RESPONSE AND STRESSES OF STRUCTURES SUBJECTED TO TWO-DIRECTIONAL EARTHQUAKE EXCITATION

by

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SYNOPSIS

Earthquake ground motions are acting in more than one horizontal direction; they are recorded in two orthogonal axis. Maximum resulting stresses from 2-D earthquake response are derived by integrating recorded time histories and compared with combinations of 1-D results. It can be shown that the shape of the cross section is an essential factor. As a model of a building a linear elastic clamped beam with a lumped mass is used. Several cross sectional shapes, representing the possible arrangement of horizontal load resisting elements, are considered.

INTRODUCTION

Earthquake ground motions are acting in more than one horizontal direction; they are recorded in two orthogonal axis. Several simple rules to combine 1-D results, or to include the 2-D effects by other means, are used for design purposes. To show the applicability and accuracy of these rules it is necessary to compare the results of 2-D time history calculations and of the simple rules. As a contribution to this problem it is the aim of this paper to present numerical results. As a model of a building a linear elastic clamped beam with a lumped mass is chosen. To represent the arrangement of the lateral load resisting elements several cross sectional shapes are considered. The comparison is done for stresses which are regarded as characteristic design limits.

CROSS SECTIONS CONSIDERED AND THEIR CHARACTERISTIC STRESSES

For the sake of clarity and notations the basic formulas are given first. The investigations are restricted to cross sections with two axis of symmetry x and y (flexural inertia I_x, I_y). For a 2-D moment loading M_x and M_y the stress f_P at a point $P(x_P, y_P)$ is given by

$$f_P = M_x/I_x \, y_P - M_y/I_y \, x_P \tag{1}$$

In design one is often interested in maximum stresses. For a given loading their magnitude and location depend upon the cross sectional shape.

<u>Circular or circular ring sections</u> do not have characteristic points. The location of the maximum stress f is therefore determined by the ratio of M_y/M_x only. It is convenient to use the resultant moment for the calculation:

$$f = \pm\sqrt{M_x^2 + M_y^2} \; r/I_x \tag{2}$$

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In an open box section, as depicted in figure 1, the resisting elements act separately in the two directions. There is no superposition of the stresses due to M_x and M_y. Stress maxima occur in the outer fibres of the elements:

$$f_x = \pm M_x b/2I_x \qquad f_y = \pm M_y a/2I_y \tag{3}$$

For stresses in rectangular, box and I-sections (Fig. 1) superposition is necessary. The maxima occur in the corners:

$$f = \pm \left[M_x b/2I_x \pm M_y a/2I_y\right] \tag{4}$$

TWO DIRECTIONAL EARTHQUAKE RESPONSE

For a one mass system with two horizontal degrees of freedom (Fig. 2) the equations of motion under earthquake excitation may be expressed as

$$\begin{aligned} \ddot{x}(t) + 2\xi\sqrt{k_x/m}\,\dot{x}(t) + k_x/m\; x(t) &= -\ddot{x}_g(t) \\ \ddot{y}(t) + 2\xi\sqrt{k_y/m}\,\dot{y}(t) + k_y/m\; y(t) &= -\ddot{y}_g(t) \end{aligned} \tag{5}$$

with $k_x = 3EI_y/l^3$, $k_y = 3EI_x/l^3$ stiffness, ξ critical damping ratio and the ground excitation (Fig. 3)

$$\begin{aligned} \ddot{x}_g(t) &= \ddot{u}(t)\cos\alpha + \ddot{v}(t)\sin\alpha \\ \ddot{y}_g(t) &= -\ddot{u}(t)\sin\alpha + \ddot{v}(t)\cos\alpha \end{aligned} \tag{6}$$

The natural periods are given by $T_x = 2\pi\sqrt{m/k_x}$, $T_y = 2\pi\sqrt{m/k_y}$
For the sake of convenience the dynamic response (solution of Eq. (5)) may be written in terms of load coefficients:

$$\begin{aligned} a_x(t) &= k_x/mg\; x(t) \\ a_y(t) &= k_y/mg\; y(t) \end{aligned} \tag{7}$$

with g acceleration of gravity. Thus the fixed-end moments are

$$\begin{aligned} M_x(t) &= l\; m\; g\; a_y(t) \\ M_y(t) &= l\; m\; g\; a_x(t) \end{aligned} \tag{8}$$

With these expressions the stresses can be calculated as shown. To depict stress response spectra reduced variables are advisable; e.g. for the box section Eq. (4) results

$$f^*(t) = \pm\left[a_x(t) \pm m_y^*/m_x^*\; a_y(t)\right] \tag{9}$$

with $m_x^* = lmga/2I_y$, $m_y^* = lmgb/2I_x$ and $f^*(t) = f(t)/m_x^*$.

The maximum combined stress during an earthquake is characterized by $\max\{f^*(t)\}$ of Eq. (9); in the simple combination rules it is replaced by a combination of the maximum components.

RESULTS

All calculations are based on the Golden Gate Park record of the San Francisco Earthquake 1957. The maximum of the larger component has been scaled to g. By using recorded time histories the natural correlation of the components is introduced. To study possible effects of the record other earthquakes will have to be analysed. The 2-D time history solution is marked by an asterisk (*) in all figures.

Fig. 4 shows a 2-D stress response spectrum for the circular section. The 2-D solution (resultant load factors) envelops the 1-D solutions. The rms of the 1-D spectrum values overestimates the stress response.

There is no combination for the stress response of the open box section, however, the stress depends on the angle α and is depicted in Fig. 5. In a response spectrum analysis these variations may be assumed to be included in the smoothed design spectrum.

In the following only the box section is discussed. The results are also representative for the rectangular and I-sections. Fig. 6 and 7 show only results for the angle $\alpha = 0^\circ$ as the variation of α has small influence for this type of cross sections. First the applicability of resultant load factors is checked (Fig. 6). Only for square sections this method gives appropriate results. For general cross sections of this type the stresses should be combined either by their absolute values or by the rms-method (Fig. 7). In this case with linear superposition the applicability of the rms-method can be expected as combination of two independent stochastic processes. Corner stresses are not covered by 1-D solutions, especially in the case of $T_y/T_x \neq 1$.

CONCLUSIONS

Maximum resulting stresses from 2-D earthquake response are derived by integrating recorded time histories. The results are compared with simplified combination rules. It can be shown that the shape of the cross section is an essential factor. Three types are distinguished and the following conclusions may be drawn.

Circular or circular ring sections: A 2-D stress response spectrum (resultant load factors) gives the correct solution. The envelope of 1-D response spectra gives a good approximation. The rms-method overestimates the stresses.

Open box section: 1-D response spectra with due regard to the angle α are sufficient.

Rectangle, box, and I-section: Combining the stresses from the two directions by the sum of the absolute values or the rms-method gives satisfactory approximations.

REFERENCES

[1] Clough, R.W. /Penzien, J.: Dynamics of Structures. New York: McGraw - Hill 1975.

[2] Zilch, K.: Bemessung von Bauwerken gegen Erdbebenbelastungen. Die Bautechnik 1974.

[3] Strong Motion Earthquake Accelerograms, Vol. II, Part A. Report EERL 71-50, California Institute of Technology, Pasadena.

[4] Newmark, N.M. /Rosenblueth, E.: Fundamentals of Earthquake Engineering. Englewood Cliffs: Prentice Hall 1971.

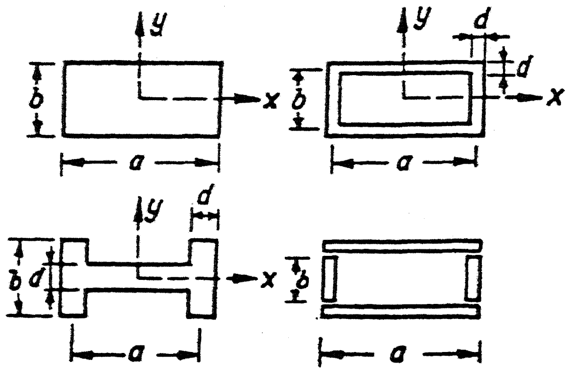

Fig. 1: Cross sectional shapes

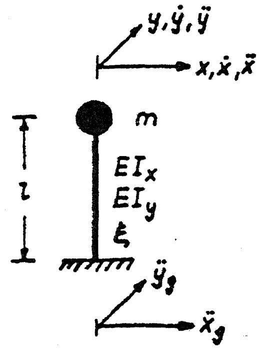

Fig. 2: 2-D idealisation of a building

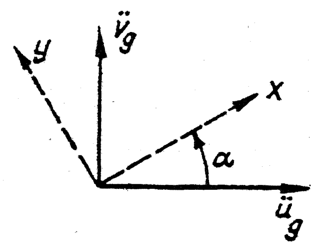

Fig. 3: Coordinate systems

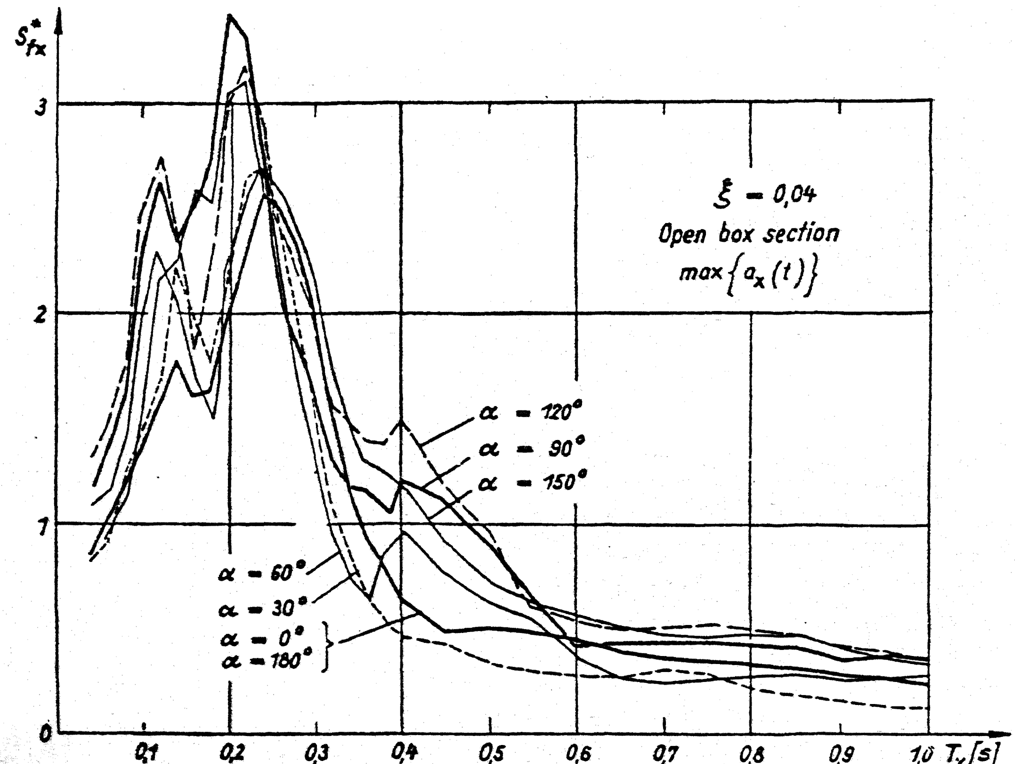

Fig. 5: Stress response spectra for open box sections
Variation of angle α

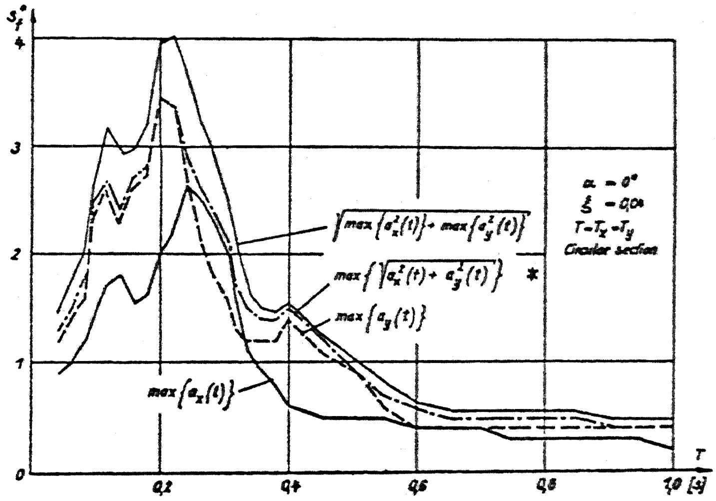

Fig. 4: Stress response spectra for circular or circular ring sections

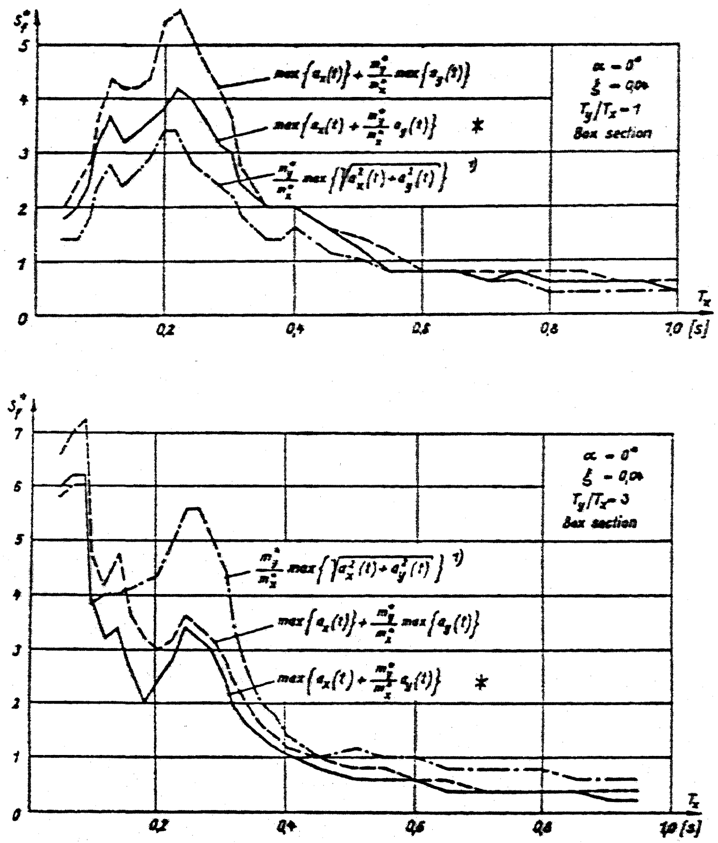

Fig. 6: Stress response spectra for box sections
1) Edge stress, for corner stress in square sections multiply by $\sqrt{2}$

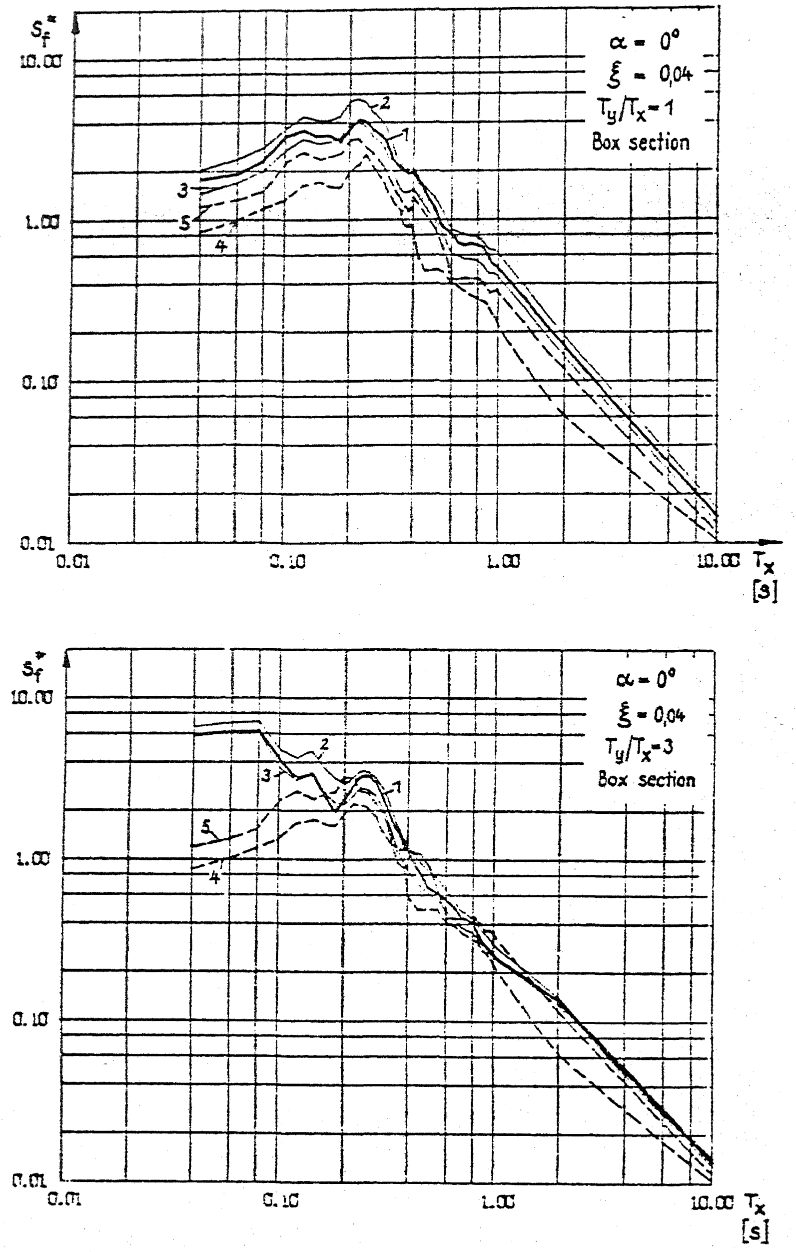

Fig. 7: Stress response spectra for box sections

1: $\max\{a_x(t) + m_y^*/m_x^* a_y(t)\}$ *

2: $\max\{a_x(t)\} + m_y^*/m_x^* \max\{a_y(t)\}$

3: $\sqrt{\max\{a_x^2(t)\} + m_y^*/m_x^* \max\{a_y^2(t)\}}$

4: $\max\{a_x(t)\}$ (1-D)

5: $\max\{a_y(t)\}$ (1-D)