AN ANALYTICAL STUDY ON RESTORING FORCE CHARACTERISTICS OF REINFORCED CONCRETE FRAMED STRUCTURES

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SYNOPSIS

An analytical approach is developed which finds out and appreciates restoring force characteristics of reinforced concrete framed structures on the basis of the geometrical properties and the material properties obtained from simple tests. Then the validity is confirmed from the comparison of some analytical results with the test ones and thereafter it is applied to several model framed structures with bracings which have some total sectional properties of bracings at each story but which are different in the arrangement of the bracings.

The results obtained are discussed mainly from the viewpoints of restoring force characteristics.

Due to the limitation of the space, we describe only the abstract of the full paper which will be published elsewhere.

ABSTRACT

The restoring force characteristics in reinforced concrete structures are so much influenced by brittle concrete, ductile steel and their interactions that it is very difficult to estimate the structural behaviors analytically. Thus, in most cases the restoring force characteristics are idealized at the structural level using test results on some elements of a structure. In such cases, however, it is also difficult to consider mutual interactive behaviors among many elements of a structure at the structural level. On the other hand, tests on the whole structure are impossible both in practice and in economy. Therefore analytical approaches to predict the whole structural behaviors from ones of structural elements or material properties without any structural test are greatly required.

In the analysis structural elements are divided longitudinally and transversally into some segments. The stress and the strain of each segment, represented at the center, are determined according to stress-strain relationships of steel and concrete with a mathematical spring model to idealize interactions between steel bar and concrete, in which steel spring and concrete spring in series to bond spring are arranged in parallel. The stiffness reduction and the propagation of concrete crack or some failure due to the increments of loading are estimated step by step.

In conclusion, this analytical approach is very useful in determining the restoring force characteristics of reinforced concrete framed structures, which are pointed out to be much influenced by the arrangement of aseismic elements such as bracings. In order to get larger stiffness and bearing capacity it is most effective to distribute bracings evenly over all the bays, though it is not always effective from the viewpoints of ductility and hysteresis damping. Furthermore, it is pointed out that the hysteresis behaviors can have remarkably unusual characteristics and thus hysteresis models must be carefully determined.

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DISCUSSION

A.K. Basak (India)

The authors presented a fine paper on restoring forcing characteristics of R.C. framed structures in a brief manner. The writer is interested to know the authors specific analytical approach and test procedure for R.C. framed structure models. It will be appreciated if the authors give a certain conclusion regarding restoring force-deflection behaviour of this type of structure.

Author's Closure

With regard to the question of Mr. Basak, we wish to state that we are appreciative of your interest in our paper, and we are very sorry that we could not describe our study in detail in that paper due to the limitation of the space. We shall be very happy if the following description will give any helpfulness to you.

Formulation of Member Stiffness

In Fig. 1, a member divided longitudinally into m parts and transversally into n parts to get mxn segments is shown, which is subjected to incremental end forces ΔN, ΔQ and ΔM at the free end, of axial force, transversal force and bending moment, respectively. ΔN_{jx}, the incremental axial force and ΔM_{jx}, the incremental bending moment acting to the cross section (X-X section) at any distance x from the fixed end within any longitudinal zone j are expressed as

$$\Delta N_{jx}=\sum_{i=1}^{m}(E_{ij}\Delta\varepsilon_{ijx})b\lambda_{ij}d=bd\left[\left\{\sum_{i=1}^{m}E_{ij}\lambda_{ij}\right\}\Delta\varepsilon_{ojx}+\left\{\sum_{i=1}^{m}E_{ij}\lambda_{ij}(\eta_{ij}-\eta_{oj})\right\}\Delta\varphi_{jx}\right]\equiv bd(p_j\Delta\varepsilon_{ojx}+r_j\Delta\varphi_{jx}) \qquad (1)$$

$$\Delta M_{jx}=\sum_{i=1}^{m}\left\{(E_{ij}\Delta\varepsilon_{ijx})b\lambda_{ij}d\cdot(\eta_{ij}-\eta_{oj})d+E_{ij}(\Delta\varphi_{jx}\cdot\frac{\lambda_{ij}}{2})\cdot\frac{b\lambda_{ij}^{2}d^{2}}{6}\right.$$

$$=bd^{2}\left[\left\{\sum_{i=1}^{m}E_{ij}(\eta_{ij}-\eta_{oj})\lambda_{ij}\right\}\Delta\varepsilon_{ojx}+\left\{\sum_{i=1}^{m}E_{ij}\left\{(\eta_{ij}-\eta_{oj})^{2}\lambda_{ij}+\frac{\lambda_{ij}^{3}}{12}\right\}\right\}\Delta\varphi_{jx}\right] \equiv bd^{2}(r_j\Delta\varepsilon_{ojx}+q_j\Delta\varphi_{jx}) \qquad (2)$$

in which $\Delta\varepsilon_{ijx}$= incremental strain at X-X section within (ij) segments, $\Delta\varphi_{jx}$ = incremental curvature at X-X section within j zone, $\Delta\varepsilon_{ojx}$= incremental strain at the centroid of X-X section within j zone, E_{ij} = equivalent tangent modulus relating $\Delta\varepsilon_{ij}$, incremental strain, to $\Delta\sigma_{ij}$, incremental stress, within (ij) segment and $\Delta\varepsilon_{ijx}=\Delta\varphi_{jx}(\eta_{ij}-\eta_{oj})+\Delta\varepsilon_{ojx}$

Noticing to $\Delta N_{jx}=\Delta N$ and $\Delta M_{jx}=\Delta M_{jx}+\Delta Q(\ell-x)$, incremental end deformations Δu, Δv and $\Delta\theta$., axial deformation, transversal deformation and rotational deformation, respectively, are obtained with use of above expressions as

$$\Delta u=\sum_{j=1}^{n}\Delta u_j=\sum_{j=1}^{n}\int_{(\xi_j-0.5a_j)\ell}^{(\xi_j+0.5a_j)\ell}\Delta\varepsilon_{ojx}(\ell d\xi)=\sum_{j=1}^{n}\left[\frac{\ell}{bd}\cdot\frac{q_j}{\beta_j}a_j \quad -\frac{\ell^2}{bd^2}\cdot\frac{r_j}{\beta_j}a_j(1-\xi_j) \quad -\frac{\ell}{bd^2}\cdot\frac{r_j}{\beta_j}a_j\right]\begin{Bmatrix}\Delta N\\ \Delta Q\\ \Delta M\end{Bmatrix} \quad (3)$$

$$\Delta v=\sum_{j=1}^{n}\Delta v_j=\sum_{j=1}^{n}\int_{(\xi_j-0.5a_j)\ell}^{(\xi_j+0.5a_j)\ell}(\Delta\varphi_{jx}/d)(1-\xi)\ell\cdot(\ell d\xi)=\sum_{j=1}^{n}\left[-\frac{\ell^2}{bd^2}\cdot\frac{r_j}{\beta_j}a_j(1-\xi_j) \quad \frac{\ell^3}{bd^3}\cdot\frac{p_j}{\beta_j}\left\{a_j(1-\xi_j)^2+\frac{a_j^3}{12}\right\} \quad \frac{\ell^2}{bd^3}\cdot\frac{p_j}{\beta_j}a_j(1-\xi_j)\right]\begin{Bmatrix}\Delta N\\ \Delta Q\\ \Delta M\end{Bmatrix} \quad (4)$$

$$\Delta\theta=\sum_{j=1}^{n}\Delta\theta_j=\sum_{j=1}^{n}\int_{(\xi_j-0.5a_j)\ell}^{(\xi_j+0.5a_j)\ell}(\Delta\varphi_{jx}/d)(\ell d\xi)=\sum_{j=1}^{n}\left[-\frac{\ell}{bd^2}\cdot\frac{r_j}{\beta_j}a_j \quad \frac{\ell^2}{bd^3}\cdot\frac{p_j}{\beta_j}a_j(1-\xi_j) \quad \frac{\ell}{bd^3}\cdot\frac{p_j}{\beta_j}a_j\right]\begin{Bmatrix}\Delta N\\ \Delta Q\\ \Delta M\end{Bmatrix} \quad (5)$$

in which $\beta_j = p_j \; q_j - r_j^2$

Eqs. 3, 4 and 5 can be written in the form of stiffness matrix as

$$\begin{Bmatrix}\Delta N\\ \Delta Q\\ \Delta M\end{Bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}\begin{Bmatrix}\Delta u\\ \Delta v\\ \Delta\theta\end{Bmatrix}$$

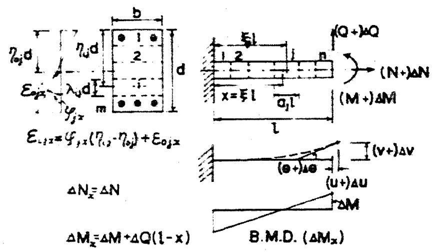

FIG. 1

in which C_{12}, C_{13}, C_{21} and C_{31} are not generally zero and thus interactive effects of axial force ΔN and bending moment ΔM on the stiffness of the member can be considered by using such a stiffness matrix.

Idealization of Material Properties

E_{ij} used in Eqs. 1 and 2 of the foregoing paragraph is E_c for a segment without any reinforcement which follows the rule illustrated in (a) of Fig. 2. On the other hand, for a segment with some reinforcements it is as follows:

in which a_s = total sectional area of reinforcements included in the segment; E_c, E_s and E_b are the tangent moduli of concrete, reinforcement steel and interaction bond spring, respectively and they follow the rules illustrated in (b), (c) and (d) of Fig. 2, while their interactive mechanism is idealized as shown in (e) of Fig. 2. Characteristics of E_b can be obtained from such tests on a covered steel bar by concrete and a plain steel bar as shown in (a) of Fig. 3, (b) of which indicates a procedure for obtaining $P_c - \delta$ relation from the test results. Fig. 4 shows an example of such test results, where $P_c = \sigma_c A_c$, and $\delta = \varepsilon l = (\varepsilon_b + \varepsilon_c)l \doteqdot \varepsilon_b l$, considering very small in comparizon with ε_b or ε_s. Although some compatibility of deformations is ignored as a result of assuming such a mechanizm, attaching importance to effects of cracking on the stiffness of the member, the influence of bond slip is limited to the neighborhood of the critical location. Therefore the assumed mechanizm is considered reasonable for the purpose of simply evaluating the stiffness of the member after concrete cracking.

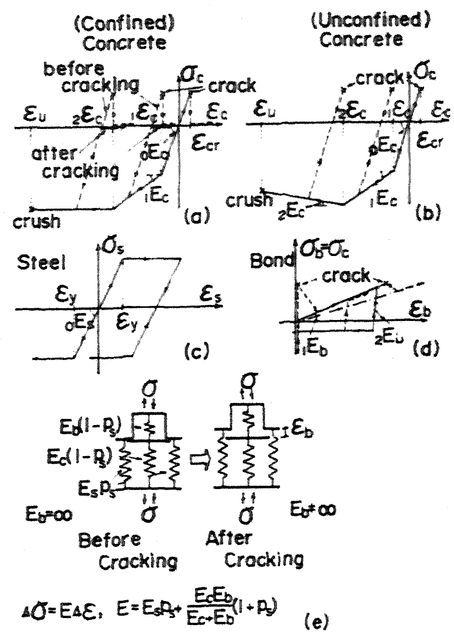

FIG. 2

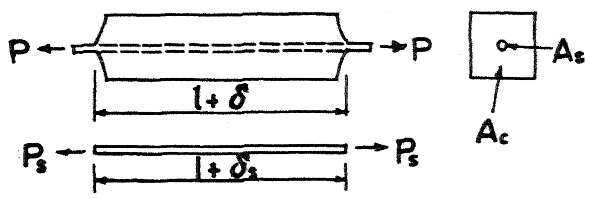

FIG. 3 (a)

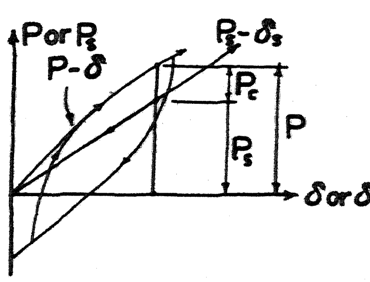

FIG. 3 (b)

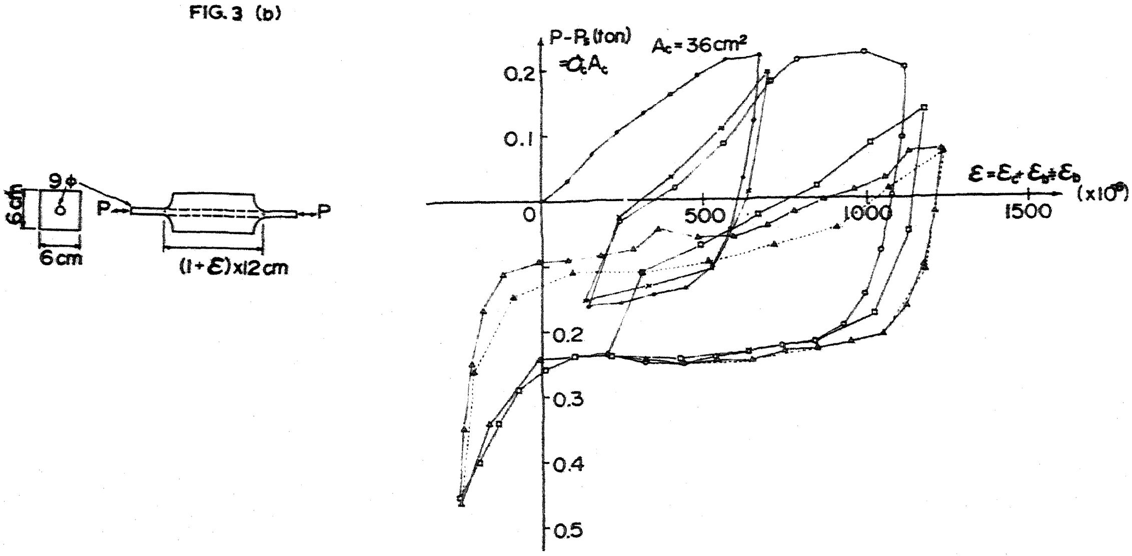

FIG. 4

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