EARTHQUAKE DESIGN OF REINFORCED CONCRETE COOLING TOWERS

by

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Reinforced concrete cooling towers of thermal power stations are space structures that can be represented as shells of revolutions with the height amounting to 150 m. The principal loads for these rather important structures under ordinary conditions are the dead load and the wind load. In the areas of high seismicity seismic loads should also be taken into account. Numerous publications (i.e. /1, 2, 3/) pay attention to the above factors within the bounds of different seismic stability theories.

A method for calculating reinforced concrete cooling towers according to the Standards in force in the USSR at present /4/ is given below.

A cooling tower is studied as a shell of revolution by the linear moment theory of thin shells with the elasticity law in a Balabuh-Novozhilov formulation /5/. The shell material is adopted homogeneous and isotropic. The middle surface of the shell is formed by rotating a smooth curve $r=r(z)$ around the axis of rotation Oz : the position of points of the middle plane is defined by coordinates z and φ . The shell may comprize two zones: the upper zone - a hyperboloid of revolution and the lower zone - a truncated cone. The following fastening conditions are assumed for the end plate sections of the shell z_a and z_b : rigid fastening, movable and immovable links, free edge, continuous linear - deformable support.

The definition of the stress-strain state of the structure within the limits of a linear spectral seismic stability theory necessitates the solution of the following problems:

1) the determination of natural vibration frequencies (periods) and modes of a shell;

2) the evaluation of seismic loads corresponding to each mode;

3) the definition of the forces and displacements corresponding to the seismic loads over each mode (static problem);

4) the estimation of the design forces and moments within a shell for several vibration modes.

By a method given below all the four problems are successively solved.

The determination of natural vibration modes and frequencies is based on a variational equation of the problem:

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$$-\omega^2 \delta T_k + \delta V_k = 0 \qquad (1)$$

where ω - natural frequency to be determined; T_k, V_k - kinetic and potential energies of the system respectively. The values of T_k and V_k are expressed in terms of the components of the displacement vector

$$\bar{u}_k = (u_k(z)\cos k\varphi,\ v_k(z)\sin k\varphi,\ w_k(z)\cos k\varphi) \qquad (2)$$

where $u_k(z)$, $v_k(z)$, $w_k(z)$ are the components of the displacement vector of a generatrix $\varphi = 0$ in the meridial, tangential and normal **directions** to the middle surface $k = 0, 1, \ldots\ldots n$ - circumferential wave number. Equation (1) is solved by the finite element method according to a scheme given in (6). The interval $[z_a, z_b]$ is subdivided into n elements. Within each element the displacements u_k, v_k are approximated by a linear function, and w_k is approximated by a cubic parabola. The substitution of the approximating polynomials into equation (1) leads to an algebraic problem of the eigenvalues for a set:

$$\begin{aligned} B_0 x_0 + C_0 x_1 &= \omega^2 (E_0 x_0 + G_0 x_1) \\ A_j x_{j-1} + B_j x_j + C_j x_{j+1} &= \omega^2 (D_j x_{j-1} + E_j x_j + G_j x_{j+1}) \\ A_n x_{n-1} + B_n x_n &= \omega^2 (D_n x_{n-1} + E_n x_n) \end{aligned} \qquad (3)$$

Here A_j, B_j, C_j, D_j, E_j, G_j - 4x4 matrices, $x_j^T = (u_{kj}, v_{kj}, w_{kj}, w'_{kj})$ (index T stands for transponency); j = 1, 2,..... n-1 - node number.

The method for the determination of the eigenvalues and the eigenvectors of the system (3) is described in /6/.

A program for the determination of free vibration modes and frequencies is worked out, allowing to evaluate the contributions of such structural features as height, shape of the a meridian, law of variation of thickness, foundation pliability, rigid and inertial effect of the side rings of the tower. A numerical procedure /7/ revealed that the foundation pliability affects most appreciably the mode and frequency spectrum. The boundary conditions of a rigid fastening or a link type yield essentially the same results. Small variations of the meridian shape, the presence of the rigidity rings and thickness variations of the lower sections of the tower produce but a negligible effect upon the spectra of natural modes and frequencies.

Contribution of seismic load in point j corresponding to the i-th vibrational mode is defined with the expression /4/:

$$S_{ij} = Q_j K_c \beta_i \eta_i \qquad (4)$$

where Q_j - the weight of the mass in point j ; K_c - seismicity coefficient ; β_i - dynamic coefficient; η_i - mode coefficient determined as a vibration mode multiplied by Fourier coefficient.

To obtain an expression η_i for a case under study, reasoning and calculations are carried out on the basis of /8/.

A equation for the induced seismic vibrations of the shell assumes the shape of:

$$L\bar{u}(t) + m(z)\ddot{\bar{u}}(t) = m(z)\ddot{\bar{A}}(t) \tag{5}$$

Here L is a differential operator of the eighth order /5/, $\bar{u}(t) = (u(t), v(t), w(t))$ - elastic displacement vector; $\ddot{\bar{A}} = \bar{F}\ddot{Y}(t)$ - base acceleration vector ($\bar{F}$ - uniy vector).

Seismic load vector

$$\bar{S} = (S^{(1)}, S^{(2)}, S^{(3)})^T = m(z)(\ddot{\bar{u}} + \ddot{\bar{A}}) \tag{6}$$

Introducing (5) in the form of

$$\bar{u}(t) = \sum_{i=1}^{\infty} \bar{u}\xi(t) \tag{7}$$

where $\bar{u}_i$ - eigenmodes; we find out that the normal coordinates satisfy the equations:

$$\ddot{\xi}_i(t) + \omega^2\xi_i(t) = D_i\ddot{Y}(t) \tag{8}$$

Here

$$D_i = \frac{\iint_\Omega m(z)\bar{F}\bar{u}^T d\Omega}{\iint_\Omega m(z)\bar{u}_i^2 d\Omega} \tag{9}$$

If a term accounting for the energy dissipation is introduced into (8), after the solution of the equation (with consideration of (6) and (7)) the following relation is obtained:

$$\bar{S}(t) = \sum_{i=1}^{\infty} \bar{S}_i(t) = = \sum_{i=1}^{\infty} m(z)\bar{u}_i D_i \omega_i \int_0^t \ddot{Y}(\tau) e^{-\frac{\gamma}{2}\omega_i(t-\tau)} \sin\omega_i(t-\tau) d\tau \tag{10}$$

γ - energy dissipation factor.

Hence

$$\bar{S}_i = max_t \,|\, \bar{S}_i(t) \,| \;= m(z) D_i \bar{u}_i C_w(T_i) \tag{11}$$

where $C_w(T_i)$ - acceleration spectrum.

Equation (11) is presented in the form corresponding to (4)

$$\bar{S}_i = K_c Q(z) \beta_i \bar{\eta}_i \tag{12}$$

where

$$\bar{\eta}_i = \bar{u}_i D_i \tag{13}$$

With account for the expression for the displacement vector under free vibrations (2), the seismic load vector components are written down as

$$\begin{aligned} S_{ik}^{(1)} &= K_c Q(z) \beta_i u_{ik} D_{ik} \cos k\varphi \\ S_{ik}^{(2)} &= K_c Q(z) \beta_i v_{ik} D_{ik} \sin k\varphi \\ S_{ik}^{(3)} &= K_c Q(z) \beta_i w_{ik} D_{ik} \cos k\varphi \end{aligned} \tag{14}$$

Under horizontal seismic effects, the expansion of the seismic load by the angular coordinate retains only one term corresponding to k = 1, and coefficient D_i assumes the form:

$$D_i = \frac{\int_0^H Q(z)(-u_i \sin\alpha + v_i + w_i \cos\alpha) A_1 \cos\psi \, dz}{\int_0^H Q(z)(u_i^2 + v_i^2 + w_i^2) A_1 \, dz} \tag{15}$$

Here α is an angle between the vertical axis and the tangent of hyperbola; ψ is an angle between the horizontal axis and the direction of the seismic impact; A_1 - Lame parameter /5/.

Under vertical vibrations of the base one term of the expansion, corresponding to k = 0, is also retained. Thus, the expression for D_i assumes the following shape:

$$D_i = \frac{\int_0^H Q(z)(u_i \cos\alpha - w_i \sin\alpha) A_1 \sin\psi \, dz}{\int_0^H Q(z)(u_i^2 + w_i^2) A_1 \, dz} \tag{16}$$

With the seismic influence arbitrarily directed in space, the Fourier series of the load expansion over the angular coordinate contains the terms corresponding to $k = 0$ and $k = 1$.

After the determination of all the seismic load components a solution for the shell under static load must be obtained (for each vibration mode).

With the loading conditions of the type (14), the solution of the problem express in form:

$$u = u_k(z)\cos k\varphi; \quad v = v_k(z)\sin k\varphi; \quad w = w_k(z)\cos k\varphi \tag{17}$$

The static problem is solved in terms of displacements by a variational method. Obtained is a variational equation of Lagrange

$$\delta(V - A) = 0 \tag{18}$$

where V - potential energy of deformation; A - the work of the external forces.

The finite element method is applied with the approximation of displacement identical to that of a free vibration problem (piece-line functions for u_k, v_k and Hermitian polynomials for w_k)

The substitution of the approximating polynomials into (18) results in a system of algebraic equations:

$$\begin{aligned} &B_0 x_0 + C_0 x_1 = f_0 \\ &A_j x_{j-1} + B_j x_j + C_j x_{j+1} = f_j \\ &A_n x_{n-1} + B_n x_n = f_n \end{aligned} \tag{19}$$

Here

$$x_j^T = (u_{ikj}, v_{ikj}, w_{ikj}, w'_{ikj}); \quad f_j^T = (f^1, f^2, f^3, f^4)$$

A_j, B_j, C_j - 4x4 matrices.

A set (19) is solved by block exclusion by Gauss method. From the solution defined are the displacements u_{ikj}, v_{ikj}, w_{ikj}, w'_{ikj} after which with the use of the elasticity law and the numerical differentiation formulas, efforts and moments in the j-th points of the structure for each i-th mode and k-th harmonic are obtained.

The design forces summed over several modes are obtained by /4/

$$N_d = \sqrt{N_{max}^2 + 0{,}5\sum_{i=1}^{n} N_i^2} \tag{20}$$

N_{max} maximum force in a design section; N_i - forces over the whole number of the vibration modes for the same section (excluding N_{max}).

In this case an exact number of modes taken into account for achieving an acceptable accuracy can be defined only by a numerical procedure (for cooling towers of different geometry and different directions of seismic impact).

Calculation example. As a calculation example studied is a problem of determining the forces developed under horizontal and vertical seismic impact in a hyperboloidal cooling tower with the following characteristic parameters: height = 90 m, radius of the base R_a = 38.2 m, thickness of the lower zone (truncated cone) variating from 0.6 m at the base up to 0.14 for the height of 20 m and remaining constant with further increase in height; parameters of hyperbola of the upper zone a = 20.3 m b = 46.28 m, elastic modulus E = 3.15x10^5 MPa, density ρ = 2.5Mg/m^3; Poisson's ratio ν = 0.16. The following boundary conditions were set: rigid fixing of a shell at the base and a free end at the top. Calculations reveal that the inclusion of the higher modes of vibration into the evaluation of meridial forces under vertical impacts is not required. The maximum meridial force under the vertical seismic impact and seismic coefficient K_c = 0.1 N_1 = 50 kN/m. Under the horizontal seismic impact and K_c = 0.1 the maximum meridial force N_1 = 70 kN/m the inclusion of the higher modes of vibration does not substantially affect the value N_1 .

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