EQUIVALENT PROPERTIES AND DUCTILITY REQUIREMENTS IN SEISMIC DYNAMIC ANALYSIS OF NONLINEAR SYSTEMS

by

O. R. Guerra[1] and L. Esteva[1]

SYNOPSIS

Ductility demands of a number of shear systems subjected to earthquake ground motion are studied.. Variables covered include natural period and distribution of stiffness and strength throughout the building height. Stories are assumed to show elastoplastic behavior in shear and the response is obtained by step-by-step integration of the equations of motion. An approximate equivalent linearization criterion is proposed and its results compared with those obtained by the mentioned step-by-step analysis.

INTRODUCTION

Seismic excitation is usually specified for design purposes in terms of spectra reduced to account for ductile behavior. These spectra are ordinarily used in combination with a linear (modal) dynamic analysis, and the assumption is implied that ductility demands throughout the system do not depart substantially form those implicit in the reduced spectra, which are based on the computed nonlinear response of single degree of freedom systems. Because ductility demands (perhaps weighted according to the number of times each value is exceeded during an earthquake) are among the variables most directly related to seismic structural behavior, it is natural to take as measures of earthquake effects. The few available results point at the need for critically examining conventional criteria for specification of seismic loads.

A brief systematic study of the nonlinear response of multistory shear buildings with elastoplastic load-deflection curves was recently carried out by Frank *et al* (1976). A number of four-story buildings was "designed" in such a manner that stories were assigned lateral strengths equal to the story shears obtained form a linear dynamic analysis that included only the fundamental mode. The design spectrum was equal to the elastoplastic spectrum for 2 per cent viscous damping and a ductility factor of 4 obtained averaging the corresponding elastoplastic spectra for two ensembles of motions: one of natural earthquake accelerograms normalized to the same peak ground acceleration and another of simulated motions.They found that the coefficients of variation of ductility are very high and that the expected values are substantially higher than the aimed-for value of 4 at the uppermost and first stories. This means that even for these simple and uniform systems conventional design criteria lead to systematic discrepancies between aimed-for and actual seismic effects.

This paper is devoted to the study of ductility demands at the stories of shear buildings for various combinations of strengths and stiffnesses. It also proposes and calibrates an equivalent linearization criterion adequate for applications to seismic design. Both objectives are pursued through application of the proposed criterion and of Wilson's step-by-step numerical integration scheme to the computation of the nonlinear response of shear buildings to three accelerograms recorded on soft soil in Mexico City. A typical accelerogram and its corresponding linear spectra for various damping ratios are shown in fig 2. The smoothed average elastic spectrum for 2 per cent damping is designated as spectrum E in the sequel.

SYSTEMS STUDIED

It is intended to study ductility demands in buildings with different natural periods and forms of variation of story-stiffness. Besides considering those buildings whose stories possess the strengths obtained through application of a conventional analysis and design criterion, it is intended to study the influence on seismic response of the variability of the safety factors with respect to story shears throughout the building height. Such variability may stem as a consequence of architectural requirements, which often lead to some stories possessing

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[1] Institute of Engineering, National University of Mexico

elements stronger that they need to be in order to comply with the adopted seismic coefficient. When this occurs, the relative contribution of each story to the hysteretic dissipation of kinetic energy changes, and those stories possessing the smallest safety factors are expected to be subjected to higher ductility demands than those that would affect them should the safety factor be uniform throughout the construction.

A number of ten-story shear buildings with elastoplastic load-deflection curves were analysed. Viscous damping of 2 per cent of critical was considered in all cases; fundamental periods were 0.5, 0.1 and 2.5 s. Non uniform overstrength factors (ratios of available to required shear capacity) were assumed in some systems, in order to simulate the often undesirable contribution of architectural elements. Assumed distributions of story stiffnesses and overstrength factors are displayed in fig 1. Cases studied are designated by two digits, the first indicating the type of stiffness distribution and the second the type of overstrength factors. Cases analysed were: 11, 12, 13, 14, 15, 21, 25, 32, 41 and 44. Lateral story-strengths were made equal to the story shears determined from a modal elastic analysis based on average values of the reduced spectra, designated by R in the sequel. These spectra are intended to represent elastoplastic acceleration spectra for 2 per cent of critical damping and ductility factor of 4. However, they were not obtained in a rigorous manner but, instead, an approximation to them was produced by dividing by 4 all ordinates of the acceleration elastic response spectra corresponding to periods longer than 2.5 s (where maximum ordinates take place) and by a factor varying linearly from 1 to 4 with respect to natural period for values of the latter comprised between 0 and 2.5 s. Because ductilities in the short period range are very sensitive to the ratio of yield strength to elastic response, adopted spectral ordinates may correspond to nominal ductilities (as defined below) significantly different from 4.

ACTUAL, NOMINAL AND PREDICTED DUCTILITIES

Ductility demands were computed for all systems by step-by-step integration of the equations of motion. Values so determined will be designated as *actual* ductilities and will be compared with *predicted* and with *nominal* ductilities. For unitary overstrength factors, *nominal* ductilities are those assumed to be valid for the selected design spectrum (in this case, 4). When the overstrength factor differs from unity, nominal ductility at each story is taken as the value implicit in the design spectrum times the ratio of the modal elastic shear to the available shear capacity. *Predicted* ductilities are those obtained according with the equivalent linearization criterion described below.

EQUIVALENT LINEARIZATION CRITERION

The problem of equivalent linearization for multidegree of freedom systems has been studied elsewhere within the framework of stochastic stationary processes for various types of load-deflection curves (Iwan, 1974). The complexity of the proposed criteria precludes their application to the approximate nonlinear analysis of civil engineering structures with moderate or large numbers of degrees of freedom. Hence the need to develop simpler, heuristically based criteria, inspired on some shown to be valid for simple systems. One such criteria was proposed by Newmark and Rosenblueth (1971) for simple systems and is extended here to multidegree-of-freedom systems.

In its extended version, the criterion consists in defining the equivalent system in such a manner that its masses are equal to those of the original system, modal damping fractions are all equal and are obtained as averages of values corresponding to all stories, and story stiffnesses are made equal to their secant values. Both damping ratio and stiffness for a given story are obtained as averages of their corresponding values when the story distortion is made to vary from 0 to its maximum amplitude. Accordingly, modal damping, as a fraction of critical, is given as follows,

$$\zeta' = \zeta + \frac{1}{2\pi n} \sum_{i=1}^{n} \frac{1}{D_i} \int_0^{D_i} \frac{H_i(x)}{x^2 K_i(x)} \, dx, \tag{1}$$

and story stiffness,

$$k_i' = \frac{1}{D_i}\int_0^{D_i} k_i(x)dx\ , \qquad (2)$$

where ζ'= equivalent modal damping ratio, ζ = viscous modal damping ratio, n = number of stories, i = index that identifies story, D_i = maximum story strain, x = story strain, H(x) = hysteretic area, K(x) = secant stiffness, k' = equivalent stiffness.

Computation of the response proceeds iteratively. It starts with an initial estimate of stiffnesses and damping values, and applies modal superposition in order to obtain a first estimate of story distorsions. New estimates of the properties of the equivalent system are then obtained and the cycle is repeated as many times as necessary to obtain nearly equal system configurations in two consecutive cycles.

ANALYSIS OF RESULTS

Figures 3-12 show some results. A complete set of figures is available in a report by the authors (Guerra and Esteva, 1976). Curves RAN show mean values and ranges of the ratios of actual to nominal ductilities for structures designed according with spectrum R and RAP show ratios of actual to predicted ductilities for the same structures. From examination of the complete set of results the following conclusions are reached:

> The proposed linearization criteria provides in general a good estimate of ductility demands, with the exception of those corresponding to stories where they are largest in systems with short natural periods: those ductilities are sistematically underestimated and errors are inadmissibly high. *Nominal* values depart from *actual* ones considerably more than those *predicted*, but this conclusion must be taken catiously, at least for short period systems, as a consequence of the manner in which *elastoplastic spectra were estimated. Having in mind the systematic nature of the* deviations shown by the proposed linearization criterion it seems worth trying to modify the manner in which system properties are estimated, as it is likely than an improved version can be obtained that will furnish acceptable results in all cases.
>
> Ductility demand at first story is usually underestimated, even in uniform systems.
>
> An effect of providing excessive strength at some sections of shear systems is to increase ductility demands at others. This increment is more significant for short period systems.

ADDITIONAL STUDIES

The results just described point at the need for studying alternate criteria for specifiying the distribution of shear capacities in an attempt to achieve uniform ductility demands. Some additional systems were studied:

Systems 11a. All properties were equal to those of systems 11 with T = 0.5 and 2.5s respectively, but with the strength of the bottom story was increased in 10 per cent. Fig 13 shows ductility demands for both the new and the original systems. It can be seen that in some cases a minor increase in the strength of the bottom story transfers substantial ductility demands to the second story. Because random deviations in strength greater than the one considered here are to be expected in actual structures, the problem of distribution of ductility demands should be studied in models that account for random deviations in structural strength.

Systems 11b. These were similar to systems 11 with T = 0.5 and 2.5s respectively, but the contribution of higher natural modes was neglected when computing design shears. Fig 14 shows significant increases of ductility demands at uper stories, despite the fact that strengths are only slightly lower than those of original system 11.

CONCLUSIONS

Two groups of conclusions can be derived from this study. The first refers to the possibility of formulating sufficiently simple equivalent linearization criteria; the second, to some characteristics of the nonlinear response of elastoplastic shear systems.

The proposed equivalent linearization criterion provides in general a good estimation of ductility demands, but it systematically underestimates those demands where they are highest. However, it is likely that the criterion can be substantially improved by simply changing the rules that are used to weight secant stiffnesses and hysteretic damping values corresponding to different response amplitudes. In view of the advantages tied to an approximate criterion for estimation of the maximum response of nonlinear systems, it seems advisable to attempt the proposed improvement.

An analysis of the actual ductility demands shows that conventional criteria of seismic analysis and design fail to provide, even for the simplest systems, an adequate control of seismic response of structures, expressed in terms of the ductilities to be developed by different elements. In some systems, where the safety factor, defined as the ratio of available strength to the internal force assumed to be acting as predicted by a linear analysis, varies appreciably from one critical section to another, ductility demands at some sections may turn out to be much higher than those that would take place at the same sections, with the same safety factors, should these be uniform throughout the structure. The implications of these results should be carried over to structural design practice.

ACKNOWLEDGEMENT

The authors are indebted to S. E. Ruiz and J. A. Avila for their assistance in the formulation of computer programs and in the performance of the computations.

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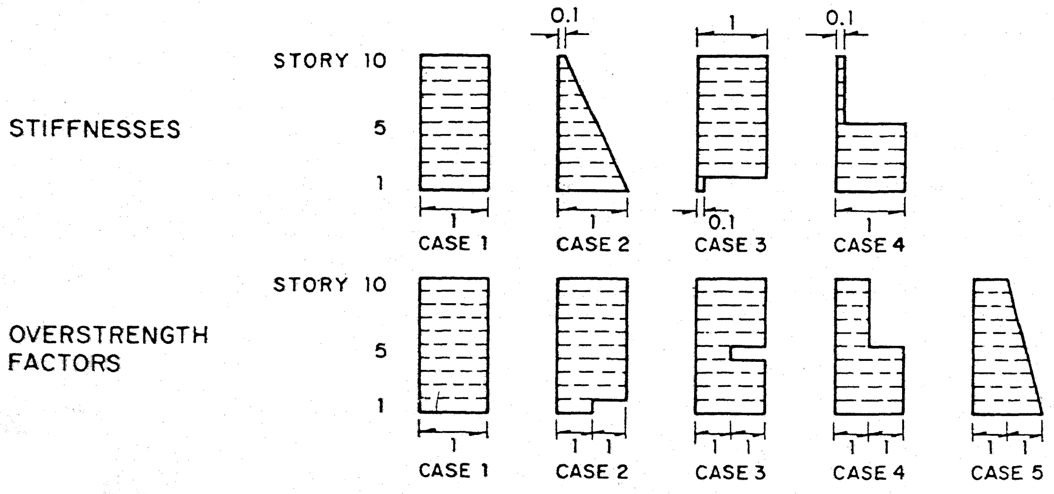

Fig 1. Variation of stiffnesses and overstrength factors

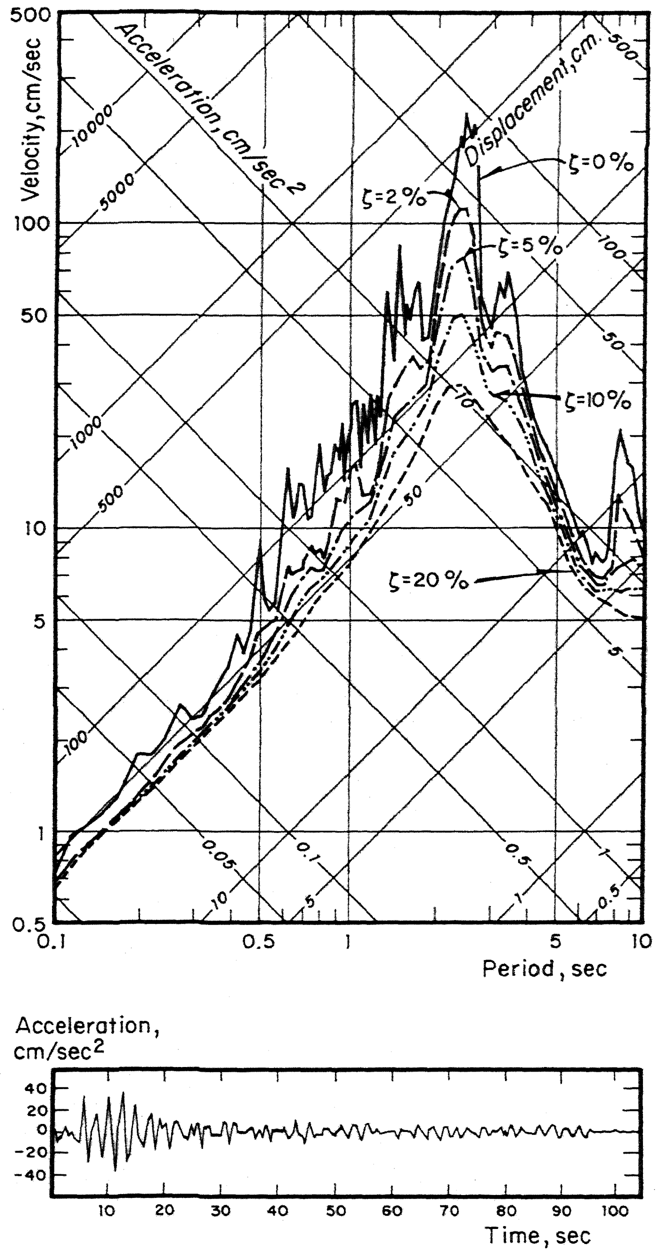

Fig 2. México city earthquake of 2 august 1964, E-W component

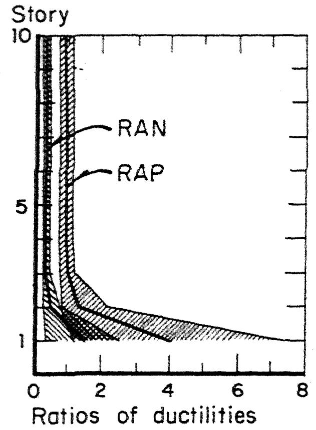

Fig 3. Case 11, T=0.5 sec

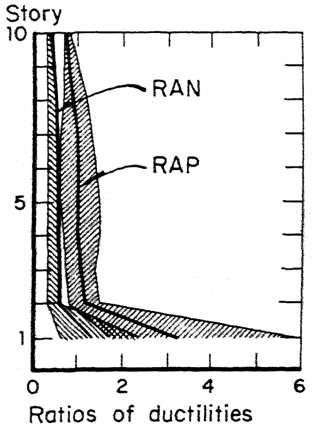

Fig 4. Case 11, T=1 sec

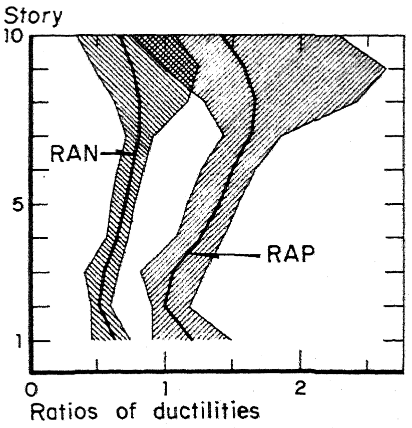

Fig 5. Case 11, T=2.5 sec

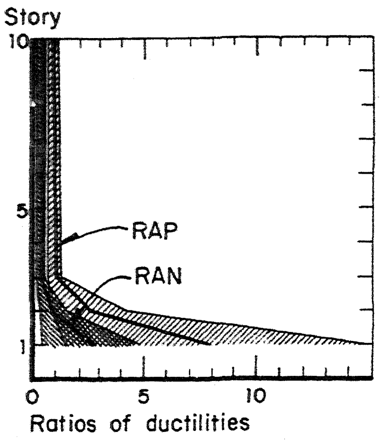

Fig 6. Case 21, T=0.5 sec

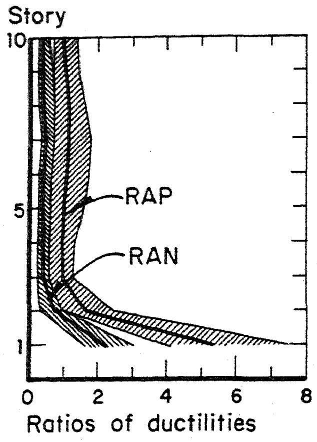

Fig 7. Case 21, T=1 sec

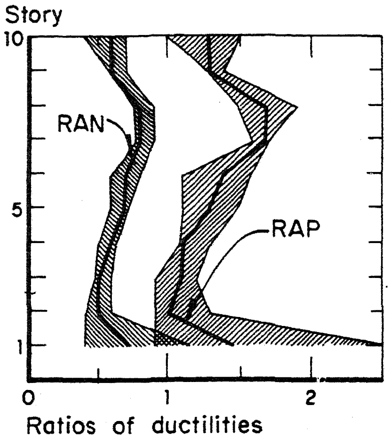

Fig 8. Case 21, T=2.5 sec

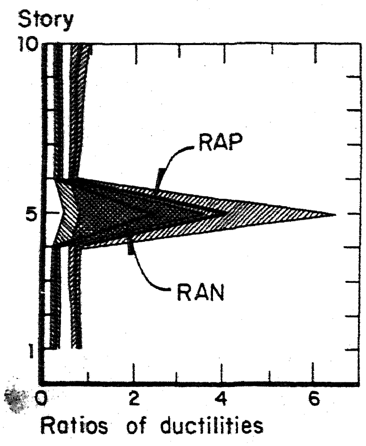

Fig 9. Case 13, T=1 sec

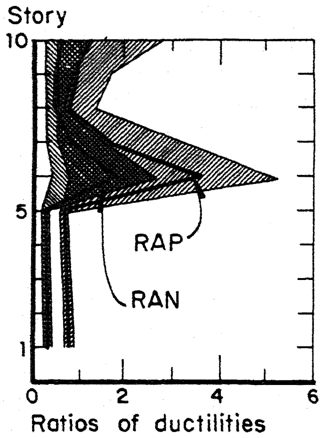

Fig 10. Case 14, T=1 sec

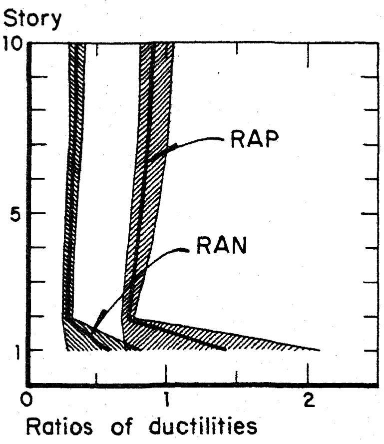

Fig 11. Case 32, T=1 sec

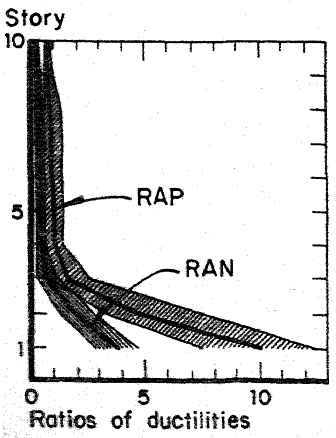

Fig 12. Case 41, T=1 sec

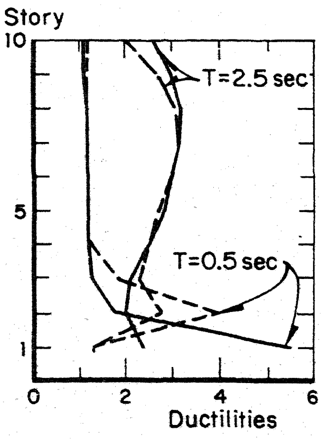

Fig 13. Case 11a

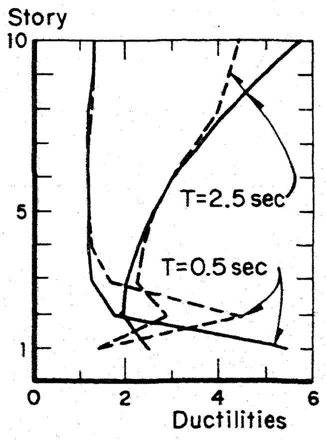

Fig 14. Case 11b

DISCUSSION

B. Chandra and A.R. Chandrasekaran (India)

Inelastic response of multistoreyed shear structures is presented and an attempt is made by the authors to work out, in an iteratative fashion, an equivalent elastic system to estimate the ductility requirements buildings. However as the authors themselves indicate the deviations in the predicted ductility and the actual one are rather large. In view of the fact that such ductilities are closely associated with the properties of ground motion, it would be important to know whether the results have been checked for any other strong motion earthquake record like El Centro 1940 or Koyna 1967 shocks.

Analysis of multistorey shear structures with elasto-plastic characteristics was studied by the writers six years ago (Ref. 1 and 2) and similar results were worked out for two strong ground motion shocks (Koyna and El Centro). For uniform structures and also for tapering structures (Stiffness wise) the ductility demands of a 10 storeyed system are shown in Fig. 1. The authors may like to compare the nature of ductility distribution obtained in the two cases.

It may also be pointed out that the accelerogram chosen for the study by the authors had peak ground acceleration of less than 4% g and the resulting ductilities are of the order of 5 or so. Obviously the requirement would be rather large in a moderate shock. What do the authors suggest in order to ensure that this ductility demand is available in the structure ?

References:

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2. Chandra, B. and Chandrasekaran, A.R., "Elasto-Plastic Behaviour of Multistorey Frames", Proc. Fourth Symposium on Earthquake Engineering, Roorkee, 1970.

S.A. Anagnostopoulos (U.S.A.)

In most of the figures in the paper large amounts of yielding is exhibited in the first story of the structure. Our experience however, has indicated that this is not an actual weakness of the structure but rather of the shear beam model. Use of more sophisticated modelling (e.g. with plastic hinges at member ends) has shown that first story yielding is not as excessive as the shear model predicts.

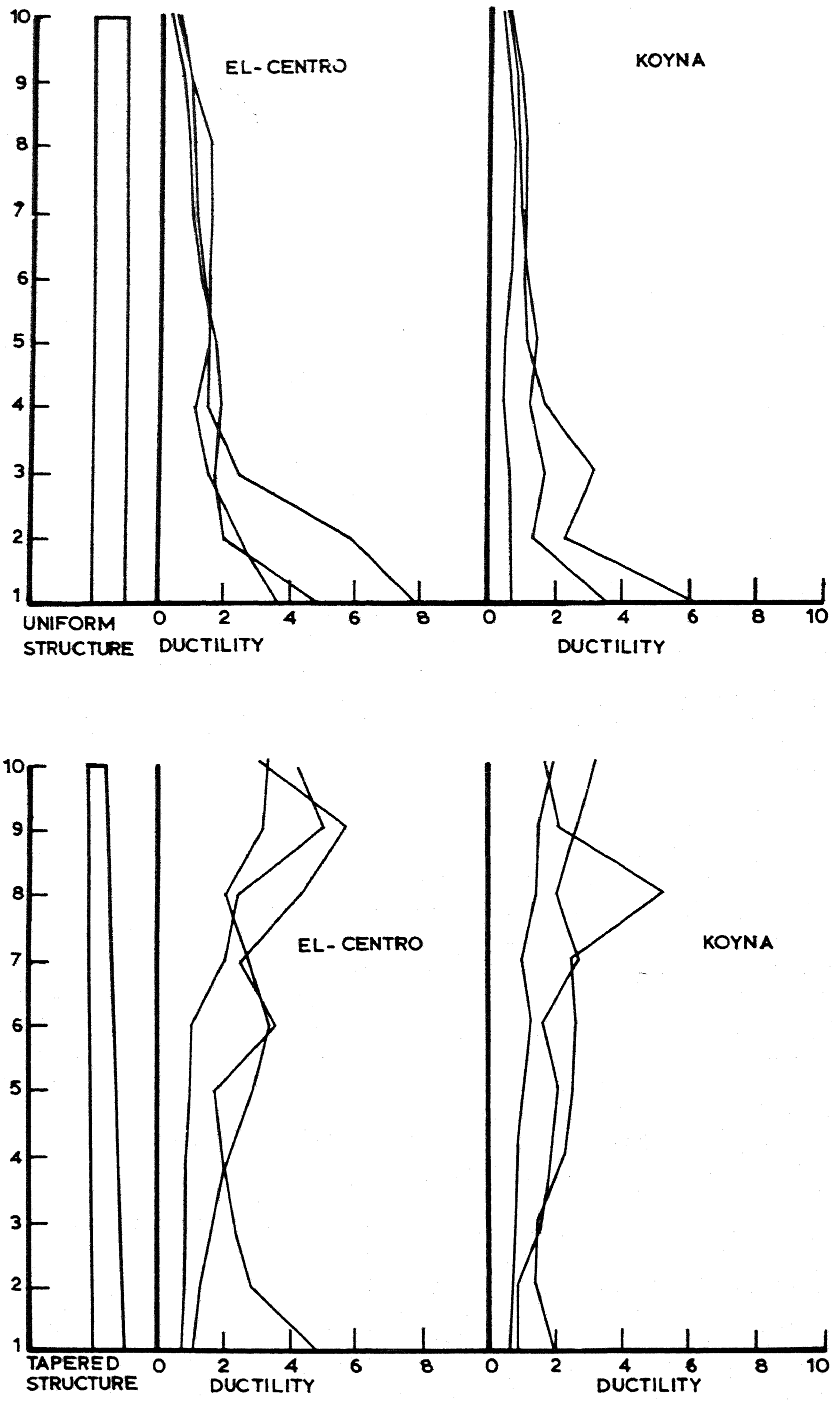

FIG. 1_ Ductility demands of Uniform and Tapering Structures for EL Centro of Koyna Shocks.

Author's Closure

Not received.