ON THE STABILITY OF A WALL-CHECKED EMBANKEMENT SUBJECTED TO EARTHQUAKE

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SYNOPSIS

The three-spherical model of soil we refer to is represented in fig.1. It is fully explained in the paper presented by Professor V. Franciosi.

Now we want to verify the theoretical results as regards to the stability of a wall-checked embankement subjected to earthquake.

1) Let us consider a three-spherical model under an earthquake impulse, given in fig. 1 by the motion

$$\Delta w_p = \Delta\eta\,\mathrm{sen.}\,\omega_p t$$

of the sphere with center C whereas the two spheres centered A and B are fixed.

The field of relative motion, associated with seismic loads charging differently the set of the upper spheres and of the lower ones ; is considered. Each disgregation is conditioned by a gradient of the seismic impulse in perpendicular direction to the seismic loads. As the model is homogeneous and uniform the set spheres-loads is sketched in fig. 1 where the given direction of the seismic impulse coincides with the axis z.

Considering the motions of small amplitude we have the components v_c, w_c, of the displacement of C and the extention $\Delta l = AC - AC' = R(1.732\varphi - 0.25\varphi^2)$

$$1.1)\quad \begin{aligned} v_c &= R(-\varphi + 0.866\varphi^2) \\ w_c &= R(1.732\varphi + 0.5\varphi^2) + \eta\,\mathrm{sen}\,\omega_p t \end{aligned}$$

$\mathcal{L} = C - E$ (C = cynetic energy ; E = potential total energy) , neglecting the terms of higher order , can be written

$$1.2)\quad \begin{aligned} \mathcal{L} = C - E = m/2\,(4R^2\dot\varphi^2 + \Delta\eta^2\omega_p^2\cos^2\omega_p t + 1.732\,R\eta\,\omega_p\dot\varphi\cos\omega_p t) + \\ - 1.5\,K\varphi^2R^2 + NR(-\varphi + 0.866\varphi^2) + TR(1.732\varphi + 0.5\varphi^2) + \\ + T\eta\,\mathrm{sen}\,\omega_p t\,; \end{aligned}$$

Hamilton's equation gives

$$1.3)\quad \begin{aligned} 4R^2m\ddot\varphi + (3KR^2 - 1.732\,NR - TR)\varphi = \\ = 0.866\,mR\Delta\eta\,\omega_p^2\,\mathrm{sen}\,\omega_p t + T\Delta\eta\,\omega_p\cos\omega_p t - NR + 1.732\,TR \end{aligned}$$

for $m = \frac{\gamma}{g}\frac{4}{3}\pi R^3$, $K = \alpha\cdot c\cdot R$, $N = 4R^2\sigma$, $T = 4R^2\tau$ 1.3.becomes

$$1.3')\quad \ddot\varphi + \omega^2\varphi = a\,\mathrm{sen}\,\omega_p t + b\cos\omega_p t + d$$

where

$$\omega^2 = \frac{g}{\gamma R^2}(0.179\,\alpha c - 0.413\,\sigma - 0.239\,\tau)$$

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II) Where m = mass of the sphere centered C

K = rigidity of the elasto-plastic brace sketching the cohesion.

From which $K\Delta l$ = strain in the elasto-plastic brace associated with the deformation energy $1,5\,K\varphi^2R^2$

1.3'') if $0.179\,\alpha c - 0.413\,\sigma - 0.239\,\tau > 0$

$$\omega^2 = -\frac{g}{\gamma R^2}\left(0.179\,\alpha c - 0.413\,\sigma - 0.239\,\tau\right)$$

1.3''') if $0.179\,\alpha c - 0.413\,\sigma - 0.239\,\tau < 0$

we have

$$a = \frac{0.217}{k}\,\Delta\eta\,\omega_p^2 \qquad d = -\frac{g}{\gamma R^2}\left(0.239\,\sigma + 0.413\,\tau\right)$$

1.4) $b = 0.239\,\frac{g}{\gamma R^3}\,\tau\,\Delta\eta\,\omega_p$

The initial conditions being $t = 0$ $\varphi = 0$, $\dot\varphi = 0$, the general integer of the 1.3'') is

1.5) $\varphi = -\frac{1}{\omega^2 - \omega_p^2}\left[\frac{\omega_p}{\omega}\,a\left(\operatorname{sen}\omega t - \frac{\omega}{\omega_p}\operatorname{sen}\omega_p t\right) + b\left(\cos\omega t - \cos\omega_p t\right) + d\left(\cos\omega t - 1\right)\right]$

of 1.3''') is

1.6) $\varphi = \frac{1}{\omega^2 + \omega_p^2}\left[(b+d)\cosh\omega t + \frac{\omega_p}{\omega}\,a\operatorname{senh}\omega t - a\operatorname{sen}\omega_p t - b\cos\omega_p t - d\right]$

We calculate the amplitude bonds φ_0 and the characteristic periods T_c with the conditions $t = T_c/4 \rightarrow \dot\varphi = 0$ and we have

1.7) $a\left(\cos\omega T_c/4 - \cos\omega_p T_c/4\right) - b\left(\omega\operatorname{sen}\omega T_c/4 - \omega_p\operatorname{sen}\omega_p T_c/4\right) + {} - d\,\omega\operatorname{sen}\omega T_c/4 = 0$

2) Referring to fig. 1 we write the equilibrium equation to the rotation round B .

2.1) $\left(N + m\ddot v_c\right)\cos(60° + \varphi) - \left(T - m\ddot w_c\right)\operatorname{sen}(60° + \varphi) + {} + K\Delta l\operatorname{sen}(60° - \varphi/2) = 0$

Considering the motions of small amplitude and neglecting the terms of higher order, the 2.1) becomes

2.2)
$$\tau = \sigma\,\frac{0.5 - 0.866\,\varphi}{0.866 + 0.5\,\varphi} + 0.433\,\alpha c\varphi\,\frac{0.866 - 0.25\,\varphi}{0.866 + 0.5\,\varphi} + {} + \frac{\gamma}{g}\frac{\pi}{3}R^2\ddot\varphi\left[\left(-1 + 1.732\,\varphi\right)\frac{0.5 - 0.866\,\varphi}{0.866 + 0.5\,\varphi} + 1.732 + \varphi\right] + {} + \frac{\gamma}{g}\frac{\pi}{3}R\,\frac{\omega_p^2\,\Delta\eta\operatorname{sen}\omega_p t}{0.866 + 0.5\,\varphi}$$

With no motion and having $\varphi = \dot{\varphi} = \ddot{\varphi} = 0$ the 2.2) becomes

$$\tau = \sigma \, tg 30^\circ + \frac{\gamma}{g} 1.209 R \omega_p^2 \Delta\eta \, sen \, \omega_p t$$

or

2.2') $$\tau = \sigma \, tg \Phi - \frac{\gamma}{g} 1.209 R \Delta\ddot{\eta}$$

where Φ is the corner of inside friction with no motion and $\Delta\ddot{\eta}$ is the seismic acceleration. Fixing $\varphi = 0$ there is no cohesion, as being $\Delta l = 0$ there is no traction load in the elasto-plastic brace. With no seismic impulse the 2.2') identifies with the well-known Coulomb's bilatera.

3) If we want to verify a wall-checked embankement according to Fellenius we must determine the rupture surface in order to calculate the smallest load multiplier K , which leads to

3.1) $$M_{rib} = M_{res}$$

where M_{rib} is the overturning bending and M_{res} is the resisting bending calculated referring to the center C of Fellenius ' curve.

The load multiplier loading to 3.1) is given by

3.2) $$K^* = \frac{M_{rib}(0) + M_{res}(0)}{M_{rib}(0) - M_{rib}(1) + M_{res}(0) - M_{res}(1)}$$

where M_{rib} and M_{res} are calculated for K = 0 and K = 1 . M_{rib} and M_{res} are:

3.3) $$M_{res} = \sum \tau_n \cdot \lambda \cdot R_o$$
$$M_{rib} = \sum (\gamma \cdot h + K p) \cdot \Delta \cdot d_i$$

where the $\sum$ refer to all the stripes dividing the embankement.

M_{res} and its corresponding value τ_n were calculated considering the real surface of rupture (tangent to Fellenius' curve).

τ_n is (fig.4)

3.4) $$\tau_n = \frac{\sigma_y \, tg \Phi - c}{1 + tg \Phi \, sen \, 2\beta + 2 \, tg^2 \Phi \, sen^2 \beta}$$

where

$$\sigma_y = -(\gamma h + K p)$$

the 3.4) of the limit tangential tension ; where Φ is the friction corner and C the cohesion of the soil, is obtained only if : a) the plane of rupture is tangent to Fellenius' curve (bent of β) ; b) the tensions σ and τ acting on the plane of rupture belong to Coulomb's bilatera ; c) the point representing the status of tension belongs to Mohr's curve tangent to Coulomb's bilatera.

The earthquake changes the mechanic characteristics of the soil and under certain circumstances the cohesion my be zero.

If $\varphi \neq 0$ for the 2.2) we have

$$3.5\ ) \qquad \Phi' = \operatorname{arctg} \frac{0.5 - 0.866\varphi}{0.866 + 0.5\varphi}$$

smaller than $\Phi' = 30°$ for $\varphi = 0$.

The earthquake produces a decreasing of the resisting bending M_{res} and of the load pultiplier K^*.

In order to determine K^* let α, c, R, γ, Φ indicate the physical and mechanic characteristics of the soil ; the characteristics of the seismic loads are the direction (β) , the amplitude of the gradient ($\Delta\eta$) the pulsation ω_p .

Referring to fig. 2 ; with Mohr's curve (fig. 3) with no earthquake , the values of σ and τ are calculated on the plane, for the point Q of Fellenius' curve, parallel to the direction of the earthquake.

Then the values of the amplitude bonds of the stripes are calculated as well as the characteristic periods T_c from the 1.5) or 1.6) for $t = T_c / 4$.

Let us verify $\varphi_o \leq \varphi_e$ where φ_e is the amplitude bond of the elasto-plastic brace.

The values of φ_o and $\ddot{\varphi}_o$ substituted in 2.2) give a new value of τ which substituted in 1.7) and 1.8) gives in second approximation new values of φ_o and T_c.

Consequently we obtain the values of the corner of inside friction and of the cohesion from

$$3.6\ ) \qquad \Phi' = \operatorname{arctg} \frac{0.5 - 0.866\varphi_o}{0.866 + 0.5\varphi_o}$$

$$c' = \tau - \sigma \operatorname{tg} \Phi' \qquad \text{(I)}$$

Substituting these values in 3.4) we have τ_n - on the plane tangent to the rupture surface-used to determine M_{res} .

(I) c' - cohesion in the presence of earthquake - is $\neq 0$ if $\varphi_o < \varphi_e$

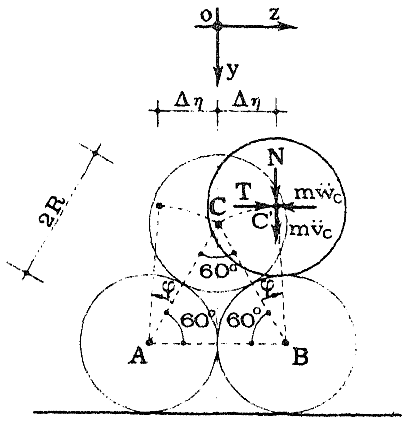


fig.1

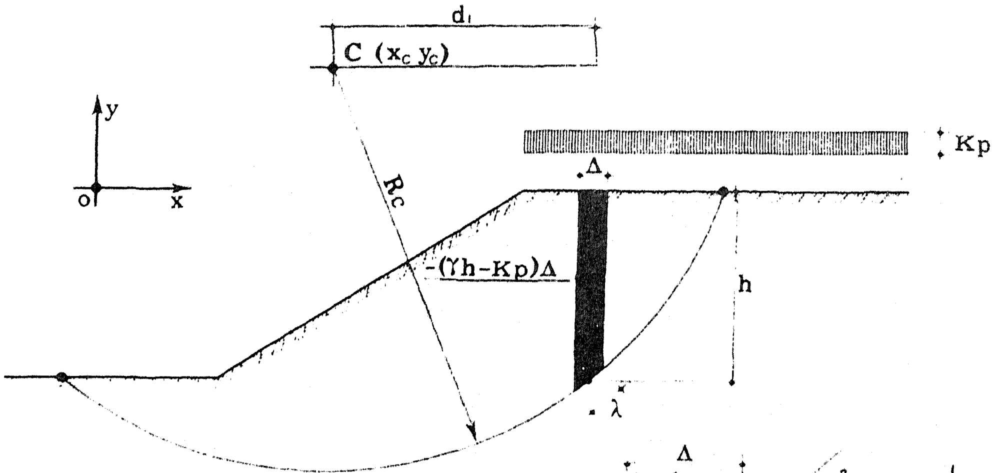
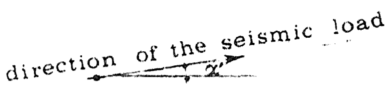


fig. 2



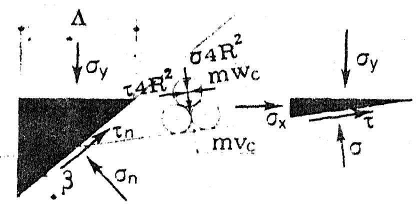


fig. 4