A STUDY OF SEISMIC INFLUENCES ON STRUCTURES CONSIDERING THEIR LENGTH AND HEIGHT

by

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SYNOPSIS

A normal seismic analysis assumes all parts of structural foundation to be subjected to the same displacements and accelerations simultaneously and the seismic disturbances to be propagating instantaneously along the structure's vertical dimension. While for small structures such an assumtion may be quite acceptable, for lengthy and high structures it is distorting the real picture of dynamic behaviour. The report deals with some problems of introducing the length of structures and non-instantaneous vertical propagation of seismic disturbance in design analysis.

EFFECTS OF EXTENSION OF STRUCTURE UNDER SEISMIC INFLUENCES

Let us assume that the different points of structural foundation undergo various displacements $y_o(t,x)$ and accelerations $y_o''(t,x)$ at a given moment of time /Fig.I/. Taking into consideration the fact that the extension of structure is much less than the epicentral distance it shall be assumed that the propagating seismic disturbance reaches a given point of foundation with a certain velocity V_o , depending upon local site conditions and remaining constant within the length of structure $y_o''(t,x) = y_o''(t - x/v_o)$. Equations of forced vibrations for the system shown in Fig.I within the time interval of seismic wave passing from support $i-1$ to support i can be presented in the form

$$-m_k(y_{o1}-y_{ki})''+(y_{ki}-y_{k-1i})\sum_{j=1}^{q}a_{kj}-(y_{k+1i}-y_{ki})\sum_{j=1}^{q}a_{k+1j}=0$$

$$K=2,3,\cdots n; \quad a_{n+1j}=0; \quad l_q=0. \qquad /1/$$

$$-m_1(y_{o1}-y_{1i})''+y_{1i}\sum_{j=1}^{q}a_{1j}-(y_{2i}-y_{1i})\sum_{j=1}^{q}a_{2j}+\sum_{j=1}^{i}y_{oj}\,a_{1j}-y_{o1}\sum_{j=1}^{q}a_{1j}=0$$

$$\sum_{j=1}^{i-1}\frac{l_i}{v_o}<t<\sum_{j=1}^{i}\frac{l_i}{v_o}, \qquad i=1,2,\cdots q$$

After the moment of time when the seismic wave reaches the last q -the support /setting in motion all support/ the equation of damped vibration may be presented as

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$$-m_k(y_{01}-y_k)''+a_k(y_k-y_{k-1})+\frac{\alpha_k}{p}a_k(y_k'-y_{k-1}')-a_{k+1}(y_{k+1}-y_k)-$$

$$-\frac{a_{k+1}}{p}\alpha_{k+1}(y_k'-y_{k+1}')=0, \qquad k=2,3,\cdots n \qquad a_{n+1}=0$$

$$-m_1(y_{01}-y_1)''+y_1a_1+a_1\frac{\alpha_1}{p}y_1'-a_2(y_2-y_1)- \qquad /2/$$

$$-a_2\frac{\alpha_2}{p}(y_2'-y_1')+\sum_{j=1}^{q}y_{0j}a_{1j}-y_{01}\sum_{j=1}^{q}a_{1j}=0, \quad t>\sum_{j=1}^{q}\frac{l_j}{v_0}$$

The systems of equations /I/ and /2/ are integrated at the following initial conditions:

$$\text{with } t=0, \quad y_{k1}=0, \quad y_{k1}'=0$$
$$\text{with } t=\sum_{j=1}^{i}\frac{l_j}{v_0}, \quad y_{ki}=y_{ki-1}, \; y_{ki}'=y_{ki-1}', \quad i=1,2,\cdots q \qquad /3/$$
$$\text{with } t=\sum_{j=1}^{q}\frac{l_j}{v_0}, \quad y_k=y_{kq}, \quad y_k'=y_{kq}'$$

where $m_k, y_k, a_k=\sum a_{kj}, \alpha_k$ - mass displacement /after the wave reaches the last support/, rigidity and damping factor for k -th floor resp., y_{0i} the bottom section displacement of the ground floor i -th column, p natural torsional frequency /y_{ki}-floor displacement at the moment when the wave reaches the i -th support/. Presenting the solution of system of equations /2/ as

$$y_k(t)=\sum_{z=1}^{n}C_{kz}U_z(t) \qquad /4/$$

where C_{kz} are the z- the natural mode vibration amplitudes at k the point, for orthogonal co-ordinates $U_z(t)$ with $\alpha_k=const$ we shall obtain

$$U_z''+P_z^2U_z+\frac{\alpha}{P_z}U_z'=y_0''(t)\frac{\sum_{j=1}^{n}m_iC_{jz}}{\sum_{j=1}^{n}m_iC_{jz}^2}+ \qquad /5/$$

$$+\frac{m_1C_1\left[\sum_{j=1}^{q}a_{1j}y_0\left(t-\sum_{i=1}^{j}\frac{l_{i-1}}{v_0}\right)\right]-y_0(t)\,m_1C_1\sum_{j=1}^{q}a_{1j}}{\sum_{j=1}^{n}m_jC_{jz}^2}. \qquad z=1,2,\cdots n$$

As it to be seen from /5/, in considering the length of structure for seismic force calculations, it is necessary in addition to earthquake accelerogram $y_0''(t)$ to have also an earthquake seismogram $y_0(t)$. To obtain the seismic response spectrum regarding the length of structure let us examine a single degree of freedom frame system /Fig.2/. In this case the linear oscillator equation will be

$$y_0''+\left(\frac{2\pi}{T}\right)^2y+\frac{2\pi}{T}\alpha y'=y_0''(t)-\frac{1}{2}\left(\frac{2\pi}{T}\right)^2\left[y_0\left(t-\frac{l}{v_0}\right)-y_0(t)\right] \qquad /6/$$

at initial conditions: $t=\frac{l}{v_0}$, $y=y_1$, $y'=y_1'$.

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The equation /6/ was integrated for the harmonic vibration

$$y_0''(t) = 100 \sin \frac{2\pi}{T_0} t, \quad T_0 = 0,2 \text{ sec} \qquad /7/$$

Fig.2 shows that sntroducing of length of the structure results in a substantial change of the resonance curve. For period range of $T<0,2$ the introduction of length increases the dynamic ratio up to two or even more. For a range of $0,2 < T < 0,4$ the dynamic ratio is decreasing 1,5-1,7 times and for $T \geq 0,4$ the length influence is negligeable. For some values of ℓ/v_0 the resonance curve takes an inversed form. It is interesting to note that the resonance amplitudes decrease and increase periodically depending on the value of ℓ/v_0 /Fig.3/. Therefore in case of a purely harmonic ground vibration for the given site conditions the length of the structure may be selected so that the oscillation phase difference will be

$$\frac{\ell}{v_0} = \frac{T_0}{2} + K T_0 \; (K = 0,1,2) \qquad /8/$$

where T_0 the dominant period of ground oscillation. In this case there'll be no dynamic effect on the structure.

In some cases practical recommendations may be worked out by employing the averaged ground accelerations and ascribing these accelerations to all structural foundation, thus assuming these accelerations to be

$$y_0''^{M}(t) = 1/\frac{\ell}{v_0} \int_0^{\ell/v_0} y_0''(t-\xi)\, d\xi \qquad /9/$$

By way of numerical integration the $y_0''^{M}(t)$ values were defined on the basis of some real accelerograms. The results shown in Fig.4 indicate that the length of structure is to be considered for structures and site conditions where the $v_0/\ell < 10$ holds. Since averaging does not result in substantial change of the actual frequency content of accelerogram, the same is expected to occur also with structural response values.

INTRODUCING THE FINITE VALUE OF VELOCITY OF SEISMIC DISTURBANCE PROPAGATION ALONG THE HIGHT OF THE STRUCTURE

A seismic study of structure usually assumes the vertical propagation of the disturbance to be instantaneous. Such an assumption is acceptable for low structures. Meanwhile the experimental studies of multietory steel, reinforced concrete, large panel and masonry buildings have proved the stroke-wave propagation velocity to be 200-1000 m/sec. /Fig.5, table 1/ which is substantially low than the similar values registered in a continuous medium. Such a phenomenon may apparently be caused by the presence of joints, floors, openings in buildings and the resulting non-hemogeneous path of wave propagation. Therefore a seismic analysis of high-rise buildings must be based on a finite value of seismic wave velocity.Presented below is a simplified method of introducing this factor. The analysis is carried out for a discrete cantilever lump-mass system. The seismic disturbance is assumed to be propagating from the foundation upward with the velocity v . Simplified kinematics of the system at consecutive mements of time are presented in Fig.6. At the moment $t_1 = h_1/v$, when the disturbance is transmitted to the ground floor, the equation of system movements is

$$\sum_{i=1}^{n} m_i y_0'' + m_1 y_{11}'' + a_1 y_{11} + \frac{a_1 \alpha_1}{p_1} y_{11}' = 0 \qquad /10/$$

Therefore at the time interval of $0 < t < \frac{h_1 + h_2}{v}$ the baha- of the system is defined as that of a single degree of freedom with the ground floor rigidity a_1 , frequency p_1 and damping factor α_1 . The initial conditions for the equation /10/ are $t=0$, $y_{11}=0$, $y_{11}'=0$. At the moment $t_k = \sum h_i/v$ the disturbance has reached the k -the floor the behaviour of the system is defined by following k equations:

$$\sum_{i=k}^{n} m_i y_0'' + \sum_{i=1}^{k} m_i y_{ki} + a_i (y_{ki} - y_{k-1i}) + \frac{a_i \alpha_i}{p_i} (y_{ki}' - y_{k-1i}') = 0 \qquad /11/$$

$$i = 1, 2, \cdots k; \quad k = 2, 3, \cdots n$$

with the initial conditions at

$$t_k = \sum_{i=1}^{k} \frac{h_i}{v}, \quad y_{ki} = y_{ki-1}, \quad y_{ki}' = y_{ki-1}'$$

$$y_{kk} = 0, \quad y_{kk}' = 0, \quad i = 1, 2, \cdots k$$

After the disturbance has reached the top floor the movements of the system are characterized by the complete set of equations:

$$\sum_{i=k}^{n} m_i y_i'' + a_k (y_k - y_{k-1}) + \frac{a_k \alpha_k}{p} (y_k' - y_{k-1}') = -\sum_{i=k}^{n} m_i y_0''(t) \qquad /12/$$

$$k = 1, 2, \cdots n$$

with the initial conditions at

$$t_n = \sum_{i=1}^{n} \frac{h_i}{v}, \quad y_k = y_{kn}, \quad y_k' = y_{kn}'$$

$$k = 1, 2, \cdots n$$

Thus although the state of the system after the disturbance has reached the top floor is described by set of usual equations, the different initial conditions for floor displacements result in a phase difference of floor inertial forces and in subsequent quantitative and qualitative changes of system's stress-strain condition. An example is presented of evaluationg the seismic forces originating in a multistorey reinforced concrete building from a seismic stroke. To simplify the analysis the floor rigidity of the structure is assumed to be constant all over the hight and the first normal mode period $T_1 = 0,1n$. The results show the non--instantaneous propagation of seismic disturbance to influence substantially the shear force maximum values and the overall stress-strain distribution along the hight of the structure /table 2, Fig.7/.

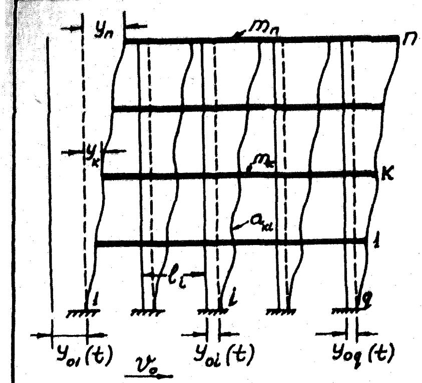

FIG. 1 DESIGN MODEL

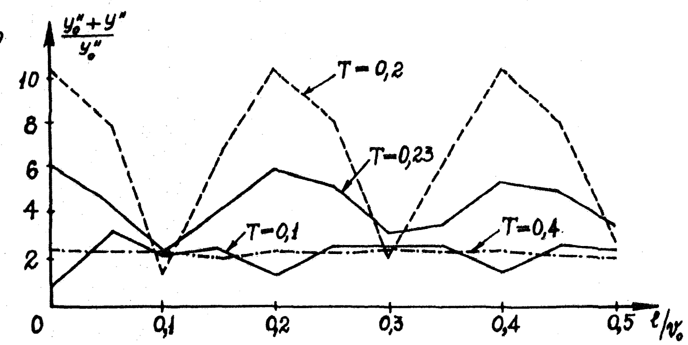

FIG. 3 DYNAMIC RATIO VARIATIONS

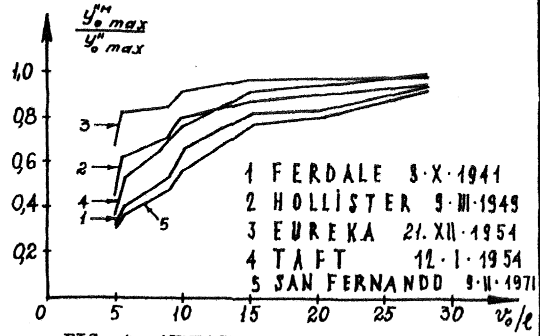

FIG. 4 AVERAGED GROUND ACCELERATIONS

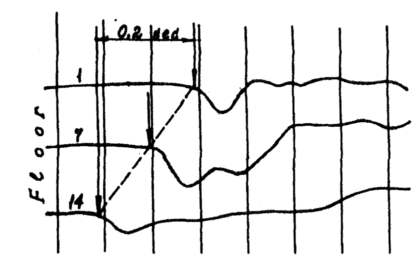

FIG. 5. INSTRUMENTAL RECORDS OF FLOOR DISPLACEMENTS RESULTING FROM A STROKE ON TOP OF THE BUILDING

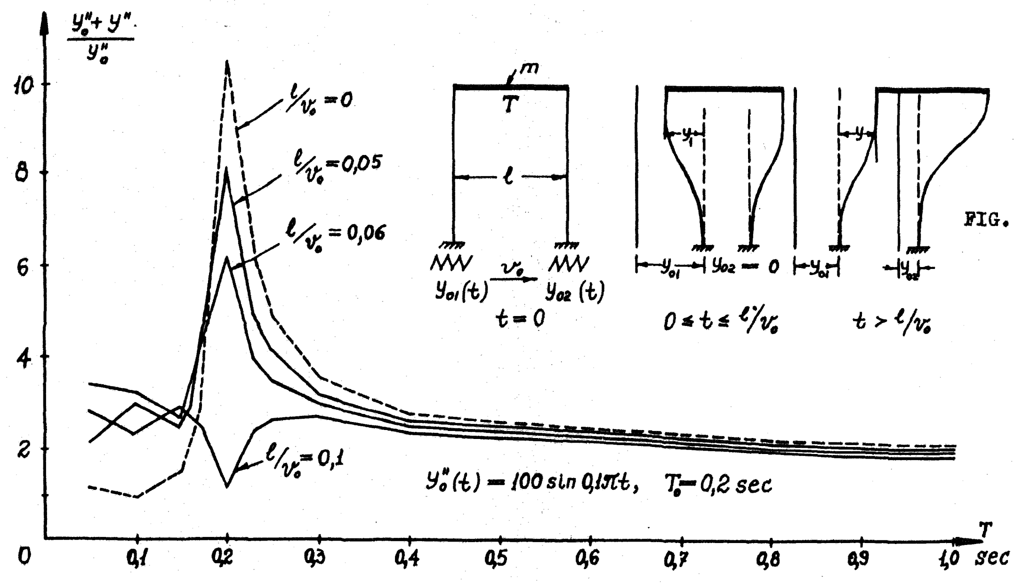

FIG. 2 RESONANCE CURVES OF THE LINEAR ASCILLATOR

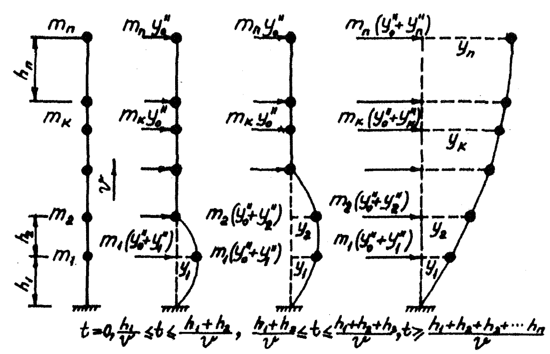

FIG. 6 SUCCESSIVE POSITIONS OF THE SYSTEM

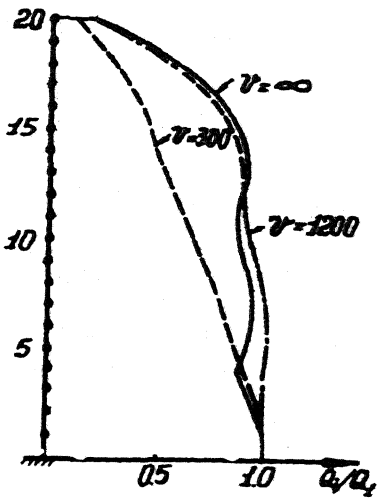

FIG. 7 SHEAR-FORCE DISTRIBUTION IN A 20 STORY BUILDING

Table 1.

Experimental values of the periods and velocities of shear waves in buildings

Type of structure	Number of storeys	Period of ambient vibration T_1	Velocity m/sec v
Buildings with large stone-block walls	5	0,3	600
	5	0,25	750
	5	0,21	690
	5	0,28	710
Large panel buildings	9	0,3	900
	9	0,36	800
	9	0,40	800
Reinforced concrete framed buildings	9	0,6	600
	10	0,8	250
	14	1,1	210
	16	1,3	330
Metal framed buildings	16	1,35	228

Table 2.

Maximum values of shear forces at the $/Q_1T_1/2\pi m\dot{y}_0'/$ ground floor in buildings of various hights $/n$ -number of storeys/

v	$n = 5$	$n = 10$	$n = 15$	$n = 20$
∞	3,73	7,39	11,01	14,63
1200	3,54	6,84	10,20	13,56
800	3,70	7,16	10,77	14,29
500	4,03	8,12	12,19	16,15
300	4,73	9,70	15,03	19,92
200	5,60	12,24	18,46	24,48