SUBSTITUTE-STRUCTURE METHOD TO DETERMINE DESIGN FORCES IN EARTHQUAKE-RESISTANT REINFORCED CONCRETE FRAMES

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SYNOPSIS

The substitute structure method to determine seismic design forces in multi-story reinforced concrete frames is described. The method, which recognizes energy dissipation in the nonlinear range of response, utilizes substitute linear models and response spectra. The paper contains (1) description of the substitute structure method, (2) an example of use to determine design forces in an eight-story frame and (3) the results of a nonlinear response analysis of the designed frame to earthquake motions.

INTRODUCTION

It has been shown that the maximum response of nonlinear hysteretic systems to earthquakes can be satisfactorily simulated by the maximum response of equivalent linear systems with reduced stiffness and increased damping determined as a function of inelastic deformation (Ref. 1, 3).

The substitute structure method was developed, based on the concept of equivalent linear response, to give a procedure for the determination of the strengths of members in reinforced concrete frames so that a tolerable response limit is not likely to be exceeded for earthquakes anticipated by design requirements (Ref. 4).

The specific advantages of the method are (1) use of linear models for dynamic analysis, (2) choice in setting limits of tolerable inelastic response in different elements of the structure and (3) deliberate consideration of displacements in the design process.

The method will be applied to structures satisfying the following.

1) The system can be analyzed in one vertical plane.
2) No abrupt changes in geometry or mass in the height of the system.
3) Inelastic action can be assumed either in beams or in columns but the tolerable limits should be the same for all beams in a given bay or all columns in a given axis.
4) All structural elements and joints are reinforced to avoid significant strength decay as a result of anticipated cyclic inelastic displacements.

SMOOTHED RESPONSE SPECTRA

The method requires a set of smoothed response spectra which represents the earthquake effects on structures under consideration. In this paper, the idealized response spectra expressed in terms of maximum ground acceleration are assumed as shown in Fig. 2 (Ref. 4).

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The relation between spectral response and damping factor is essential because in the substitute structure method inelastic energy dissipation is taken into account in the form of equivalent damping factor. The simple relation shown in Fig. 3 is assumed to relate spectral response for a certain damping factor β to that for a damping factor of 0.02.

OUTLINE OF THE METHOD

The main operations of the method are divided into the following three steps.

1) Based on tolerable limits of inelastic response, determine the reduced stiffness (substitute stiffness) and increased damping (substitute damping) of each member for the substitute frame.
2) Calculate modal frequencies and modal damping factors for the substitute frame.
3) Determine design forces from the modal analysis of the substitute frame using linear response spectra.

It is assumed that preliminary member sizes of the actual frame are known from gravity loads, functional requirements or a previous trial.

Substitute Frame. The flexural stiffnesses of substitute-frame elements are related to those of actual-frame elements in accordance with Eq. 1.

$$(EI)_{si} = (EI)_{ai} / \mu_i \qquad \text{--- 1)}$$

where $(EI)_{si}$ and $(EI)_{ai}$ are cross-sectional flexural stiffnesses of the i-th element in the substitute-frame and actual-frame, respectively and μ_i is the selected damage ratio for the i-th element.

$(EI)_{ai}$ is calculated from the fully-cracked section and represents the member stiffnesses when the member-end moments reach the yield points under the moment pattern considered (Fig. 1).

The damage ratio which is used to represent the extent of tolerable inelastic deformation is defined as the ratio of the cracked stiffness at yield to the equivalent stiffness at the maximum rotation (Fig. 1). It is comparable but not exactly the same as the ductility factor based on the maximum and yield rotations. They are identical for elasto-plastic response.

The substitute damping factor β_{si} which represents the characteristics of inelastic energy dissipation in each substitute-frame element is given by Eq. 2 in terms of the assumed damage ratio μ_i.

$$\beta_{si} = 0.2(1 - 1/\sqrt{\mu_i}) + 0.02 \qquad \text{--- 2)}$$

Eq. 2 was derived from the studies on inelastic energy absorption based on dynamic experiments as well as earthquake response analysis of reinforced concrete structures (Ref. 1).

Modal Frequencies and Modal Damping Factors. Periods or frequencies, modal shapes and modal member forces are calculated with sufficient accuracy from the undamped substitute frame.

Modal damping factors for the substitute-frame can be estimated by taking the weighted average of substitute damping factors of individual members assuming that each element contributes to the modal damping factor in proportion to its relative flexural strain energy in member associated with the mode shape. Thus, the modal damping factor of the substitute frame for the m-th mode, β_m, is obtained as follows.

$$\beta_m = \frac{1}{4\pi} \cdot \frac{\sum_i \Delta P_i}{\sum_i P_i} = \sum_i \frac{P_i}{\sum_i P_i} \cdot \beta_{si} \qquad \text{--- 3)}$$

$$\Delta P_i = 4\pi \cdot \beta_{si} \cdot P_i \qquad \text{--- 4)}$$

$$P_i = (L_i / 6(EI)_{si}) \cdot (M_{Ai}^2 + M_{Bi}^2 - M_{Ai} \cdot M_{Bi}) \qquad \text{--- 5)}$$

where ΔP_i is the energy dissipation in the i-th element, P_i is the flexural member potential energy, L_i is the member length, $(EI)_{si}$ is the substitute stiffness, and M_{Ai}, M_{Bi} are the member-end moments of the i-th element in the substitute frame for mode m.

Design Forces. Design forces in individual elements are determined on the assumption that the root-sum-square combination of member moments in the substitute frame represent the required member strengths to limit the damage to tolerable levels. A safety factor given in terms of the base shears is considered to cover the uncertainty in the mode-superposition and other factors.

$$F_i = F_{irss} \cdot \left(\frac{V_{rss} + V_{abs}}{2 V_{rss}} \right) \qquad \text{--- 6)}$$

where F_i is the design forces in the i-th element, F_{irss} is the member moments based on the root-sum-square, and V_{rss} and V_{abs} are the root-sum-square and the absolute-sum base shears, respectively.

Design yield moments should be appropriately modified so as to realize the assumed yield pattern. For example, in order to avoid excessive inelastic action in columns in case of weak-beam strong-column design, the column design moments from Eq. 6 should be amplified by a factor of 1.2 (Ref. 4).

DESIGN OF AN EIGHT-STORY FRAME

The design forces corresponding to earthquake resistance in eight-story, three-bay reinforced concrete model frame (Fig. 4) are to be determined using the substitute structure method.

Initial uncracked stiffnesses are calculated from the gross concrete sections. Cracked stiffnesses at yield are assumed to be 1/3 of uncracked stiffnesses for beams and 1/2 for columns, respectively. To take account of the effect of slab to increase the sectional stiffness, beam stiffnesses are multiplied by 2.0.

Allowable damage ratios are taken as six for beams and one for columns assuming the "weak-beam strong-column" criterion. Substitute stiffnesses and substitute damping factors for individual elements in the substitute frame are calculated from Eq. 1 and 2.

Periods of model frames with initial uncracked stiffness, cracked stiffness at yield and substitute stiffness, respectively, are shown in Table 1, together with the calculated modal damping factors for substitute-frame.

Root-sum-square moments (five modes) of members in the substitute-frame are determined using the design spectra in Fig. 2 for the maximum ground acceleration of 0.3G and the relation for damping effect in Fig. 3. Values of design spectral acceleration for substitute-frame are given in Table 1. The safety factor based on the root-sum-square and the absolute-sum (the largest two modes) base shears is 1.05. Column moments from Eq. 6 are amplified by a factor of 1.2 to reduce the risk of column yielding.

PERFORMANCE OF THE DESIGNED FRAME

The model frame having flexural strengths equal to the moments determined from the substitute structure method was "subjected" to four ground motions, i.e., the two components of El Centro 1940 and Taft 1952 earthquakes normalized to have the maximum accelerations of 0.3G. It was assumed that all beams or all columns at each story have the same yield capacity equal to the maximum design forces within that story, respectively. Response calculation was made using the inelastic dynamic analysis program, SAKE (Ref. 2). Hysteresis rules defined in Ref. 5 were assumed ignoring the tensile strength of concrete. Internal viscous damping of 0.02 for the first mode was considered.

Fig. 5 shows the calculated maximum damage ratios in beams and columns for four ground motions. It is seen that the beam damage ratios are generally close to the target value of six along the height of structure except a little lower values at the top, while the column damage ratios are generally within the elastic range though a few columns exhibit damage ratios up to 1.5.

CONCLUSIONS

The outline of the substitute structure method for the seismic design of reinforced concrete frames is described. Application of the method to an eight-story moment-resisting frame was shown. The member damage ratios of the designed frame calculated from the response analysis were close to the anticipated tolerable limits.

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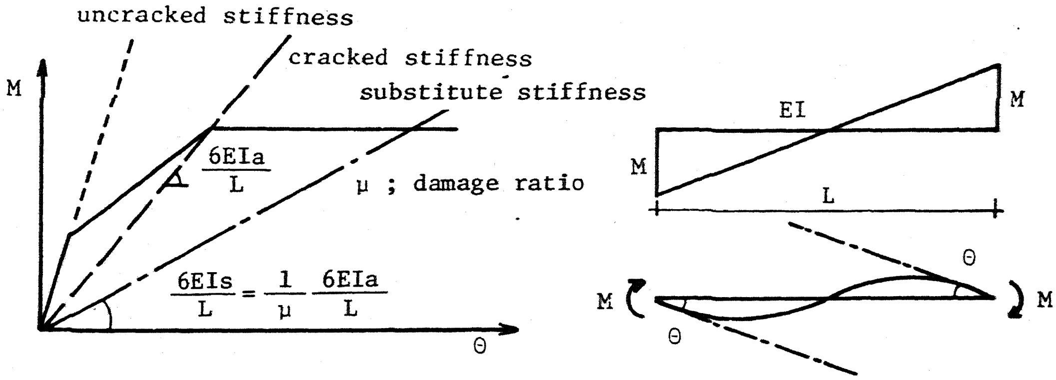


Fig. 1 Damage Ratio and Substitute Stiffness

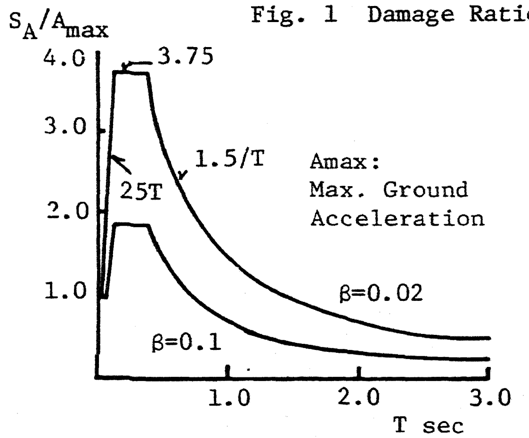


Fig. 2 Smoothed Response Spectra

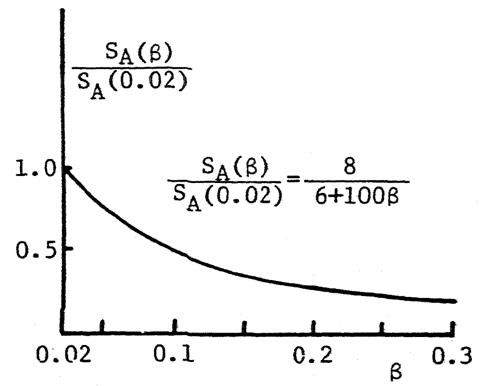
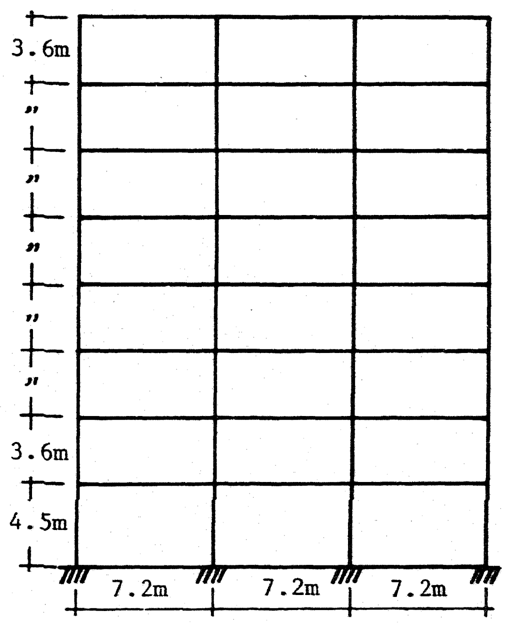


Fig. 3 Damping Effect on Spectral Response



Member Sizes

Story	Beam	Column
8,7	40 cm x 60 cm	60 cm x 60 cm
6,5	40 x 70	70 x 70
4,3	40 x 80	80 x 80
2,1	40 x 90	90 x 90

Weight per floor 150^{ton}

Young's Modulus: 210 ton/cm^2
of Concrete

Fig. 4 Eight-Story Model Frame

Table 1 Dynamic Properties of Eight-Story Model Frame

	Uncracked Stiffness	Cracked Stiffness	Substitute Stiffness		
Mode	Period	Period	Period	Damping Factor	Design Spectral Acceleration
1	0.73 sec	1.19 sec	2.47 sec	0.116	0.08G
2	0.28	0.45	0.83	0.094	0.27G
3	0.16	0.25	0.41	0.070	0.65G
4	0.11	0.16	0.23	0.053	0.79G
5	0.08	0.12	0.14	0.037	0.91G

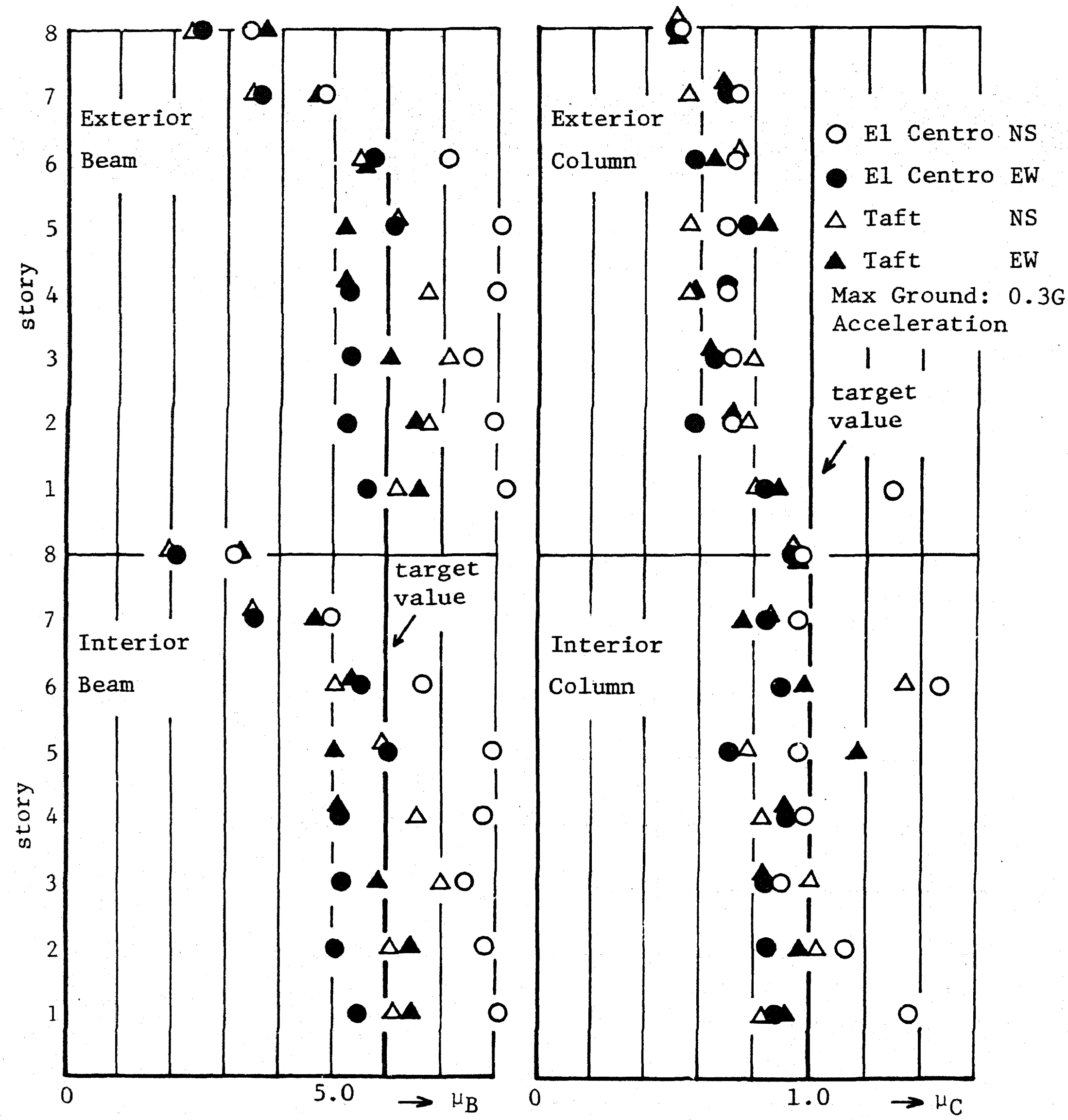


Fig. 5 Maximum Damage Ratios in Members