A STUDY ON THE EARTHQUAKE RESISTANCE DESIGN OF BRACED MULTI-STORY STEEL FRAMES

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1. Synopsis

This paper describes how to obtain the vertical distribution of the ratios (hereafter termed β) of story shear force shared by bracings to total story shear force along the height of the frame when the ultimate factored design loads are given. The effect of horizontal deflection caused by column shortening and elongation of the braced bay on the ratios β is taken into account in the process of designing.

Design example according to the present method is illustrated for ten-story three-bay braced frame. Overall inelastic static and dynamic behaviors of this frame are examined, and these response quantities are compared with those of allowable stress designed braced frame and plastically designed open frame.

2. Determination of Distribution of β

To determine the value of β_j in each story when ultimate loads are given, the following condition is introduced, that is,"Tensile yielding in bracings occurs simultaneously at each story under specified factored loads." Columns and beams are assumed to be elastic at the time of brace yieldings. This condition is expressed as follows:

$$\beta_j \lambda_k Q_{wj} + Q_{Fj} = \lambda_B Q_{wj} \quad (j=1,2,\cdots,n) \tag{1}$$

where, j is a story number, λ_k is a seismic load factor, Q_{wj} is working story shear force, Q_{Fj} is story shear force shared by columns, λ_B is a load factor at the time of brace yieldings and n is a number of story. Trial and error method for determining the value of β_j which satisfy Eq.(1) is described below.

(1) At the outset, the values of β_{jo} are assumed as the initial value of β_j.

(2) Bracings are designed to carry the story shear force $\beta_{jo}\lambda_k Q_{wj}$, and columns and beams are so designed as to resist the remaining factored design load $(1-\beta_{jo})\lambda_k Q_{wj}$.

(3) Considering the total story rotation R is sum of the terms due to brace elongation R_S and to column shortening and elongation R_B, rotations of all columns and beams at the time of brace yielding are obtained.

(4) Moment distribution in columns and beams corresponding to these rotations is obtained by the relaxation method. Hence, story shear force Q_{Fjo} shared by columns in jth story is obtained.

(5) Load factors λ_{Bjo} corresponding to β_{jo} are obtained from following equations.

$$\beta_{jo}\lambda_k Q_{wj} + Q_{Fjo} = \lambda_{Bjo} Q_{wj} \quad (j=1,2,\cdots,n) \tag{2}$$

in which, suffix o indicates the quantities corresponding to β_{jo}

(6) If the values of λ_{Bjo} in Eq.(2) agree with the specified load factor λ_B, columns and beams arranged above are examined with factored gravity loads.

If they do not agree, another distribution of β_j is selected according to the technique described below.

Now, the relation between elastic sway stiffness K_{Fj} of open frame consisting of columns and beams, and story shear force Q_{Fj} is

$$K_{Fj}(R_{Sj} + R_{Bj}) = Q_{Fj} \quad (j=1,2,\cdots,n) \ . \tag{3}$$

The additional column axial force caused by bracings is given by

$$N_j = \sum_{i=j}^{n} \beta_i \lambda_k Q_{wi} H_i / L \quad \text{(H:Story Height, L:Span Length)} \ .$$

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Therefore, relative rotation ρ_j of jth beam to (j-1)th beam is

$$\rho_j = 2N_jH_j/A_jEL = 2\lambda_k H_j \beta_j \sum_{i=j}^{n} b_i Q_{wi} H_i/A_j EL^2$$

in which $b_i=\beta_i/\beta_j$. Hereon, following conditions are introduced.

(i) $b_i = b_{io}$ $(i=j,j+1,\cdots,n)$ (ii) $A_j = Ajo$ $(j=1,2,\cdots,n)$

Condition(i) implies that the shape of the vertical distribution of β is same as that of preceding step. Condition (ii) is based on the fact that the larger the β becomes the smaller the shear force shared by columns, and the larger the additional axial force due to bracings. So it can be considered that the change in the value of β does not result in that in the column section in the braced bay. According to the conditions (i) and (ii), ρ_j can be written in the following form.

$$\rho_j = (\beta_j/\beta_{jo})\rho_{jo}$$

Therefore, story rotation due to column shortening and elongation is

$$R_{Bj} = \sum_{i=1}^{j-1}\rho_j = \sum_{i=1}^{j-1}(\beta_i/\beta_{io})\rho_{io} \quad . \tag{4}$$

Moreover, considering the relation between the elastic stiffness and the ultimate strength of open frame as linear, the following equation is obtained,

$$K_{Fj}/K_{Fjo} = (1-\beta_j)/(1-\beta_{jo}) \quad . \tag{5}$$

According to the equations from (1) to (5), β_j is written as follows:

$$\beta_j = \frac{\lambda_B(1-\beta_{jo}) - D_j(\lambda_{Bjo}-\beta_{jo}\lambda_k)}{\lambda_k(1-\beta_{jo}) - D_j(\lambda_{Bjo}-\beta_{jo}\lambda_k)} \quad (j=1,2,\cdots,n) \tag{6}$$

in which $D_j = [R_{Sj} + \sum_{i=1}^{j-1}(\beta_i/\beta_{io})\rho_{io}]/(R_{Sj} + R_{Bjo})$.

β_j obtained from Eq.(6) is used as the initial value for the succeeding step. The same procedure should be repeated until λ_{Bjo} agrees with λ_B at every story. When $\lambda_B=\lambda_k$, it is clear from Eq.(6) that β_j becomes unity at each story. Namely, the total horizontal force is resisted by bracings alone.

3. Design of Members

In designing individual members, the member sizes are treated as continuous quantity which is defined in the following equations:

$$A = \alpha_A Z_p^{2/3} \quad , \quad I = \alpha_I Z_p^{4/3} \tag{7}$$

in which, A, I and Z_p are respectively the area, the moment of inertia and the plastic modulus. Coefficients α_A and α_I in Eq.(7) are from all the standardized wide flange shapes by the least square method. Results are shown in Table-1.

Columns and beams are designed according to the method in the reference [1]. For all column members, section (a) shown in Table-1 is used, and for all beams, section (b) is used. Plastic moment modified to include the effect of the axial force is given as:

$$|M| = M_p \ (|N|/N_p \le 0.123), \quad |M| = 1.14M_p(1-|N|/N_p) \ (|N|/N_p > 0.123) \tag{8}$$

where M_p and N_p are respectively the full plastic moment and the yield axial force.

When a bracing member is subjected to repeated axial tension and compression in the elastic-plastic range, both the maximum tensile and the maximum compressive strength of the member decrease gradually. Consequently, as the number of loading cycle increases, energy absorption capacity tends to decrease gradually. Therefore, tensile and compressive bracing members are designed respectively by the following plastic design formulas based upon the reduction in energy absorption capacity[2]:

$$N_T = N_p \ (\lambda \leqq 0.534), \quad N_T = N_p/(1.056\lambda - 0.436) \ (\lambda > 0.534)$$
$$N_C = N_p \ (\lambda \leqq 0.282), \quad N_C = N_p/(7.198\lambda - 1.029) \ (\lambda > 0.282) \qquad (9)$$

where N_T and N_C are the tensile and compressive axial strength, respectively, and $\lambda=\sqrt{\varepsilon_y A/I}\cdot l/\pi$. Eqs.(9) are described in Fig.1. Section (c) shown in Table-1 is for all bracings, and bracings are assumed to be pinended.

4. Example of Braced Frame Design

By the procedure described above, a ten-story three-bay braced frame is designed here. The general layout and the assumed working loads of this frame are given in Fig.2. Working lateral loads with base shear coefficient 0.2 were obtained from shear force coefficient distribution proposed by T. Kobori and R. Minai. The frame is designed for combinations of dead, live and seismic loads. The value 1.0 is used for the load factor λ_B, which means that the all bracings are designed to yield simultaneously under the working load.

As shown in Fig.3, the process of designing starts with the initial value 0.5 of β_j at each story. The stiffness of the columns at the 1st story is so rigid due to the base being fixed that the solution of Eq.(6) does not exist. Therefore, β_1 is equated to β_2. As is seen in Fig.3, after repeating twice, the specified load factor λ_B is satisfied to the accuracy of less than 2% except the 1st story. The final converged values of β_j and the slenderness ratios of bracings are shown in Table-2.

5. Method of Inelastic Response Analysis

In order to study the inelastic behaviors of braced frames in which sway deflection due to column shortening and elongation is not neglible, an overall analysis is essential. In determining the inelastic member stiffness, interaction of bending moment and axial force at a plastic hinge should be considered, since members of a braced frame are accompanied by variable axial forces. Moreover, the component of plastic axial deformation at a plastic hinge works significantly on the post-buckling behavior of a bracing member.

Hence, a generalized plastic hinge method[3] is used to evaluate member stiffness, and Prager's rule of kinematic hardening[4] is introduced to take the effect of strain hardening into account.

According to this method, inelastic member stiffness equation is written in incremental form as follows:

$$\{\Delta p\} = [Kp]\{\Delta d\} \qquad (10)$$
$$[Kp] = [Ke]([I] - (1-\tau)[\Phi][C]^{-1}[\Phi]^t[Ke]), \quad [C] = [\Phi]^t[Ke][\Phi]$$

where $\{\Delta p\}$ and $\{\Delta d\}$ denote the member end force and the deformation vector increment, respectively. $[Ke]$ is a stiffness matrix for the elastic portion of a member, $[I]$ is a unit matrix, τ is a strain hardening factor, and $[\Phi]$ is a matrix consisted of the exterior normal vectors at the point on a yield surface specified by the force vector. $[Ke]$ is derived from the slope-deflection equation with stability function expressed in the incremental form[5] in order to include the buckling phenomenon. In the numerical analysis described below, τ is given as 0.01. For columns and beams, Eq.(8) is used as the yield surface equation, while, for bracing members, the following equation is used.

$$|M|/M_p + (N/N_p)^2 - 1=0 \qquad (11)$$

The modified displacement incremental method[5] is applied to static analysis of the frame subjected to proportional lateral loading under constant gravity loads.

In carrying out dynamic analysis, the mass matrix is derived from the consistent mass method[6]. The damping matrix is derived from the method suggested by Rayleigh, and damping factors in the first two modes are given as 0.01. Newmark generalized acceleration method is used for the integration of equation of motion, and time interval is chosen as 0.01sec.

6. The Structures and Earthquake Accelerogram

The structures considered in this study are listed in Table-3. In this table, XB-P series are the plastically designed braced frames according to the present method. F-P series are the plastically designed open frames according to the method in the reference[1]. And XB-E is the braced frame designed according to the conventional allowable stress design method in which bracing members are designed as compression members. XB-P-1.00-2nd and F-P-2nd are the frames designed with shear force response distributions of XB-P-1.00 and F-P, respectively.

The first 8 seconds of the El Centro 1940 earthquake(N-S component) with the acceleration amplitude 500gal is used to study the response of structures.

7. Results of Static Analysis

Fig.4 shows the load factor to Effective Structural Rotation[7](λ-θ_{EF}) relations of the three frames under proportional lateral loads shown in Fig.2. The plastically designed frames satisfy the seismic load factor. While the load carrying capacity of allowable stress designed frame XB-E is comparatively small and its maximum load factor is 1.066. This is due to the post-buckling deterioration of the load carrying capacity in the bracings which mostly carry the lateral loads.

The cyclic λ-θ_{EF} relation of XB-P-1.00 is shown in Fig.5. This curve shows that the maximum strength does not decrease beyond the factored design load, whereas the curve admits a gradual cyclic deterioration of the maximum strength due to the post-buckling behavior of bracings.

Fig.6 shows the distribution of ratios β_j when the top-displacement takes the maximum value in each cycle. The vertical distribution of β_j along the height of the frame agrees approximately with that obtained in the process of designing. Furthermore, values of β_j gradually approach the design values along with the increase in the number of loading cycles.

8. Results of Dynamic Analysis

Fig.7(a)-(c) show the relations between story shear force(Q/Q_w) and story rotation of XB-P-1.00, XB-E and F-P, respectively. Fig.8 shows an example of axial force-axial deformation relation of a bracing member in XB-P-1.00.

Maximum story rotations are shown in Fig.9. It is clear that the response of XB-P-1.00 is small and uniform in comparison with the other frames, and the response of F-P in upper stories is comparatively large due to plastic drift as is seen in Fig.7(c). It may be considered that the hysteretic characteristics of a bracing member control the plastic drift as already suggested in Ref.[8]. Besides, it should be noted that XB-P-1.00-2nd shows the response in remarkable uniformity compared with that of XB-P-1.00, whereas no such difference between F-P and F-P-2nd can be recognized.

Fig.10(a)-(c) show the time histories of internal work of members. It is clear that the most of input energy is dissipated by bracings in XB-P-1.00.

9. Conclusions

An earthquake resistant design method of a braced frame is proposed. Ten-story three-bay braced frames are designed according to the present method, and the inelastic behaviors of these frames are investigated. The results may be summarized as:

(1). The distribution of ratios of story shear force shared by the bracings to total story shear force, obtained from static inelastic analysis, agrees approximately with that given by the present method.

(2). Inelastic dynamic responses are relatively small and uniform along the height of the frame. From this, the present mehtod points to the practical utility.

Reference

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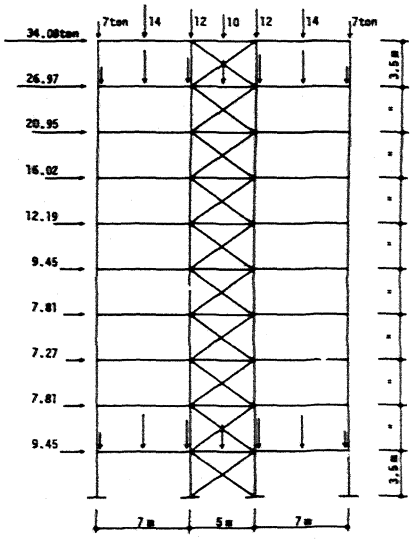


Fig.2 Frame Geometry and Working Loads.

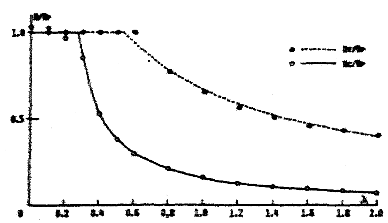


Fig.1

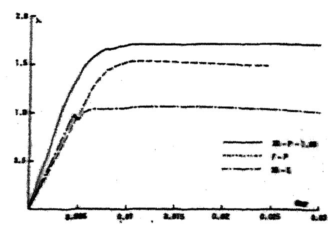


Fig.4 λ-θ_{EF} Relation

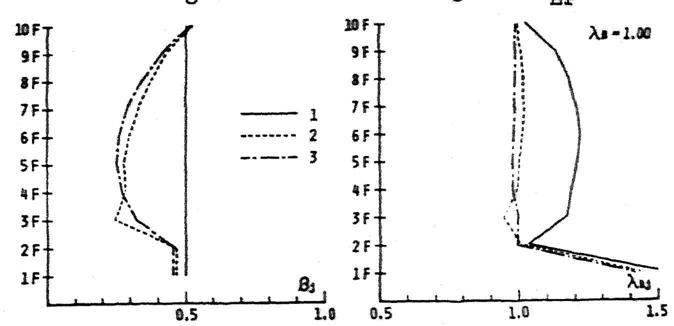


Fig.3 Design Process of XB-P-1.00

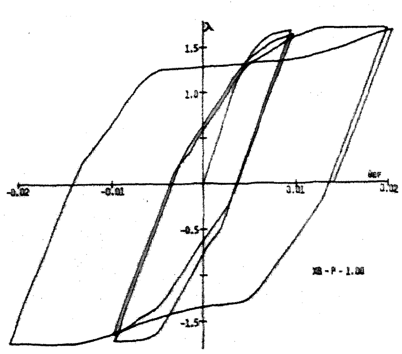


Fig.5 λ-θ_{EF} Relation

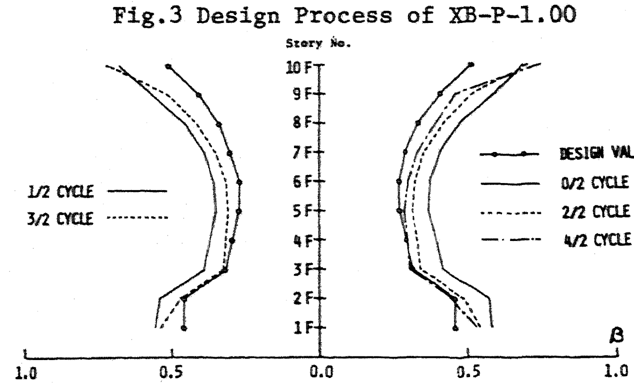
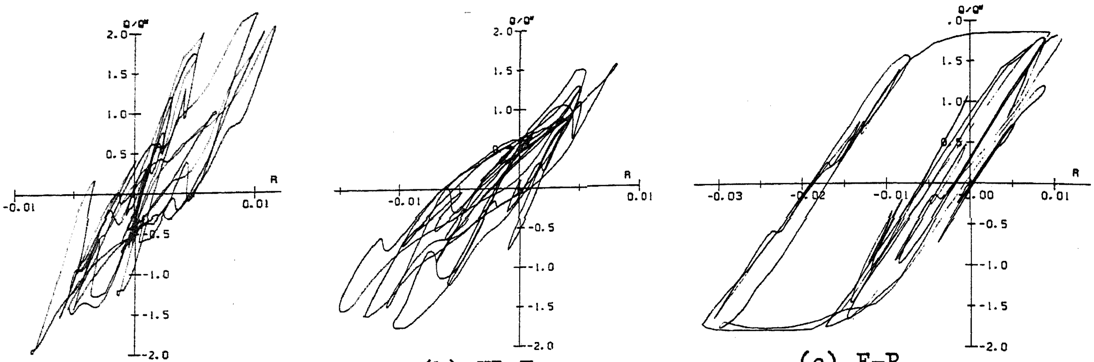


Fig.6 Distribution of β_j



(a) XB-P-1.00 (b) XB-E (c) F-P

Fig.7 Story Shear Force to Story Rotation Relation in 10th Story

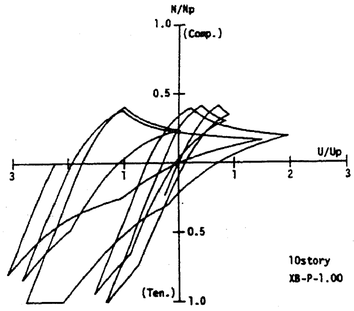


Fig.8 Axial Force-Axial Deformation Relation of Bracing for El Centro NS(Max=500gal)

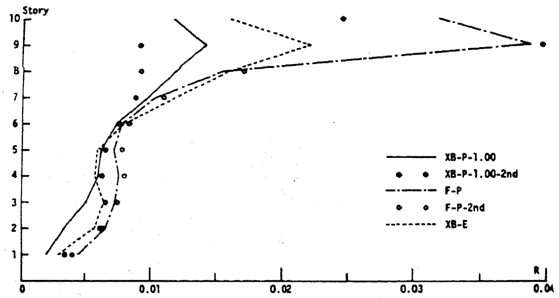
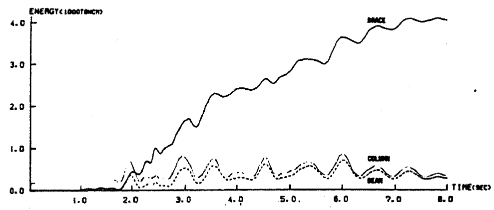
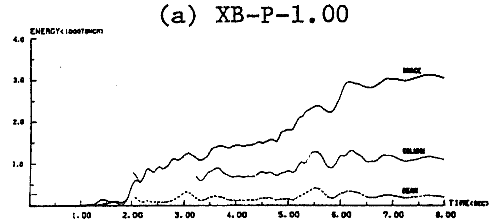


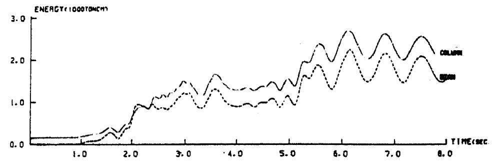
Fig.9 Maximum Story Rotation



(a) XB-P-1.00



(b) XB-E



(c) F-P

Fig.10 Time History of Internal Work

Table-1

section	(a)	(b)	(c)	(d)
α_A	1.029	0.723	1.679	2.063
α_I	1.059	1.578	1.001	0.975

Table-2

Story	β	Slenderness Ratio
10	0.519	115.0
9	0.414	101.2
8	0.343	97.2
7	0.295	96.2
6	0.273	94.8
5	0.275	91.8
4	0.297	86.9
3	0.319	83.0
2	0.460	71.1
1	0.460	69.4

Table-3

Frame	Design Method	C	Load Factor			Natural Period	
			λ_k	λ_v	λ_B	1st	2nd
XB-P-1.00	Present Method	0.2	1.5	1.65	1.0	1.089	0.348
XB-P-1.00-2nd	"	0.2	1.5	1.65	1.0	1.056	0.336
F-P	Reference [1]	0.2	1.5	1.65	—	1.380	0.564
F-P-2nd	"	0.2	1.5	1.65	—	1.412	0.561
XB-E	Allowable Stress Design	0.2	—	—	—	1.292	0.408

C:Base Shear Coefficient λ_k:Seismic Load Factor, λ_v:Gravity Load Factor for Working Loads,