ULTIMATE ANALYSIS OF PLASTIC STRUCTURES IN A SEISMIC ENVIRONMENT

by
Alessandro Baratta[I]

SYNOPSIS

The problem of progressive weakening of structures under subsequent earthquakes is approached with the purpose to define a "ultimate threshold" of the structural damage. This threshold is defined as the maximum damage the structure can suffer compatibly with a sufficiently small probability to collapse under the next quake. In the numerical example it is shown the practical possibility to calculate this admissible damage as a function of the seismic strength of the virgin structure; moreover, it is pointed out that a lower bound of the seismic strength exists such that a small amount of damage may be allowed before condemning the building.

INTRODUCTION

In a modern seismic structure in which a certain amount of plastic deformations is commonly allowed to develop during strong motion earthquakes, the possibility of alternate plasticity and/or of destabilizing permanent displacements may be regarded as one of the principal and not negligible forms of progressive weakening. Weakening occurs as the result of the cumulation of damage due to subsequent earthquakes preparing the structure to undergo the decisive quake, i.e. the quake that definitely puts the structure off.

At the present state, attempts have been made [1,2,3] to approach the problem of cumulative damage from a probabilistic point of view, by modeling the history of the structure as a stochastic process. Even leaving out all computational difficulties connected with such approach, it is evident that a good knowledge of long-term statistics of earthquakes and of sequences of seismic events would be required.

In the paper, another approach to the problem of cumulation is investigated, by directly studying the response of the structure to the decisive quake.

BASIC IDEAS

In a recent paper [4] the Author investigated the behavior of a single-bay shear-type frame (Fig.1.a) of ideal non-deteriorating and perfectly ductile material (i.e. neglecting fatigue effects) subject to the idealized symmetrical quake of Fig.1.b, where a(t) is the soil acceleration at time t. Fig.1.c shows the qualitative response of the structure, w being the ratio of the plastic displacement the structure undergoes at time t to

I Assistant professor of Structural Engineering, Faculty of Engineering, University of Naples, Naples, ITALY.

the critical displacement, coincident in this specific case with the displacement at the position of static collapse. Obviously, w can be assumed as a measure of damage, and the structure collapses when w=1 (line (1)).

The proceeding of the function w(t) suggests to distinguish two fundamental phases in the diagram: the deterioration phase, from t=0 to $t=t_d$, in which the structure does not suffer apparently much damage, but, with all evidence, the strength is so seriously weakened that in the successive period, the destructive phase, the structure is led to collapse in a time $t_c = t_D - t_d$, shorter than t_d itself. In the same paper, the Author derived an expression approximating the response of the frame in the destructive phase (line (2)). The response calculated by following this line can then be interpreted as the behavior of a structure already deteriorated by previous earthquakes; therefore it should suitably describe the effects of a destructive quake.

That being stated, the problem of cumulation of damage can be approached by investigating the relation between the initial seismic strength of the virgin structure, say a_r, and the maximum damage the structure can suffer saving an acceptably small probability to collapse under the next quake.

NUMERICAL EXAMPLE

In order to evaluate what this approach means from the practical point of view, an attempt is made to adequately calculate the admissible damage of the frame in Fig. 1.a, assuming that the decisive quake can be modeled as a stochastic sequence of statistically independent bi-triangular symmetrical cycles (Fig. 2), each cycle exhibiting random peak acceleration $\tilde{a}_n$ distributed according to an exponential law

$$\text{Prob}\ (\tilde{a}_n \geq a) = \exp(-a/a_m) \qquad (1)$$

and period T_n deterministically related to a_n by an expression of the type

$$T_n = \alpha\, a_n^{\beta} \qquad (T_n\text{:secs}\ ;\ a_n\text{: cm/sec}^2) \qquad (2)$$

while the number $\tilde{n}$ of cycles in the quake is governed by (neglecting the normalizing factor for large λ)

$$\text{Prob}\ (\tilde{n} = n) = \exp(-\lambda)\frac{\lambda^n}{n!} \qquad (3)$$

These assumptions have been compared with the digital registrations of a number of really occurred earthquakes, in order to get a realistic calibration of the parameters involved in eqs.(1),(2),(3), although these expressions can eventually be replaced by more pertinent assumptions.

However, a good fit was found setting

$$a_m = 65\ \text{cm/sec}^2\ ; \quad \alpha = \beta = 0.16\ ; \quad \lambda = 70 \qquad (4)$$

The stochastic response of the frame to such quakes has been calculated by the Markov chains theory [5] , and the probability distribution function of the final displacement w_f after the quake and starting with w=0, is shown in Fig. 3 for a number of values of the "safety factor" $s=a_r/a_m$, defined as the ratio of the initial strength a_r to the average intensity a_m of the decisive quake. By the same technique, the probability of collapse P_c has been calculated in the hypothesis that the structure is already damaged at the start of the quake: in Fig. 4 P_c is quoted versus the initial damage w_o for a number of values of s. In Fig. 5, finally, the "admissible damage" w_a is quoted as a function of the safety factor for a number of prescribed values of the probability P_c to fail at the next quake. The diagrams clearly show that , allowing P_c to be about $10^{-4} \div 10^{-6}$, if the safety factor is smaller than 1.7, no admissible damage exists, and, as a consequence, the transition from the deterioration phase to the destructive phase can probably happen at once.

CONCLUSIONS

A criterion has been presented to design aseismic structures against the peril of progressive weakening, by direct inspection of the behavior of the already damaged structure under a single earthquake. The numerical example shows that, at least for the simple pattern studied in the paper, it is practically possible to handle the problem, although, of course, many questions would arise when treating more complex structures, more realistic models of seismicity, and more comprehensive definitions of damage.

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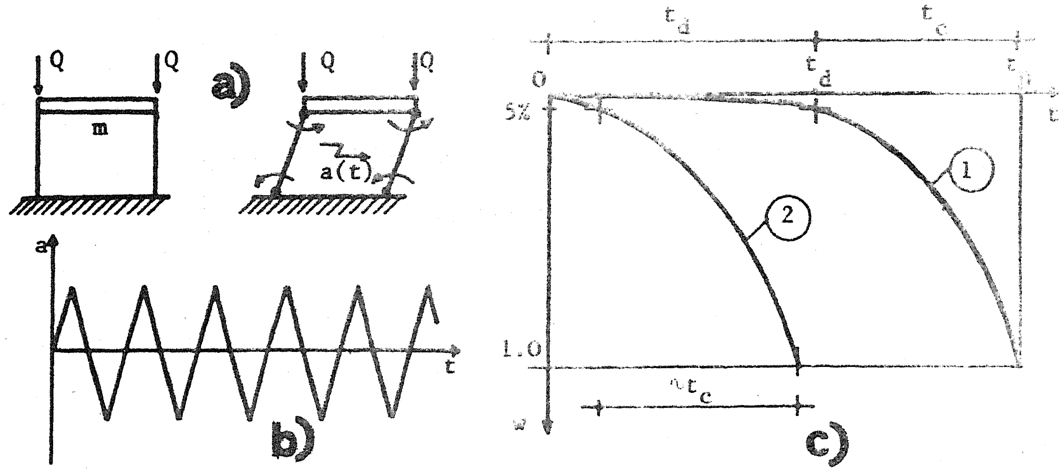

FIG. 1: a)Model structure; b)Soil acceleration; c)Response of structure

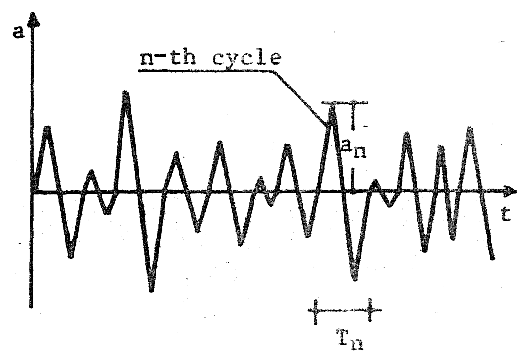

FIG. 2: Sample quake

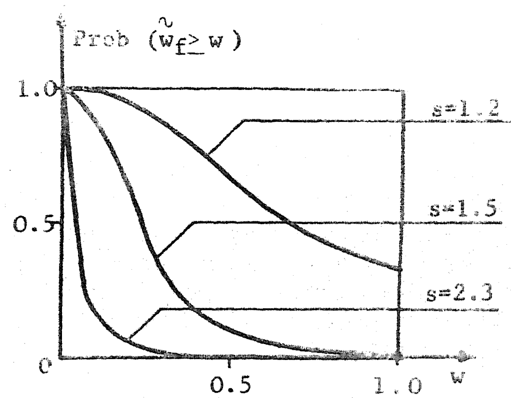

FIG. 3: Probability distribution of w_f

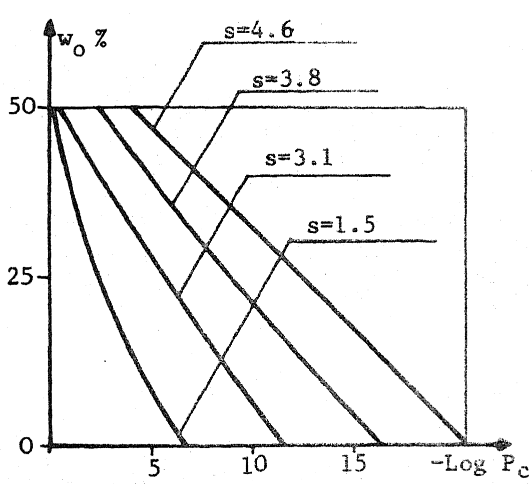

FIG. 4: Current damage versus P_c

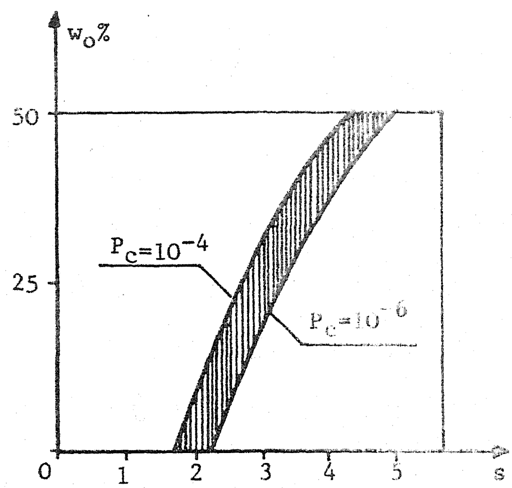

FIG. 5: Admissible damage versus s