

# RESPONSE SPECTRA FOR OCEAN STRUCTURES

by

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## SYNOPSIS

The lateral forces caused by earthquakes on offshore structures include fluid-structure interaction effects in addition to structural inertia. The interaction forces include the added mass force, friction, and radiation damping. The study describes the effect of these additional forces on the response spectrum. Also presented are hydrodynamic parameter spectra for determining the kinematical quantities for use in selecting coefficients associated with these forces.

## INTRODUCTION

Response spectra are used widely in the design of land-based structures to resist earthquake motions. Response spectra are also useful in the design of offshore structures if adjustments are made to include the effects associated with fluid-structure interaction. Submerged portions of an offshore structure are subjected continuously to forces associated with water waves and currents. When the structure responds to earthquake excitation, additional fluid forces are generated by the motion of the structure with respect to the water. These forces include the "added" or "hydrodynamic" mass effects, additional drag and lift forces, and "wave making" or "radiation damping" forces. Analytical representation of these fluid-structure interaction forces usually includes empirical coefficients which depend upon basic dimensionless hydrodynamic parameters involving fluid properties, geometry of structural members, and kinematical quantities.

The studies described here were conducted to answer two related questions. First, what effects, if any, do the additional fluid-structure interaction forces have on the structural response spectrum? Second, what are the ranges of the basic dimensionless hydrodynamic parameters which occur during earthquakes? The latter question was intended to provide a basis for future experiments since available experimental data for the coefficients are not very extensive. The approach used in the study was to compute various response spectra which include the fluid-structure interaction terms in the equation of motion of the oscillator. The results of the computations are used to develop approximate methods of computing response and other quantities appropriate to hydrodynamic parameters. The excitations used were various earthquake records as well as regular patterns of base motion.

## FLUID-STRUCTURE INTERACTION FORCES

Structures of interest here primarily are "jacket" or "template" structures used for offshore production platforms. They are space frames

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constructed of tubular members that have small diameters compared to the depths of water involved. Accordingly, fluid-structure interaction forces are computed from Morison's formula; a formula developed originally for static structures loaded by ocean waves. If Morison's equation is used for computing fluid-structure interaction effects, it may be written as follows when waves and currents are ignored

$$F = + K_D \dot{x} |\dot{x}| + K_m \ddot{x} + K_r \dot{x} \quad (1)$$

where  $F$  is a fluid-structure interaction force acting on part of the structure. The quantities  $\dot{x}$  and  $\ddot{x}$  are absolute structural velocity and acceleration respectively as defined in Fig. 1. The quantities  $K_D$ ,  $K_m$ , and  $K_r$  are coefficients of drag, added mass, and radiation damping respectively.

Dimensional analysis suggests that the coefficients may be a function of at least the Reynold's Number and the ratio of maximum structural displacement,  $x_m$ , to member radius,  $R$ , i.e.,  $x_m/R$ . The nature of the latter parameter may be illustrated as follows. When  $x_m/R$  is small no significant flow separation occurs and the values are essentially those for unseparated flow. For larger values of the ratio, partial vortex shedding leads to unsymmetrical flow patterns which may cause substantial variations in the coefficients. For very large values of the maximum structure displacement with respect to member radius, the wave appears more comparable to that in steady flow and  $K_D$ , at least should approach the steady flow value. Thus the coefficients in Eq. (1) depend on kinematical quantities such as absolute structure velocity and displacement. It should be noted that these coefficients may depend also on the ambient flow, i.e., waves and currents.

When the added mass term is included in the equation of motion for a structure, conventional practice is to combine it with the inertia force of the structure to produce a force equal to the virtual or total mass multiplied by the acceleration of the structure. Thus the effect of the added mass term is to alter the natural frequencies and mode shapes of the structure. Estimates of the added mass coefficients can be made for simple geometries in a fashion similar to that given by Newmark and Rosenblueth (2) and for more complex cases by numerical methods similar to those developed by Garrison and Berklite (1). In the presence of the free water surface, the ideal fluid added mass coefficient is in fact frequency dependent, but methods noted are adequate so long as the ratio  $hf/g$  exceeds about 1.0. Here  $h$  is the depth of water,  $f$  the frequency of the ground motion in Hertz, and  $g$  is the gravitational acceleration. For the structures considered here, this criterion is likely to be satisfied except for very low frequency structures in fairly shallow water. Caution need be exercised, however, since real fluid effects may alter the added mass coefficient in a manner similar to that described above. That is, if the maximum value of the total displacement of parts of the structure exceeds some fraction of the member diameter, partial flow separation may alter the coefficient from the value predicted by ideal fluid theory. Additional comments on the wave making force are given below.

#### ELASTIC RESPONSE SPECTRA

Elastic response spectra may be defined to include the fluid-structure interaction effects. Consider first the case where the force given by Eq. (1) consists only of the inertia force and velocity-squared force, i.e.,

$K_m \ddot{x} + K_D \dot{x} |\dot{x}|$ . Then the equation of motion of the simple oscillator shown in Fig. 1 is

$$\ddot{z} + 2\beta\omega\dot{z} + \omega^2 z = -\ddot{y} - \Gamma \frac{\omega^2}{a} \dot{x} |\dot{x}| \quad (2)$$

where  $\beta$  is structural damping,  $\omega$  is the natural frequency including the sum of the actual mass and  $K_D$ ,  $\Gamma$  is a dimensionless parameter which equals  $(K_D a)/K$ , and  $a$  is maximum ground acceleration.

Fig. 2 shows the effect of the parameter  $\Gamma$  on idealized response spectra. In the low and mid-frequency regions, the velocity drag produces effects comparable to viscous damping. In the high frequency range, the parameter causes a "tail" parallel to a line of constant displacement. In the region of the tail the pseudo-velocity,  $V'$ , is given by the following equation.

$$V' = \Gamma \frac{v^2}{a} \omega \quad (3)$$

Here  $v$  is the maximum ground velocity.

The non-linear hydraulic drag term can be linearized by replacing the term  $K_D \dot{x} |\dot{x}|$  by a term  $K_{DL} \dot{x}$  where  $K_{DL}$  is selected to make the work done by the forces to be equal when averaged over the duration of the earthquake motion,  $t_1$ . Accordingly,  $K_{DL}$  is given by the following equation.

$$K_{DL} = K_D \frac{\int_0^{t_1} \dot{x}^2 |\dot{x}| dt}{\int_0^{t_1} \dot{x}^2 dt} \quad (4)$$

An equivalent linear hydraulic damping ratio can be defined as  $\beta' = K_{DL}/(2M\omega)$ . Accordingly, the linearized equation of motion becomes

$$\ddot{z} + 2(\beta + \beta')\omega\dot{z} + \omega^2 z = -\ddot{y} - 2\beta'\omega\dot{y} \quad (5)$$

Eq. (5) provides a slightly conservative estimate of response up to a frequency slightly below that for which the high frequency tail intersects the acceleration-amplified region of the spectrum. In the tail region, the linearized solution can be substantially in error; in fact, the ratio of pseudo velocity in this region from Eq. (5) to that for Eq. (3) is  $K_{DL}/(K_D v)$  where  $v$  is the maximum ground velocity. Apparently, the upper bound frequency for which Eq. (5) is valid can be taken as the intersection of the tail and acceleration-amplified region of the spectrum.

If the term  $2\beta'\omega\dot{y}$  on the right hand side of Eq. (5) can be ignored, the equation reduces to that normally used for land structures, except that the damping factor is the sum of structural and hydraulic damping. Empirical studies show that it may be ignored provided the total damping,  $\beta + \beta'$ , is less than about 10 percent. Up to that limit the error in computed displacements is less than about 5 percent. It is extremely useful,

therefore, to be able to estimate  $\beta'$  without using Eq. (4). Empirical studies show the hydraulic damping can be estimated conservatively as follows.

$$\beta' \sim \frac{\pi}{2} \Gamma \frac{vf}{a} \quad (6)$$

where  $f$  is the natural frequency in Hertz. For low frequency systems Eq. (6) overestimates the amount of equivalent damping and therefore will yield unconservative spectral values for  $f < \sim 0.2$  Hertz. For very high frequencies Eq. (3) gives an accurate estimate of the pseudo-velocity.

In accordance with the discussion above, approximate response spectra can be constructed using ground motion amplification factors such as those given by Newmark and Rosenblueth plus modifications in the very low and high frequency regions. In this procedure the displacement amplification factor is selected on the basis of the structural damping factor. The velocity and acceleration amplification factors are based on an amount of damping equal to the sum of the structural and linearized hydraulic damping factors. The pseudo-velocity in the high frequency region is computed, if necessary, from Eq. (3). This is illustrated on Fig. 2 where line 1 is computed for the structural damping, lines 2 and 3 for the sum  $\beta + \beta'$ , and line 4 from Eq. (3). Methods for combining modal responses in multidegree-of-freedom are discussed elsewhere (3).

Consider next the case when radiation damping is present and the velocity drag term is absent, i.e.,  $F = K\ddot{x} + K_r\dot{x}$ . The equation of motion becomes the same as Eq. (5) except that  $\beta'$  is replaced by  $\beta''$  which is the radiation damping coefficient. If  $\beta''$  is taken as a constant, the resulting spectrum is identical to that described above for linearized hydraulic damping. The pseudo-velocity in the tail region is given by  $2\beta''v$  instead of by Eq. (3). Available data are not adequate to determine the radiation damping factor. Ideal fluid theory may be used, but the computations are complicated. An estimate which is adequate for high frequency systems, based on inviscid fluid theory is as follows.

$$\beta'' \cong 2\pi^3 \lambda \left(\frac{hf^2}{g}\right) \left(\frac{R}{h}\right)^2 \quad (7)$$

where  $\lambda$  is the ratio of the mass of water displaced by the structure to the sum of the actual mass and added mass,  $h$  is the depth of water,  $g$  is the gravitational acceleration and  $R$  is radius of the members. Eq. (7) is based on the assumption that the structure moves as a rigid body in translation and that the ground motion is periodic. Since  $\beta''$  is sensitive to the displacements near the water surface, it is easily an order of magnitude less than the value given by Eq. (7) for low frequency systems having small absolute displacements near the free water surface.

#### SPECTRA FOR HYDRODYNAMIC PARAMETERS

Quantities such as the absolute maximum displacement of the mass,  $x_m$ , and its absolute maximum velocity,  $\dot{x}_m$ , are necessary in order to compute Reynold's Number and the Kuelegan-Carpenter modulus. Response spectra for these quantities can be estimated using amplification factors and maximum ground motion quantities. The amplification factors depend only on the

total damping, i.e., the sum of structural and hydraulic damping. Amplification factors are given in Table I. Fig. 3 shows the hydrodynamic parameter response spectrum for the maximum absolute displacement. The "velocity" scale is  $\omega x_m$ . The line labeled 1 is the result of multiplying the maximum ground displacement by the displacement amplification factor. The line labeled 2 is the amplified value of the maximum ground velocity. The line labeled 3 is the maximum ground displacement.

Fig. 4 shows the idealized spectrum for  $\dot{x}_m$ . Lines labeled 1 and 2 are amplified maximum ground displacement and velocity respectively. The line labeled 3 is amplified maximum ground acceleration. The line labeled 4 is the maximum ground velocity. Methods for approximate analysis of multidegree of freedom are given elsewhere (4).

#### SUMMARY

Methods are presented for determining approximate response spectra and hydrodynamic parameter spectra for ocean structures such as production platforms. These include added mass, drag resistance, and radiation damping.

#### ACKNOWLEDGEMENT

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#### REFERENCES

1. Garrison, C. J. and Berklite, R. B., "Hydrodynamic Loads Induced by Earthquakes," OTC 1554, Proceedings, Offshore Technology Conference, 1972.
2. Newmark, N. M. and Rosenblueth, E., Fundamentals of Earthquake Engineering, Prentice-Hall, Inc., Chap. 6.
3. Kirkley, O. M., "Earthquake Response of Fixed Offshore Structures," Ph.D. Thesis, University of Illinois at Urbana-Champaign.
4. Kirkley, O. M. and Murtha, J. P., "Earthquake Response of Offshore Structures," Presented at Civil Engineering in the Ocean III, University of Delaware, Newark, Delaware, June 1975.

TABLE I  
Amplification Factors for Hydrodynamic Parameter Spectra

Total Damping (percent)	Maximum Displacement Spectrum		Maximum Velocity Spectrum		
	Displacement	Velocity	Displacement	Velocity	Acceleration
0	2.4	5.2	2.2	4.1	7.1
1	2.2	4.4	1.9	3.1	4.9
2	2.1	3.6	1.6	2.6	3.7
5	1.8	3.1	1.4	2.0	3.1
10	1.6	2.1	1.2	1.8	2.6

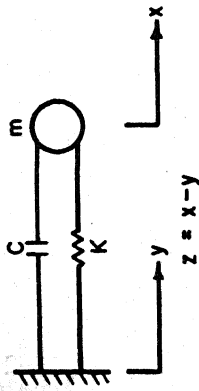


FIG. 1 DEFINITION SKETCH

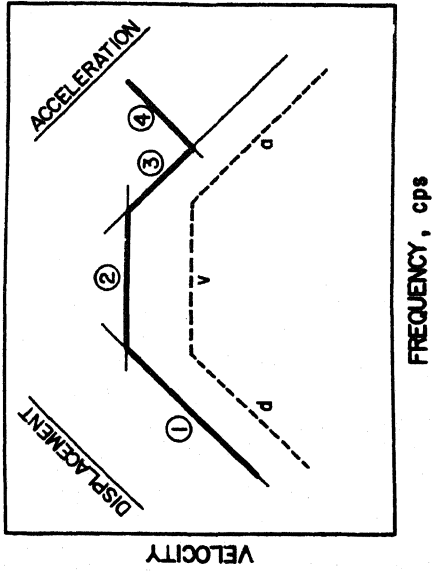


FIG. 2 IDEALIZED ELASTIC DEFORMATION RESPONSE SPECTRUM

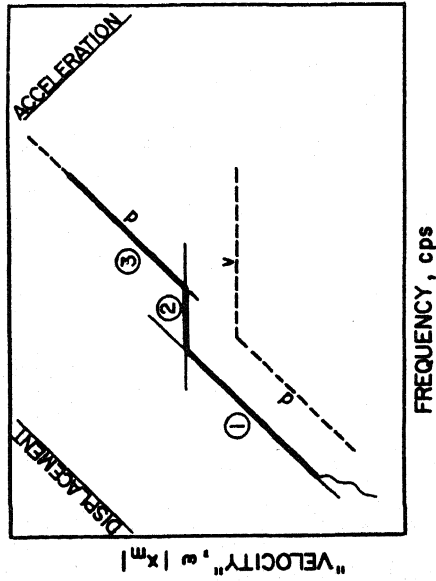


FIG. 3 CONSTRUCTION OF IDEALIZED HYDRODYNAMIC PARAMETER RESPONSE SPECTRUM FOR MAXIMUM STRUCTURAL DISPLACEMENT

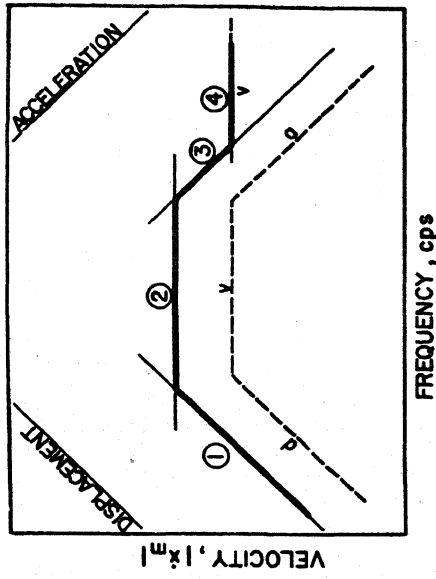


FIG. 4 IDEALIZED HYDRODYNAMIC PARAMETER RESPONSE SPECTRUM FOR MAXIMUM STRUCTURAL VELOCITY