EXPERIMENTAL AND THEORETICAL STUDIES OF TIME VARIATIONS OF STRUCTURAL PROPERTIES DURING STRONG GROUND SHAKING

by

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SYNOPSIS

Studies on the variation of structural properties during the period of actual ground shaking as indicated from strong motion data recorded in structures have been carried out. After modeling the structure as a linear time variant single degree of freedom system, a systematic methodology for the analysis of such behaviour from a knowledge of the basement and roof records has been developed. Utilizing such noisy records, and incorporating any data available from pre-and for post-earthquake vibration tests, the time varying coefficients of the differential equation, used to model the structure, have been identified using a minimum variance sequential filter. Polynomial forms of stiffness and damping variations have been investigated.

The technique has been used to analyze the response of a nine-story reinforced concrete structure during the San Fernando, California earthquake of February 9, 1971. The results indicate that significant changes in the stiffness and damping characteristics, from those obtained during vibration tests, could ensue during the actual progress of strong ground shaking and that a proper knowledge of the pre-earthquake characteristics may be very useful in identification using strong ground motion records.

INTRODUCTION

It has been known for some time now [1] that building structures exhibit, in general, time-variant nonlinear responses to strong earthquake ground shaking. Based on data obtained from the San Fernando Earthquake of Feb. 9, 1971 studies in the past have been carried out to characterize such behavior in terms of quasi-time-invariant linear formulations [2]. The method basically consists of 1) looking at different segments of the earthquake excitation, and the corresponding segments of structural response, 2) modelling the structural system as a linear system during each of these different time segments, and 3) determining on the basis of the excitation-response data of each segment, the properties appropriate to a linear system.

Basically, if $y(t)$ is the structural response to the excitation $x(t)$ between the times $t_o-T/2$ and $t_o+T/2$, then the system is assumed to be-have linearly during that time interval $I(t_o,T)$ and the response is expressed as (Fig. 1)

$$Y(\omega,t_o,T) = H(\omega,t_o,T)\ X(\omega,t_o,T) \qquad (1)$$

where $H(\omega,t_o,T)$ is the transfer function of the system calculated from the Fourier Transforms $X(\omega,t_o,T)$ and $Y(\omega,t_o,T)$ of $x(t)$ and $y(t)$ in the

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interval $I(t_o, T)$. $H(\omega, t_o, T)$ would be then interpreted as an average characterization of the structural system during the time window $t_o - T/2$ to $t_o + T/2$, from which information about the dampings and fundamental frequencies can be extracted. When determined for various t_o values it yields a time variant quasilinear characterization of the structural system.

The major drawbacks of such a technique are that 1) it yields an 'average' characterization over a period T, and that 2) it generally ignores the 'initial conditions' prevailing during the time window T. To illustrate this, consider a single degree of freedom nonlinear system with mass m, approximated as a time variant linear system (Fig. 2)

$$m\ddot{y} + c'(t)\dot{y} + k'(t)y = -m\ddot{x} \tag{2}$$

where y is the relative response of the oscillator, c(t) and k(t) are time variant structural properties and $\ddot{x}(t)$ is the base acceleration. Normalizing with respect to the mass, equation (2) can be rewritten as

$$\ddot{y} + c(t)\dot{y} + k(t)y = -\ddot{x}(t) \tag{3}$$

Observing the system from a time t_o to t_o+T during which it is assumed that $c(t) \approx c_o$ and $k(t) \approx k_o$ we then have

$$\ddot{y} + c_o\dot{y} + k_o y = -\ddot{x} \qquad t \varepsilon [t_o, t_o+T]$$

with

$$y(t_o) = y_o,\ \dot{y}(t_o) = \dot{y}_o,\ x(t_o) = x_o, \text{ and } \dot{x}(t_o) = \dot{x}_o.$$

Taking Fourier transforms we have

$$Y(\omega) = \frac{-\omega^2}{(-\omega^2 + i\omega c_o + k_o)} X(\omega) + \frac{i\omega(y_o + x_o) + (\dot{y}_o + \dot{x}_o) + c_o y_o}{(-\omega^2 + i\omega c_o + k_o)}$$

Equation (1) is, we observe in error by the second term on the right hand side, the neglect of which may lead to substantial deviations in the damping estimates (band width of transfer function) though the frequency estimates (location of the zeros) may be left substantially unchanged. Though this error regarding the initial conditions can be presumably corrected for, uncertainties in the values of initial displacements and velocities obtained from actual earthquake records may be quite substantial making the damping estimates from such an analysis very inaccurate.

The first drawback mentioned above is a more serious one in that the time T must be short enough so as to be capable of capturing the variations in k(t) and c(t) and yet must be long enough so as to yield reliable estimates. The resolution, Δf, with which a frequency f can be identified in a time signal of duration T is given by $\Delta f T = 1$. Thus for a resolution of 0.1 cps, T would need to be 10 seconds, a duration often far too long to adequately capture the variation of natural frequencies with time.

In order to eliminate these drawbacks, in this paper, we model a structure as a single degree of freedom oscillator and indicate a methodology for determining c(t) and k(t) utilizing the input and response records. The functions c(t) and k(t) are expressed functionally as

$$c(t) = \psi(\varphi_i, t), \qquad k(t) = \chi(\varphi_i, t)$$

where φ_i are constant parameters and ψ, χ define the functional dependence of c(t), k(t) on φ_i. The time variant identification essentially leads to the estimation of the coefficients φ_i from a sequence of observations $\theta(t)$.

THEORY

We write the observation θ_k at time t_k as a non-linear function

$$\theta_k = h_k(\varphi, t_k) + v_k$$

where φ is the n-vector of constant parameters to be estimated and v_k is additive white noise such that

$$E(v_k) = 0, \quad E(v_k v_k') = R_k$$

Linearizing the equation about some nominal set of values, to first order

$$\Delta\theta_k = \frac{\partial h_k}{\partial \varphi}\Delta\varphi + v_k$$

or, writing z_k, H_k and q for $\Delta\theta_k$, $\partial h_k/\partial\varphi$ and $\Delta\varphi$,

$$z_k = H_k q + v_k \tag{4}$$

An estimate of q can be obtained from the linearized observation equation (4) using the sequential filter formulation developed by Kalman [3]. This can be derived in several different ways [4]. To present a simplified approach, we seek a linear unbiased estimate, $\hat{q}_k$, of q having a minimum variance of error, i.e. min J_k where

$$J_k = E(\tilde{q}_k' \tilde{q}_k)$$

$$\tilde{q}_k \triangleq \hat{q}_k - q \tag{5}$$

Expressing $\hat{q}_k$ as a linear function of the observation z_k and the prior estimate $\hat{q}_{k-1}$, we have

$$\hat{q}_k = L_k\hat{q}_{k-1} + K_k z_k$$

$$= L_k\hat{q}_{k-1} + K_k(H_k q + v_k) \tag{6}$$

Taking expectations and setting $E(v_k) = 0$,

$$E(\hat{q}_k) = L_k E(\hat{q}_{k-1}) + K_k H_k E(q) \tag{7}$$

Unbiasedness of the estimate requires

$$E(\hat{q}_k) = E(\hat{q}_{k-1}) = E(q)$$

which substituted into Eq.(7) yields

$$L_k = I - K_k H_k,$$

where I is the identity matrix. Substituting into Eq.(6) and using Eq. (5),

$$\tilde{q}_i = (I - K_k H_k)\tilde{q}_{k-1} + K_k v_k$$

The covariance of the estimate error is therefore given by

$$P_k \triangleq E(\tilde{q}_k \tilde{q}_k') = (I - K_k H_k) P_{k-1} (I - K_k H_k)' + K_k R_k K_k \tag{8}$$

since the measurement errors are uncorrelated.

The criterion of optimality is that the variance of estimate error, J_k, be a minimum. Now

$$J_k = E(\tilde{q}_k' \tilde{q}_k) = \text{Trace}(P_k)$$

The first variation must vanish for the estimate to be optimal, yielding

$$K_k = P_{k-1} H_k' (H_k P_{k-1} H_k' + R_k)^{-1} \tag{9}$$

as the expression for the optimal gain matrix. Also, the second variation of J_k can be shown to be positive indicating that K_k given by (9) minimizes J_k.

Substituting Eq. (9) into Eq. (8) and rearranging gives

$$P_k = (I - K_k H_k) P_{k-1}$$

Constraints: If some linear constraints need to be imposed, these can be implemented easily through a transformation of variables. Thus, if it is desired to impose a probabilistically stated constraint

$$E\{Cq\} = d, \quad E\{Cqq' C'\} = \Lambda,$$

where d is an m-vector and the first m columns of C are assumed to be linearly independent, we define an m-vector of variables P_1 such that

$$Cq \triangleq P_1$$

Partitioning the left hand side

$$[C_1 \; C_2] \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = P_1$$

where C_1, C_2 are $m \times m$, $m \times (n-m)$ matrices respectively and q_1, q_2 are m-, (n-m)-vectors respectively. Defining p and A, we can write

$$p \triangleq \begin{Bmatrix} p_1 \\ q_2 \end{Bmatrix} = \begin{bmatrix} C_1 & C_2 \\ 0 & I \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \triangleq Aq$$

$$q = A^{-1} p = \begin{bmatrix} C_1^{-1} & -C_1^{-1} C_2 \\ 0 & I \end{bmatrix} \begin{Bmatrix} p_1 \\ q_2 \end{Bmatrix} \tag{10}$$

Substituting into equation (4) gives

$$z_k = H_k A^{-1} p_k + v_k = \tilde{H}_k p_k + v_k$$

The problem reduces to the estimation of p; the final estimate $\hat{p}_k$ is transformed to the original variables using Eq.(10).

APPLICATION TO AN R-C STRUCTURE

The technique described above was applied to a nine-story reinforced concrete structure using the EW translational components from the San Fernando earthquake of 1971. Ambient vibration tests indicate that the second EW translation mode of vibration of the structure occurs at about 6 cps and that the lowest torsional mode occurs at about 3 cps. The base acceleration and the roof responses, $\ddot{x}(t)$ and $\ddot{Y}(t)$, were low pass filtered to 2.5 cps thereby minimizing the effect of all vibration modes other than the lowest EW translational mode. These low passed filtered records were then used to generate the relative velocity and displacement time histories (Fig. 3), $\dot{y}(t)$ and $y(t)$. At time t_k writing z_k for the measured value of the low-passed $\ddot{r}(t)$, using Eq. (3)

$$z_k = [-k(t)y(t) - c(t)\dot{y}(t) + v(t)]_{t=t_k}$$

Here $v(t_k)$ represents the combination of the errors and for simplicity, ignoring all other effects, is taken to be white noise with $\sigma_v = 5\text{cm/sec}^2$, a value estimated using previous studies of digitization errors.

Several forms of stiffness and damping variation were investigated. In particular expansions in terms of Chebychev polynomials, $\beta_j(t)$, were used, i.e.

$$k(t) = \Sigma\, a_j\beta_j(t), \quad c(t) = \Sigma\, b_j\beta_j(t)$$

where coefficients a_j, b_j were to be estimated. Pre-and post-earthquake vibration test data [1] were used to constrain the initial and final values of the stiffness and damping along with realistic variances. At the end of the estimation the parameter covariance was mapped into the standard deviations $\sigma_k(t)$, $\sigma_c(t)$ for the stiffness and damping respectively.

The model and measured responses are shown in the figure along with a description of the model itself through the functions $f_n(t)/f_n(0)$ and $c(t)/(4\pi\, f(0))$. It is observed that the fundamental frequency drops to about two third its value during the strong motion part of the excitation. A gradual recovery of this frequency follows until the post-earthquake ambient vibration frequency is obtained at the end of the record. We note that the constraints on the frequencyat the beginning and the end of the record lead to smaller variances at the two ends. The small variance of the estimate of $f_n(t)$ observed between about 5 seconds and 30 seconds is due to the high signal to noise ratio in that interval.

The damping estimates have a larger variance than the stiffness indicating that recovery of such data from earthquake records is harder than the recovery of fundamental frequency changes. It is worth noting that whereas $f_n(t)/f_n(0)$ is a smoothly varying function which reduces initially and gradually increases, the damping varies with time in a much more oscillatory manner. After the initial increase in damping following the large strains induced by the strong ground motions the damping estimates keep oscillating with time.

CONCLUSIONS AND DISCUSSION

1) The analysis of building structural response as carried out here indicates that marked changes in the stiffness and damping properties occur in a building structure during the period of strong ground shaking.

2) The technique illustrated utilizes earthquake data as a test input to study large amplitude response characterizations of structures, and incorporates, in a systematic manner, knowledge of both the pre-and post-earthquake ambient vibrations test results. The use of such low level test results is shown to be extremely useful in reducing the uncertainty in the estimates of both the stiffness and damping variations with time.

3) The actual $c(t)$ and $k(t)$ are found to be relatively insensitive to the order of the Chebychev polynomials chosen, provided that the order is high enough. In the example illustrated an eighth order model was considered adequate.

4) During the strong motion part of the earthquake excitation (which generally lasts for not more than about 30 seconds) the signal to noise ratio is high and the uncertainties in the estimates of c(t) and k(t) are small, thus yielding a large amount of information on the structural system during the crucial period of the excitation.

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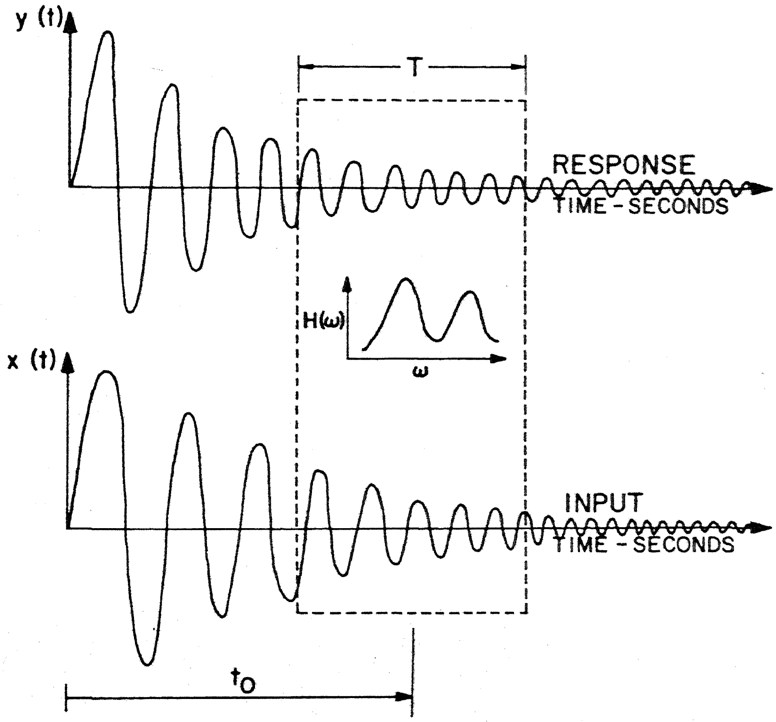

Fig. 1; Quasi-time invariant formulation

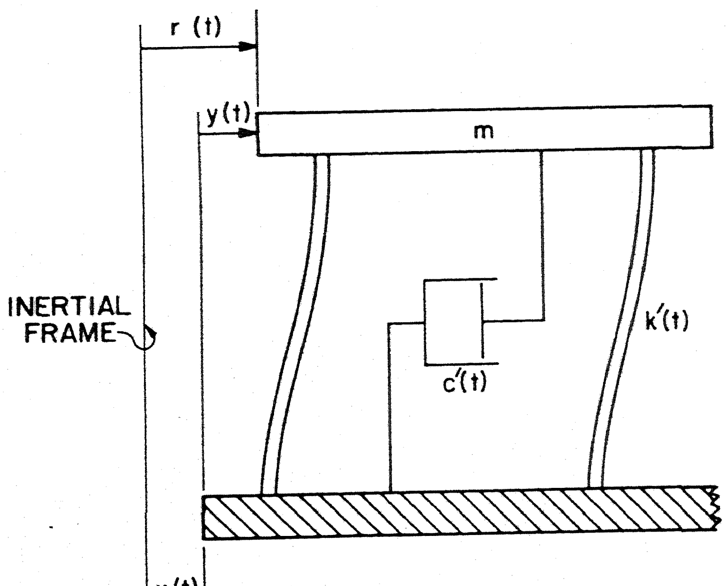

Fig. 2; Second order model.

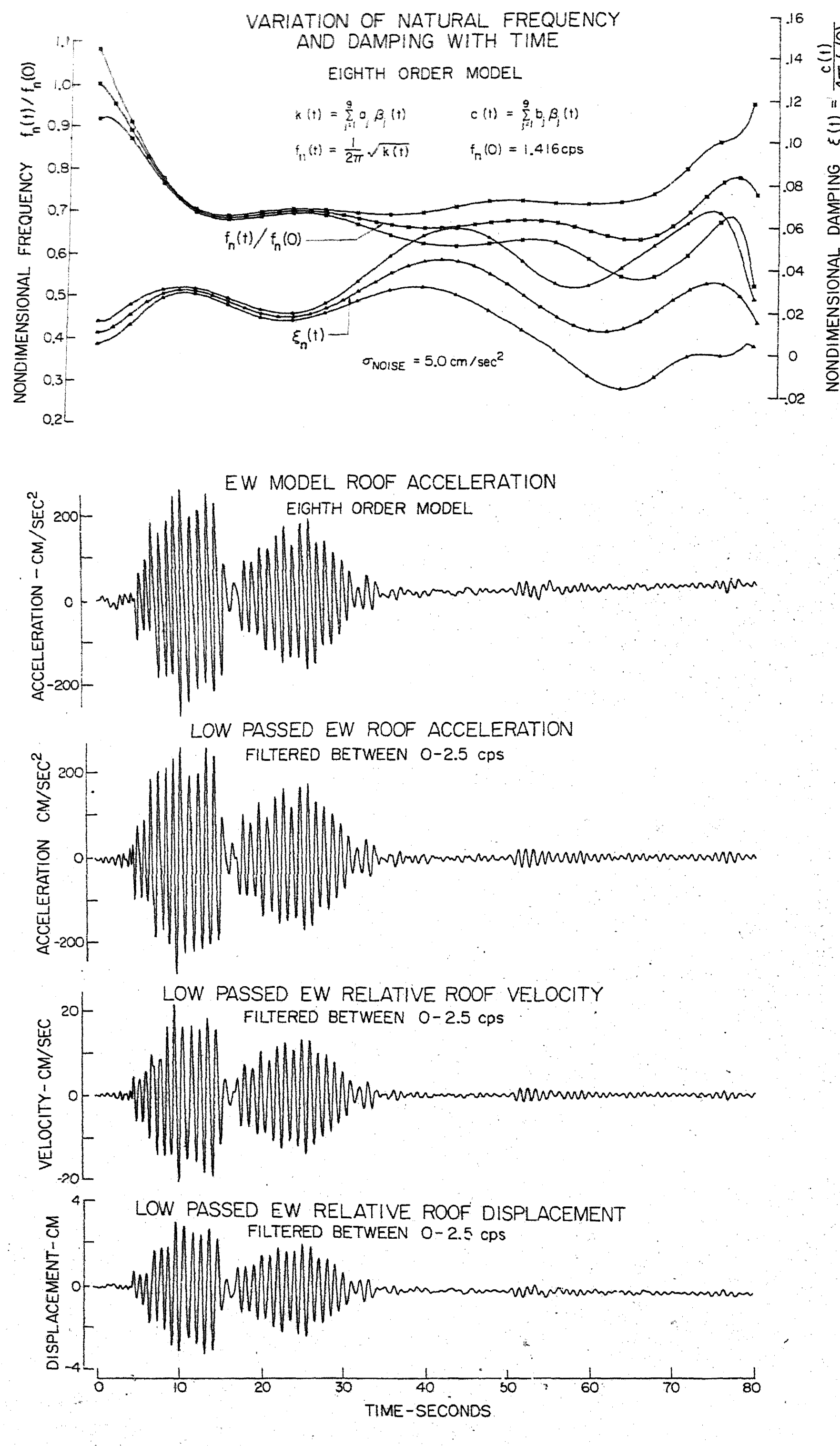

Fig. 3; Time-variant results