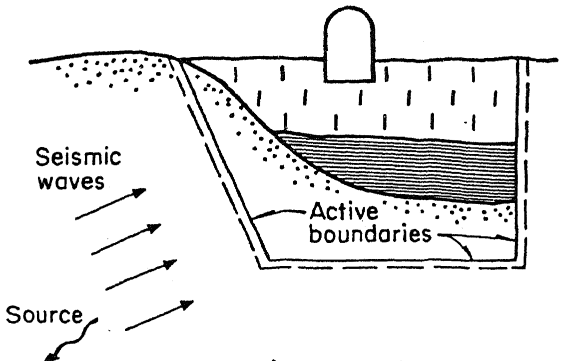

Fig 1. Active boundary

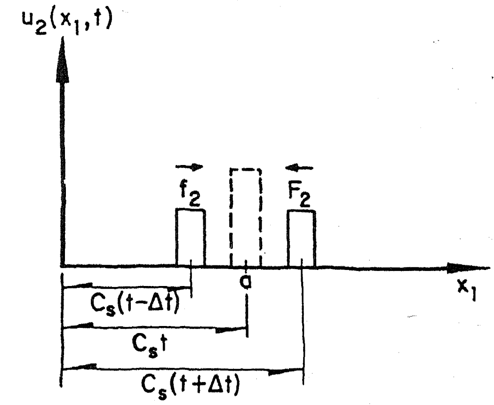

Fig 2. Shear waves propagating along x_1 axis

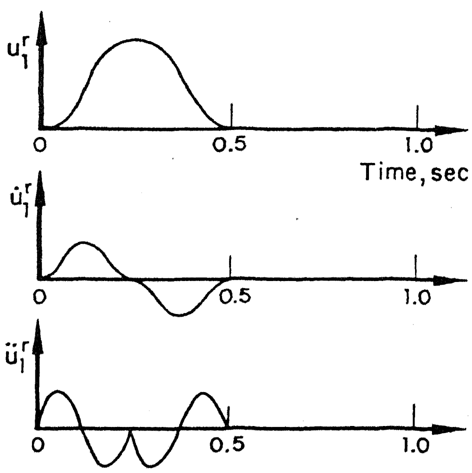

Fig 3. Excitation used in numerical examples

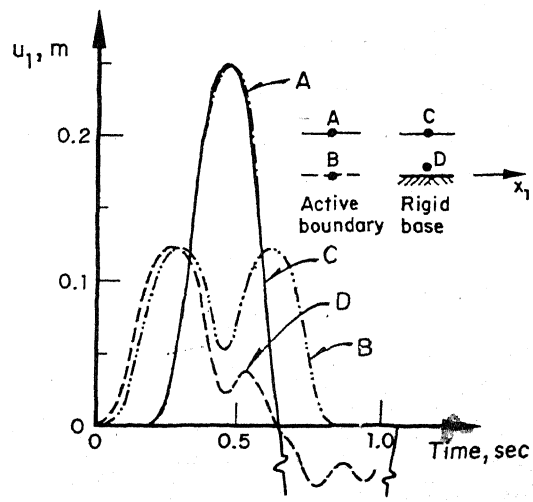

Fig 4. One dimensional response model

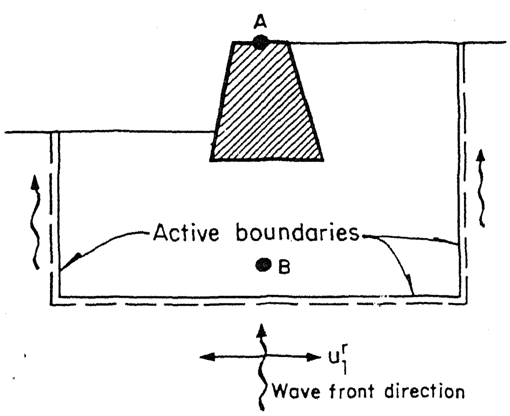

Fig 5. Two dimensional example

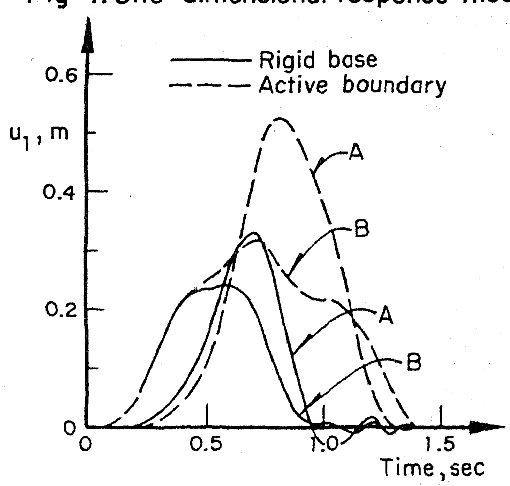

Fig 6. Two dimensional response model