SAFETY OF REINFORCED CONCRETE BUILDINGS TO EARTHQUAKES

by

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SYNOPSIS

The safety of buildings to earthquakes of specified intensity is evaluated in terms of probability of failure. Probabilities of the first sign of distress (such as yielding) are assessed for reinforced concrete buildings designed in accordance with current earthquake-resistant codes.

INTRODUCTION

This paper is based on a previous study (1), aimed at the determination of the level of risk underlying the design of reinforced concrete buildings against earthquake forces. Failure probabilities are estimated under the following conditions and assumptions: 1) Only linear systems are considered; therefore, failure refers only to the first inelastic action or yielding in a structural member. It does not necessarily mean serious damage or collapse, unless the members are not provided with adequate ductility. 2) Dead, live and earthquake loads are assumed to act simultaneously; these loads, as well as their induced effects, are assumed to be statistically independent. Moreover, only the sustained portion of the live load is considered in examining the combined action of live and earthquake loads.

The response spectrum approach is used in estimating the earthquake load effects. A simplified response spectrum curve, as proposed by Mohraz, Hall and Newmark (2), having the shape shown in Fig. 1 is used for this purpose. The means and variances of the amplification factors (denoted here by α_d, α_v and α_a), obtained from the analysis of 28 records in the horizontal direction (2) are shown in Table 1.

Failure probabilities are estimated for earthquakes of prescribed intensity, and thus are conditional probabilities.

LOAD EFFECTS

Dead Load. For a dead load intensity D, the dead load effect may be expressed as $S_D = C_D D$, where C_D is an influence coefficient. The mean dead load μ_D may be estimated from the geometry of the structure and the unit weight of the material. A coefficient of variation of $\Omega_D = 0.12$ has been estimated in Refs. (1,3). Ascribing a prediction error of 0.10 to the method of static structural analysis, the total uncertainty in the dead load effect is $\Omega_{S_D} = \sqrt{(0.12)^2 + (0.10)^2} = 0.16$; whereas $\mu_{S_D} = C_D \mu_D$.

Live Load. Denote the live load intensity at a point (x,y) of a given floor by (4,5)

$$w_L(x,y) = m_L + \gamma_{bld} + \gamma_{flr} + \varepsilon(x,y) \tag{1}$$

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where m_L is the mean load and γ_{bld}, γ_{flr} and $\varepsilon(x,y)$ are zero-mean statistically independent random variables, representing, respectively, the variation of the average load from one building to another, from one floor to another, and from one location to another. The live load effect may be expressed as

$$S_L = \iint_A w_L(x,y)\ I(x,y)\ dxdy \tag{2}$$

where $I(x,y)$ is an influence surface and A is the influence area (5). If $\varepsilon(x,y)$ is assumed to be a "white-noise" process, it follows that

$$\mu_{S_L} = m_L\ A \iint_0^1 I(x,y)\ dxdy \tag{3}$$

and

$$\Omega_{S_L} = \left[\frac{1}{m_L^2}\left(\sigma^2_{\gamma_{bld}} + \sigma^2_{\gamma_{flr}} + \frac{\sigma^2_\varepsilon}{A} u^2\right) + (0.10)^2\right]^{1/2} \tag{4}$$

in which $u^2 = \iint_0^1 I^2(x,y)dxdy/[\iint_0^1 I(x,y)dxdy]^2$. On the basis of the data reported by Mitchell and Woodgate (6), m_L = 11.8 (psf), $\sigma^2_{\gamma_{bld}}$ = 3.0 (psf)^2, $\sigma^2_{\gamma_{flr}}$ = 17.25 (psf)^2 and σ^2_ε = 8230 (psf)^2 are estimated in Refs. (4,5). The value of u^2 depends on the type of load effect. A good approximation for all load effects is obtained by taking u^2 = 2.20 (1,5). The c.o.v. of the axial load in a column supporting n floors is obtained by dividing $\sigma^2_{\gamma_{flr}}$ and σ^2_ε in Eq. 4 by n.

Earthquake Load. The earthquake load effect in a given member of a structure is expressed by

$$S_E = \left[\sum_{i=1}^{n} (C_E \gamma_i \{\phi_i\} D_i)^2\right]^{1/2} \tag{5}$$

in which γ_i, $\{\phi_i\}$, and D_i are, respectively, the participation factor, modal shape, and spectral displacement corresponding to the i^{th} mode, and C_E is an influence coefficient that translates the relative displacement of the floors into the desired load effect. In Ref. (1) it is shown that the floor masses, as well as the member stiffnesses, may be assumed to be perfectly correlated, with equal coefficients of variation. It follows then, that γ_i and $\{\phi_i\}$ are deterministic quantities, depending only on the means of the mass and stiffness matrices (denoted hereafter by $[\mu_M]$ and $[\mu_K]$). If D_i and D_j are assumed to be perfectly correlated, then

$$\mu_{S_E} = \left[\sum_{i=1}^{n} (C_E\ \gamma_i\ \{\phi_i\}\ \mu_{D_i})^2\right]^{1/2} \tag{6}$$

and

$$\Omega^2_{S_E} = \frac{1}{\mu^4_{S_E}} \left[\sum_{i=1}^{n} (C_E \gamma_i \{\phi_i\} \mu_{D_i})^2\ \Omega_{D_i}\right]^2 + (0.15)^2 \tag{7}$$

where an uncertainty of 0.15 is assigned for the inaccuracies of the method of dynamic analysis. D_i may be expressed, with reference to Fig. 1, as

$$D_i = \frac{\alpha_{p_i} P}{\omega_i^{j}} \tag{8}$$

in which α_{p_i} is the amplification factor, in the i^{th} mode, corresponding to the p^{th} ground motion component, ω_i is the i^{th} natural frequency, and j is equal to 0, 1 or 2 depending on the value of ω_i; e.g. j=0 for $2\pi\omega_i<f_1$; j=1 for $f_1<2\pi\omega_i<f_2$ etc. (See Fig. 1). The statistics of D_i may be found from Eq. 8 once the statistics of ω_i and α_{p_i} are known. On the basis of the assumptions above, it can be shown that

$$\mu_{\omega_i} = \left[\frac{\{\phi_i\}^T[\mu_K]\{\phi_i\}}{\{\phi_i\}^T[\mu_M]\{\phi_i\}}\right]^{1/2} \tag{9}$$

and $\Omega_{\omega_i} = \sqrt{\Omega_M^2 + \Omega_K^2}$. With $\Omega_M = 0.12$ and $\Omega_K = 0.34$ (1), $\Omega_{\omega i} = 0.18$. The statistics of α_{p_i} may be found on the basis of the data reported in Ref. (2), from which the means and variances of the amplification factors are estimated for given values of damping. The mean and c.o.v. of α_{p_i} may be given as (1)

$$\mu_{\alpha_{p_i}} = f_{p_1}(\mu_{\beta_i}) \tag{10}$$

and

$$\Omega_{\alpha_{p_i}} = \frac{1}{f_{p_1}(\mu_{\beta_i})}\left[f_{p_2}^2(\mu_{\beta_i}) + \left(\frac{\partial f_{p_1}}{\partial \beta_i}\right)^2 \mu_{\beta_i}^2\, \Omega_{\beta_i}^2\right]^{1/2} \tag{11}$$

in which $f_{p_1}(\beta_i) = E[\alpha_{p_i}/\beta_i]$ and $f_{p_2}^2(\beta_i) = Var[\alpha_{p_i}/\beta_i]$. On the basis of the results of dynamic tests of full-scale and model structures, the mean and c.o.v. of the modal damping coefficients were estimated to be 4 or 5 % of critical and 0.55, respectively (1). Mathematical expressions for $f_{p_1}(\beta_i)$, necessary to estimate $\Omega_{\alpha_{p_i}}$ from Eq. 11, were obtained by assuming an equation of the form

$$f_{p_1}(\beta_i) = a_1[1+a_2\beta_i]^{a_3} \tag{12}$$

and calculating a_1, a_2 and a_3 by fitting the data given in Ref. (2). These values are shown in Table 2; their applicability is limited to damping values ranging from 0.5 to 10% of critical.

RESISTANCE MODELS

Flexural resistances of structural members are obtained on the basis of the ultimate strength theory of reinforced concrete. For rectangular sections, the moment resistance is expressed as

$$M_T = [P + A_s f_s - A_s'(f_s' - k_3 f_c')]\left[1 - \frac{k_2}{k_1 k_3}\,\frac{P + A_s f_s - A_s'(f_s' - k_3 f_c')}{f_c' bd}\right] d + A_s'(f_s' - k_3 f_c')(d-d') - P(d-d')/2 \tag{13}$$

in which conventional ACI notation is used, and k_1, k_2 and k_3 are coefficients describing the characteristics of the concrete stress block distribution. Fig. 2 shows the c.o.v. of M_T obtained on the basis of Eq. 13 using the variabilities given in Ref. (2) (as reported in Table 3). It is assumed that the axial load and bending moment are statistically independent. In the case of interior columns, this approximation should be good since the major portion of the axial load is due to the dead and live loads, whereas the moment is almost exclusively caused by the earthquake load. For exterior columns, however, this would be an approximation.

The shear strength is calculated on the basis of the truss analogy; that is

$$V_T = v_c bd + \frac{d}{s} A_v f_y \tag{14}$$

The nominal shear strength is estimated through Eqs. 11.4 and 11.7 of the ACI code. Uncertainties in the shear strength are estimated on the basis of the variabilities shown in Table 3 and test data reported in the literature. In formulating the shear strength, it is assumed that the longitudinal reinforcement is kept within the elastic limit. Fig. 3 summarizes the c.o.v. of the shear strength.

RELIABILITY OF CURRENT DESIGNS

On the basis of the information summarized above, current earthquake-resistant designs are evaluated in terms of the estimated probability of failure. For this purpose, a 10-story structure designed according to the 1974 SEAOC code is used (values of Z, I and S in formula 1-1 of the code, are taken respectively to be 1.0, 1.0 and 1.5, and uniformly distributed loads equal to 50, 20 and 10 psf are used for the live load, partitions and mechanical and ceiling). The plan and elevations of the structure are shown in Fig. 4. The eqrthquake is assumed to act in one horizontal direction only and no interaction with the other directions is considered. The probabilities of failure are calculated on the basis of prescribed lognormal distributions.

Probabilities of failure in flexure and shear for all the members of the frame, when subjected to a ground acceleration of 0.100g are shown in Fig. 5. In this regard, the earthquake load effect is calculated on the basis of a ground spectrum with v/a and ad/v^2 equal to 47 in/sec/g and 6.0, respectively. It may be observed that the probabilities of flexural failure of the beams are practically constant at all story levels, except at the roof where the minimum reinforcement limitations dictated the design. The probability of column flexural failures, however, increases for the upper stories, indicating that the code-specified equivalent lateral forces tend to produce weak columns for the higher stories.

The comparison between the risk levels for beams and columns is important. It is implied in the code that a structure should have a strong-column-weak-beam design, so that yielding will occur first in the beams. However, it may be observed from Fig. 5 that this is not the case, even when the provisions of the code are fully satisfied. Therefore, a more conservative column design is necessary if first yielding is to be confined to the beams.

From Fig. 5 it may also be observed that the probability of failure in shear is lower than the probability of failure in flexure. On this basis, it appears that the code gives sufficiently conservative designs for shear as to avoid premature shear failures.

Figure 6 shows the probability of failure in flexure and shear for the interior and exterior columns, and for the interior beam of level 5, as a function of the maximum ground acceleration (the discontinuity in the shear failure probability curve for the columns observed in the figure, is due to the change of the governing equation in estimating the concrete shear strength). These curves are typical of most of the members in the structure.

CONCLUSIONS

On the basis of the calculations performed for a 10-story reinforced concrete building designed in accordance with an existing code, failure probabilities of major structural components to specified earthquake intensities are as follows: In the case of beams, failure probabilities are fairly constant at all story levels, except where minimum reinforcement limitations control the design. For a maximum ground acceleration of 0.1 g, probabilities of failure in flexure are of the order of 0.03, whereas those in shear ranges from 10^{-3} to 10^{-4}. In the case of columns, the probabilities of failure in combined axial load and flexure increase with story level; these range from 0.02 for the lower stories to about 0.10 for the higher stories. Corresponding probabilities of failure in combined axial load and shear range from 10^{-8} to 10^{-5}.

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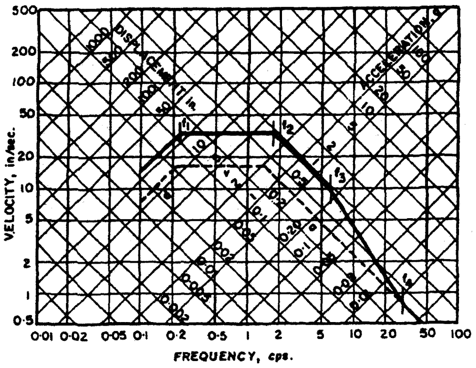

FIG. 1 BASIC SHAPE OF THE RESPONSE SPECTRUM [2]

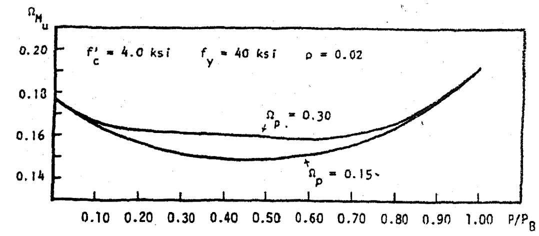

FIG. 2 COEFFICIENT OF VARIATION IN FLEXURAL CAPACITY SYMMETRICALLY REINFORCED COLUMN

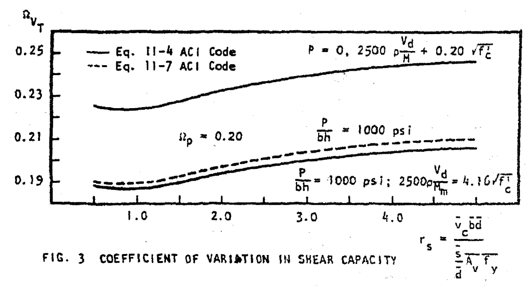

FIG. 3 COEFFICIENT OF VARIATION IN SHEAR CAPACITY

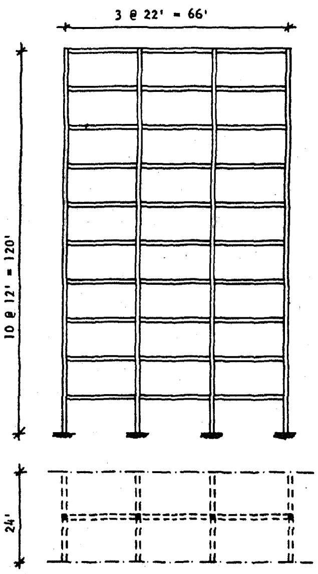

FIG. 4 TYPICAL PLAN AND ELEVATION OF BUILDING CONSIDERED

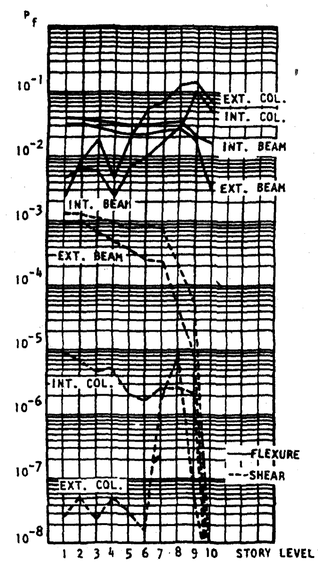

FIG. 5. PROBABILITY OF FAILURE OF SEAOC DESIGN FOR a = 0.1 g

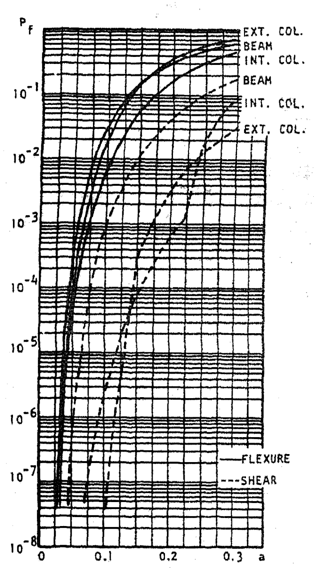

FIG. 6 PROBABILITY OF FAILURE FOR THE INTERIOR BEAM AND COLUMNS OF LEVEL 5

TABLE 1
STATISTICAL VALUES OF THE AMPLIFICATION FACTORS
MEAN (STD. DEV.)

β(%)	α_d	α_v	α_a
0.5	1.97(1.02)	2.58(1.23)	3.67(1.45)
2.0	1.68(0.83)	2.06(0.92)	2.76(0.89)
5.0	1.40(0.64)	1.66(0.66)	2.11(0.56)
10.0	1.15(0.47)	1.34(0.47)	1.65(0.36)

TABLE 2
SUMMARY OF COEFFICIENTS IN EQUATION 12

	a_1	a_2	a_3
ACCEL.	4.40	130	-0.365
VELOC.	2.98	120	-0.300
DISPL.	2.13	58	-0.310

TABLE 3
UNCERTAINTIES IN DESIGN PARAMETERS

PARAMETER	MEAN	Ω
f_y (nominal 40 ksi)	47.7 ksi	0.150
f_y (nominal 60 ksi)	64.0 ksi	0.139
f_c (nominal 3 ksi)	3.5 ksi	0.216
f_c (nominal 4 ksi)	4.7 ksi	0.216
A_s		0.036
b		0.045
d		0.086
h		0.045
k_1k_3	0.72	0.130
k_2/k_1k_3	0.59	0.050
ϵ_{c_u}	0.004	0.156
E_c	$57000\sqrt{uf'_c}$	0.108