EARTHQUAKE RESISTANT DESIGN FOR STEEL BUILDINGS

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SYNOPSIS

The ultimate strength design method for steel-frame buildings subjected to severe earthquakes is presented. The proposed method is based on current knowledges on the inelastic deformability of steel skeletons and the elementary response characteristics of inelastic vibrational systems observed through a vast amount of numerical analyses. An underlying concept is the energy concept similar to that proposed by Housner. The structure is capable of resisting to an earthquake when the energy absorption capacity is greater than the energy input by the earthquake. The concentration of the energy input into a story is the most important matter to be considered in the design of multi-story structures.

1. INTRODUCTION

Although the current design practice is not more than to proportion the structure to resist to the lateral shear forces prescribed in the Code, it is recognized that the inelastic deformability is an important resistance of the structure to earthquakes and together with nonlinear response analyses, enormous amount of experimental works on the deformability of structural elements have been made. These experimental data, however, have not been fully utilized to formulate the design method, because most of researchers have shown less interest in evaluating the difference of restoring force characteristics of individual structures and the model adopted for the response analysis was sometimes too idealized or too specialized to discern the essential collapse mechanism of the structure.

Housner's energy concept, inspite of it's simplicity and high consistency, has not been followed by it's advanced application to more detailed design practices(1),(2).

In this paper, to fill the gap between the response analysis and the experiment, the elementary formulation of the ultimate strength design method is sought after. Through response analyses based on the realistic restoring force characteristics it was made certain that the energy concept is applicable to general structures and the total energy input exerted by an earthquake is an invariable quantity scarcely affected by both the strength of the structure and the configuration of restoring force characteristics. Total energy input balances with the energy absorption of the structure. To controle vertical distribution of damages is one of the essentials in the aseismic design of multi-story buildings. When the strength distribution is well balanced, the damages may be evenly apportioned. Otherwise, the damages concentrate into a relatively weak story. It was found that the distribution of damages is also influenced by the configuration of the restoring force characteristics and this makes it difficult to obtain the unified design criterion applicable to any type of structures.

In this paper, the object is limited to the shear-type rigid frame structures whose inelastic behaviors are well understood, and an emphasis is placed on evaluating the effect of the strain-hardening of steels. Basic philosophy concerning to the safety is as follows: The structures must be prevented from the ultimate collapse under the expected severest earthquake and the structure must be kept from such damages as cause the fracture of non-structural components such as claddings under moderate earthquakes.

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2. BASIC CONSIDERATION

The basic consideration on which the proposed design method is built is briefly summarized as follows.

1) The structure can be divided into plane frames and the plane frame can be considered to be a collection of story-frames as shown in Fig.1. The isolated story-frame acquires a proper restoring force characteristics. Thus, the structure can be reduced to an idealized lumped mass system.

2) The deformation characteristics of the story-frame are such as illustrated in Fig.2. In the figure, Q_i denotes the lateral shear force applied to the story and δ_i denotes the relative story displacement. $Q_i - \delta_i$ relation under a monotonic loading can be expressed by a bi-linear relation as shown in Fig.2(a). Beyond the yield strength of the story, Q_{yi}, the story proceeds into plastic region, where most of structural members in the story are stressed plastically and the rest remain still elastic. The line with a slope, k_{si} is a basal line which represents the effect of gravity loading(P-Δ effect) and the elastic components of the story, and the slope, k_{si} is written as

$$k_{si} = k_{ei} - \frac{W\delta_i}{H}, \qquad (1)$$

where k_{ei}=rigidity of the elastic component of the story, W=total weight above the story, H=story height.

The inelastic behavior is remarkably affected by the strain-hardening of steels. The contribution of the strain-hardening is evaluated by k_{hi}.

$$k_{hi} = k_{pi} - k_{si} \,. \qquad (2)$$

k_{pi} is rigidity in inelastic range. Usually, k_{hi} reaches a several percent of k_i (3). There exists a definite correspondence between the $Q_i - \delta_i$ relation under the monotonic loading and that under an arbitrarily changing deformation history (4). In Fig.2(b), $Q_i - \delta_i$ relation divided by the basal line a-a are indepedent each other. The curve in each domain traces so that the each segment in every cycle connected sequentially coincides with the monotonic curve. Thus the ultimate state under cyclic loading is reached when the accumulated plastic deformation in one-side($\overline{AB}+\overline{BC}+\overline{CD}$) reaches the ultimate plastic deformation under monotonic loading, δ_{ulti}.

3) The total energy input, E exerted by an earthquake depends on the total mass of the structure, M and the fundamental natural period of the structure, T, and is scarcely affected by the strength and the configuration of the restoring force characteristics of the structure(5)(6). Introducing equivalent velocity, V_E, the total energy input can be expressed as

$$E = \frac{MV_E^2}{2} \qquad (3)$$

V_E in an undamped elastic system with single degree of freedom coincides with the power spectral density of the earthquake motion. Eventually, for the purpose of the ultimate strength design, the seismic loading should be specified by V_E proportional to the averaged power spectral density of strong earthquakes. The total energy input must be in equilibrium with the energy absorption capacity of the structure. Therefore,

$$W_p + W_h + W_e = E \qquad (4)$$

where W_p=energy absorption due to the accumulated plastic work(structural damages), W_e=elastic vibrational energy, W_h=energy absorption due to damping.

Non-dimensionalyzing with $(MT^2 g^2 / 4\pi^2)$, the above equation is rewritten as

$$A_p + A_h + A_e = A_E \qquad (5)$$

A_p corresponds to the structural damages. The following relations roughly hold between them(5).

$$A_p+A_e=A_E/(1+3h+1.2\sqrt{h})^2,\quad 0<A_e\leq 0.5\alpha_{min}^2, \qquad (6)$$

where h=fraction of critical damping, α_i=yield shear coefficient=$Q_{yi}/\sum_{j=i}^{N} m_j g$, α_{min}= the minimum value of α_i, N=total number of story, g= acceleration of gravity.

4) Denoting the accumulated plastic work in each story by W_{pi}, the total damage W_p can be expressed as

$$W_p = \sum_i W_{pi}. \qquad (7)$$

Non-dimensionalyzing with $(MT^2 g^2/4\pi^2)$, the equation is rewritten as

$$A_p = \sum_i A_{pi}. \qquad (8)$$

For the elastic-plastic system with such a restoring force characteristics as shown in Fig.3, A_{pi} can be related to the plastic deformation as follows.

$$A_{pi}=(\sum_{j=i}^{N} m_j/M)^2(\alpha_i^2/\kappa_i)\eta_i = c_i\alpha_i^2\eta_i, \qquad (9)$$

where $\kappa_i=k_i/K$, $K=4\pi^2 M/T$, η_i=accumulated ductility ratio=$\sum\Delta\delta_p/\delta_{yi}$. $\Delta\delta_p$= increment of plastic deformation, δ_{yi}=the deformation at the elastic limit, $c_i=(\sum_{j=i}^{N} m_j/M)^2\alpha_i^2/\kappa_i$.

For the general system with such a restoring force characteristics as shown in Fig.2, the accumulated ductility ratio in the system, η can be converted into that of the elastic-plastic system with the same yield strength by taking account of equivalency in the energy absorption. Referring to Fig.4, upon the assumption that the same amount of the accumulated plastic deformation occurs in both positive and negative directions, the conversion is made by

$$\eta_{ep}=(1-\frac{k_p}{k})(1+\frac{k_p-k_s}{2(k-k_s)}\eta)\eta. \qquad (10)$$

Thus, the fundamental description of the behavior of the structure can be made by using the elastic-plastic system.

5) For the elastic-plastic system, we can find out the optimum distribution of α_i which allows the damage, η_i to occur evenly over entire stories. The result of numerical analysis is shown in Fig.5, where the optimum coefficient ratio, $\alpha_{opti}/\alpha_{opt1}$ is expressed by a single curve and the nodal point obtained by dividing the scale, L with N gives the ordinate relative to the number of the story. This curve is obtained for the case of uniform mass distribution. Some deviation of the mass distribution, however, may not affect the result, and the curve is scarcely affected by the stiffness distribution.

6) When the distribution of α_i is out of the optimum distribution, the input energy flows into a relatively weak story(7). The concentration of damages due to a slight deviation of α_i from α_{opti} is so remarkable that considering the scatter of the materials, it seems impossible to practically attain an even distribution of η_i. α_i can be generally expressed as

$$\alpha_i = p_i\alpha_{opti},\quad p_1=1. \qquad (11)$$

When p_i=1(the optimum case), the allotted damages to i-th story is given by

$$A_{popti}=(\sum_{j=i}^{N} m_j/M)^2(\alpha_{opti}^2/\kappa_i)\eta_o,\quad \eta_o\text{ =a common response.} \qquad (12)$$

An important rule governing the concentration of damages was found empirically to be: When $p_i \neq 1$, A_{pi} varies from A_{popti} so as to satisfy the next relation.

$$A_{pi}/A_{p1}=(A_{popti}/A_{popt1})p_i^{-12}=s_i p_i^{-12}, \qquad (13)$$

where $s_i=(\sum_{j=i}^{N} m_j/M)^2(\alpha_{opti}/\alpha_{opt1})^2(k_1/k_i)$.

Therefore, substituting A_{pi} in eq(13) into eq(8), we get

$$A_{pi}=(s_i p_i^{-12}/\sum_{j=1}^{N} s_j p_j^{-12})A_p. \qquad (14)$$

p_i in actually designed buildings is dominated by an artificial factor in the process of proportioning and unavoidable scatter in materials. The later must be covered by introducing a factor of safety.

Thus, for the design purpose, p_i could be specified as

$$p_i=p_{oi}+\Delta p_{si}, \qquad (15)$$

where $p_{oi}=1+\Delta p_{oi}$, Δp_{oi} = a deviation expected by the designer, Δp_{si} = a deviation due to the scatter of the material.

When the damages in i*-th story is to be evaluated, a conservative estimation could be made by taking

$$\Delta p_{si} = \Delta p_{so} = \text{a constant, for } i \neq i^*, \text{ and } \Delta p_{si^*} = 0 \,. \tag{16}$$

7) The concentration of damages is mitigated by the effect of the strain-hardening. In compensation for introducing a constant value of Δp_s in any story except for i^*-th story where the damages are expected to concentrate, the preferable effect of the strain-hardening in i^*-th story could be naturally taken into account. As is seen in Fig.4, the yield level of a story can increase as the plastic deformation increases at least by an amount given by

$$\frac{\Delta Q_y}{Q_y} = \frac{k_h' \eta}{2} = \frac{(k_p - k_s)}{k} \frac{\eta}{2} = \frac{k_h \eta}{2k}, \text{ for } k_s \geq 0, \text{ or } \frac{k_p \eta}{2k}, \text{ for } k_s < 0. \tag{17}$$

As the plastic deformation in i^*-th story proceeds, p_{i^*} and α_{i^*} are altered to

$$p_{i^*}' = p_{i^*}(1 + 0.5k_h' \eta_{i^*}), \quad \alpha_{i^*}' = \alpha_{i^*}(1 + 0.5k_h' \eta_{i^*}). \tag{18}$$

Analogously to eq(9) and eq(13), the following relation may hold.

$$dA_{pi^*} = c_{i^*} \alpha_{i^*}^2 d\eta_{i^*} = s_{i^*} p_{i^*}^{-12} dA_p / (s_{i^*} p_{i^*}^{-12} + \sum_{j \neq i^*} s_j p_j^{-12}). \tag{19}$$

Therefore,

$$c_{i^*} \alpha_{i^*}^2 (1+0.5k_h' \eta_{i^*})^2 \{1 + \sum_{j \neq i^*} s_j p_j^{-12} (1+0.5k_h' \eta_{i^*})^{12} / s_{i^*} p_{i^*}^{-12}\} d\eta_{i^*} = dA_p. \tag{20}$$

Integrating the above equation, the response reduced by the strain-hardening, η_{i^*} is determined by

$$\frac{2c_{i^*}\alpha_{i^*}^2}{15k_h'} \{5(1+\frac{k_h'}{2}\eta_{i^*})^3 + (1+\frac{k_h'}{2}\eta_{i^*})^{15} \sum_{j \neq i^*} s_j p_j^{-12} / s_{i^*} p_{i^*}^{-12} - (5 + \sum_{j \neq i^*} s_j p_j^{-12} / s_{i^*} p_{i^*}^{-12})\} = A_p. \tag{21}$$

η_{i^*} can be converted into the response of the elastic-plastic system, η_{epi^*} by taking account of equivalency in the absorbed energy:

$$\int_0^{\eta_{i^*}} c_{i^*} \alpha_{i^*}^2 (1+0.5k_h' \eta_{i^*})^2 d\eta_{i^*} = \int_0^{\eta_{epi^*}} c_{i^*} \alpha_{i^*}^2 d\eta_{epi^*}. \tag{22}$$

Thus,

$$\frac{2}{3k_h'} \{(1 + \frac{k_h' \eta_{i^*}}{2})^3 - 1\} = \eta_{epi^*}. \tag{23}$$

8) Apparent plastic deformations as shown with δ_{pm+} and δ_{pm-} in Fig.3 can be related to the accumulated plastic deformation with the following equation(3).

$$\frac{\mu}{\eta} = a_s = 0.5 - \frac{k_s}{k}(1 - 10\frac{k_h}{k}), \text{ for } \frac{k_s}{k} \geq 0, \text{ and } 0.5 - 10\frac{k_s}{k}, \text{ for } \frac{k_s}{k} < 0,$$

$$\text{and also not greater than } 1.0, \tag{24}$$

where $\mu = (\delta_{pm+} + \delta_{pm-})/\delta_y$, $\quad \eta = \sum \Delta\delta_p / \delta_y$.

9) When the degradation in the load carrying capacity initiates in a story, the concentration of damages also takes place. Therefore, even a slight degradation of the restoring force must be prevented especially in multi-story structures. Referring to Fig.6, the tolerable degradation can be specified by the following empirical formula, and the deteriorating path can be replaced by the flat path shown by a broken line in the figure(7).

$$g = \frac{\Delta Q}{Q_{max}} = \frac{1}{3N^{0.7}}, \tag{25}$$

where g=critical degrading ratio. ΔQ= degradation in strength.

10) Logically, the accumulated plastic deformation in one-side of the basal line as shown in Fig.2(b) lies between $\eta_i/2$ and η_i. Averagingly, the accumulated deformation in one-side, η_{ri} could be specified by

$$\eta_{ri} = 0.75\eta_i. \tag{26}$$

3. DESIGN CRITERIA

The design criteria for the rigid frame structures is proposed as follows: The deformation capacity of i^*-th story must satisfy the following criteria.

$$\eta_{ulti^*} \geq \eta_{ri^*}, \tag{27}$$

$$\theta_a \geq a_{si^*} \eta_{ri^*} / r, \tag{28}$$

where η_{ulti^*}=the ultimate plastic deformability of i^*-th story under the monotonic loading, converted into that of the elastic-plastic system.(refer to eq(10) and eq(25))

$\eta_{ri*}=0.75\eta_{epi*}$=the response of i*-th story (refer to eq(21),eq(23) and eq(26)), θ_a= deformation index for claddings, r=A_{Emax}/A_{Em}, A_{Emax} = the expected maximum energy input, A_{Em}=the energy input under the moderate earthquake, a_{si*}=a reduction factor to estimate the apparent plastic deformation(refer to eq(24)).

The safety against the ultimate collapse is guaranteed by the first criterion. The second criterion secures the structure from any damages except for a slight damage of non-structural components.

The total energy input expressed by V_E can be specified by a bi-linear relation as shown in Fig.7. β_1 being an importance factor of the structure, β_2 is a seismicity factor and β_3 is a site factor relative to foundation conditions. V_o is a standard value of V_E.The adopted curve reveals the averaged feature of the power spectral density of earthquake motions.

4. AN ILLUSTRATIVE EXAMPLE

How to calculate η_{ri*} is demonstrated with an example of a five story building under a prescription: V_o=130cm/sec, $\beta_1=\beta_2=\beta_3$=1.0, T_o=0.4sec, Δp_{so}=0.2, θ_a=0.5, r=10.

Parameters of the building: N=5, h=0.02, T=1.0sec, (k_i/k_1)=(1.0, 0.87, 0.73, 0.6, 0.4), $\mathcal{H}_1$=3.074, k_{pi}/k_i=0.05, k_{s1}/k_1=−0.02, k_{s5}/k_5=0,
(p_{oi})=(1.0, 1.14, 1.05, 1.02, 1.05).

Calculation procedure: $(\alpha_{opt\,i}/\alpha_{opt\,1})$=(1.0, 1.1, 1.25, 1.53, 2.0)(Fig.5), A_e neglected, $A_p=(2V_E^2\pi^2/g^2T^2)/(1+3h+1.2\sqrt{h})^2$ =0.23, (s_i)=(1.0, 0.89, 0.77, 0.62, 0.40), (c_i)=(0.325, 0.239,0.160, 0.087, 0.0325).

1) Damages in first story, i*=1: $(p_i)=(p_{oi}+\Delta p_{si})$=(1.0, 1.34, 1.25,1.22,1.25) k_{h1}'/k_1=0.05(eq(17)), a_{s1}=0.7(eq(24)). η_{ep1} can be calculated through eq(21) and eq(23). $\eta_{r1}=0.75\eta_{ep1}$.

2) Damages in fifth story, i*=5: (p_i)=(1.2, 1.34, 1.25, 1.22, 1.05), k_{h5}'/k_5=0.05, a_{s5}=0.5. η_{ep5} can be calculated through eq(21) and eq(23). $\eta_{r5}=0.75\eta_{ep5}$.

Obtained results are shown by solid lines in Fig.8. The broken lines show the responses for the case of the increase of the yield strength due to the strain -hardening dismissed. It can be seen in the figure that the concentration of damages can be considerably mitigated by the effect of the strain-hardening.

BIBLIOGRAPHY

(1) Housner,G.W.:Behavior of Structures During Earthquakes,ASCE, EM4, 1959.
(2) Housner,G.W.:Limit Design of Structures to Resist Earthquakes, 1st WCEE 1956.
(3) Kato,B. and Akiyama,H.:An Application of High-strength Steels to Earthquake Resistant Buildings, Preliminary Pub. 10th Congress of IABSE, 1976.
(4) Kato,B. and Akiyama,H.:Inelastic Bar Subjected to Thrust and Cyclic Bending, ASCE, ST1, 1969.
(5) Kato,B. and Akiyama,H.:Energy Input and Damages in Structures Subjected to Severe Earthquakes, Trans. of Architectural Inst. of Japan, NO.235, 1975(in Japanese).
(6) Kato,B. and Akiyama, H.:Aseismic Limit Design of Steel Rigid Frames, Trans. of AIJ, NO.237, 1975(in Japanese).
(7) Kato,B. and Akiyama,H.:Collapse Criterion for Multi-storied Shear-type Buildings under Earthquakes, Trans. of AIJ, NO.244, 1976(in Japanese).

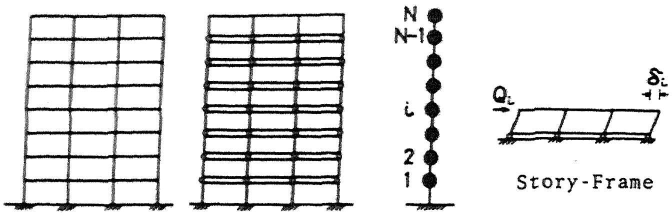

Fig.1 Structural Model.

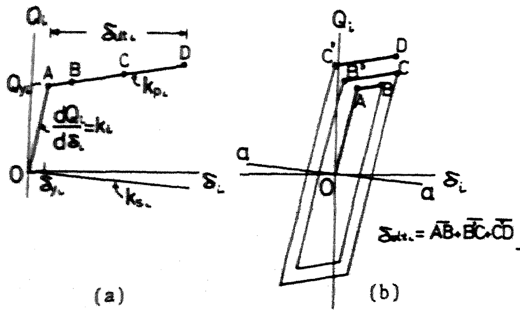

Fig.2 Load Deformation Characteristics of Steel Frames.

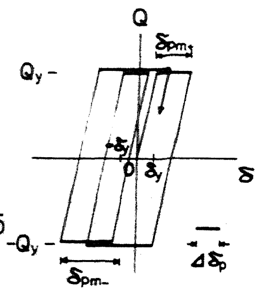

Fig.3 Elastic-Plastic System.

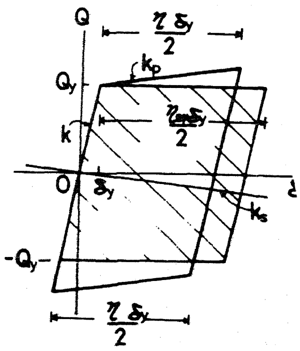

Fig.4 Equivalency in Energy Absorption.

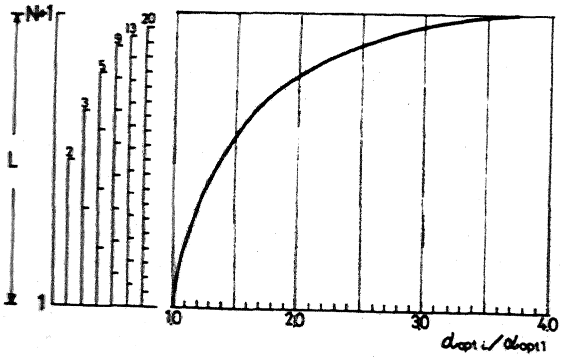

Fig.5 Optimum Yield Shear Coefficient.

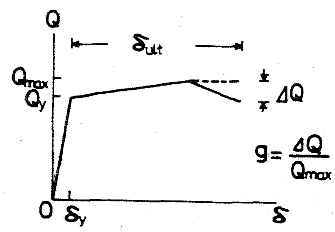

Fig.6 Allowable Limit of Degradation.

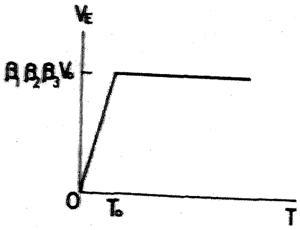

Fig.7 Seismic Loading.

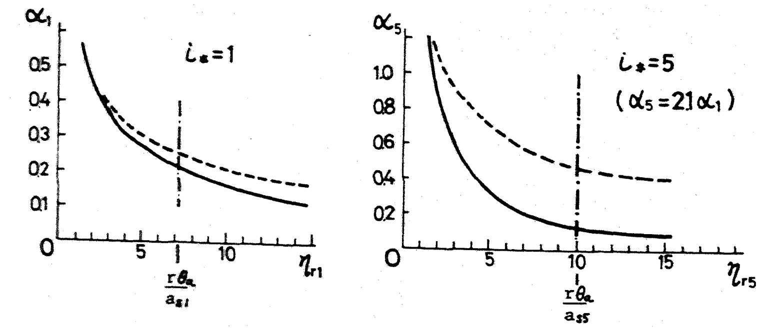

Fig.8 Responses of Five-Story Buildings.