ENERGY APPROACH TO EARTHQUAKE RESISTANT DESIGN

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This paper examines the seismic behaviour of engineering structures through the mechanics of energy absorption which is governed by the force deflection relation. An attempt has been made to relate the seismic energy demand with the associated ductility for elasto plastic systems. Two parameters namely 'Energy Coefficient' and 'Sway Ratio' are suggested to study their earthquake behaviour and a method of design using these parameters has been proposed.

ENERGY ANALYSIS

The governing differential equation for a single degree freedom system is

$$m\ddot{Z} + c\dot{Z} + R(Z) = - m\ddot{y} \qquad \ldots (1)$$

where $\ddot{y}$ is the ground acceleration, m is the mass, c is the coefficient of viscous damping and Z is the displacement relative to the ground. The solution of Eq. (1) for elastic and elasto plastic systems is obtained by Runge Kutta method to determine the response at the end of each time step, by defining appropriately, the resistance function R(Z) as shown in Fig 1. Multiplying Eq. (1) by $\dot{Z}/m$ and integrating from t to t + Δt, one obtains the incremental energy equation as follows:

$$\int_t^{t+\Delta t} \left[\ddot{Z}\dot{Z} + 2p\xi\dot{Z}^2 + \frac{1}{m}\,\dot{Z}R(Z) \right] dt = - \int_t^{t+\Delta t} \ddot{y}\dot{Z}\,dt \qquad \ldots (2)$$

$$\text{or} \quad \Delta E_k + \Delta E_d + \Delta E_p = \Delta E_i \qquad \ldots (3)$$

in which ξ is the damping factor and p is the circular natural frequency.

The terms in Eq. (3) represent increments per unit mass in, kinetic energy E_k, energy dissipated in damping E_d, the total strain energy E_p and the input energy E_i to the system respectively. The maximum values of these, during the passage of earthquake motion are found and are denoted by $\bar{E}k$, $\bar{E}d$, $\bar{E}p$ and $\bar{E}i$. Seismic energy demand, defined as the maximum value of $(E_k + E_p)$ has been found to be nearly equal to $\bar{E}p$. In the short period range, $\bar{E}p$ for elasto plastic systems has been found to be greater than the elastic kinetic energy value $\bar{E}k$. At zero period, the energy demand of elasto plastic system with 5% damping, tends to assume a value of $v^2/2$, where v is the peak ground velocity in an earthquake. For ξ = 0.05, energy $\bar{E}p$ for elasto-plastic system has been compared with smoothened elastic kinetic energy spectra, considering additional damping of 2% to account for

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hysteretic behaviour (1). This is shown in Fig. 2 for the longitudinal component of Koyna Dec. 11, 1967 shock and for the north south component of El Centro May 18, 1940 earthquake. Two parameters, 'Energy Coefficient' and 'Sway Ratio' have been obtained from the response computations. These are explained in following sections.

Energy Coefficient:- To study the nature of energy absorption in elasto plastic systems, the ratio (λ), of the maximum total strain energy $\bar{E}_p$ to twice the area under the force deflection curve, on either side of the origin, upto the maximum positive and negative deflections attained during response, was determined. It may be written as:-

$$\lambda = \frac{\bar{E}_p}{2(A_1 + A_2)} \qquad \ldots (4)$$

where,

$$A_1 = \frac{Q_y Z_y}{2} (1 + 2\nu^+); \quad A_2 = \frac{Q_y Z_y}{2} (1 + 2\nu^-)$$

in which Q_y, Z_y, ν^+ and ν^- are as defined in Fig 1.

Calling $\nu_s = \nu^+ + \nu^-$, λ is obtained as,

$$\lambda = \bar{E}_p / 2 \; Q_y Z_y (1 + \nu_s) \qquad \ldots (5)$$

Energy coefficients λ, obtained for elasto-plastic systems have been plotted against period T of the structure for the two earthquakes and for two yield levels of 0.10 and 0.15 in Figs. 3 for 5% damping. Two significant observations are made regarding the value of λ. At zero period, λ assumes a value of $\lambda_c = \alpha/q$, where α is the peak ground acceleration and q is the yield level of the structure, both expressed as fraction of g. The other point on these curves has been located at a period equal to twice the predominant period (T_{ps}) of the accelerogram where λ is found to have a value of unity. Here T_{ps} is the predominant period of the accelerogram as obtained from the Fourier Spectra. Thus as the period of the structure increases, λ decreases and tends to attain a minimum value of 0.25. λ may be interpreted as the minimum number of cycles of hysteretic action experienced by the structure in an earthquake. The maximum value of λ_o has been found to be not greater than 6(2). From these studies empirical relationship between λ and period T has been obtained for elasto-plastic system as follows:

$$\lambda = \lambda_o - bT^{1/3} \qquad \ldots (6)$$

in which the constant b is determined from the condition that $\lambda = 1$ at $T = 2 T_{ps}$.

SWAY RATIO

The nature of vibratory motion of structures during an earthquake has been studied by plotting values ν_m against ν_s (Fig.4) where ν_m is taken as the maximum of either ν^+ or ν^- and ν_s is the maximum width of the hysteretic loop as defined earlier. This figure shows variation of ν_m with ν_s for elasto-plastic systems with 5% damping for the entire period range. Fig 4a shows result for two yield levels of 0.10 and 0.15 for Koyna earthquake. Fig 4b shows similar results for El Centro earthquake. It is seen that for values of ν_s less than 5, value of Sway Ratio ($\eta = \nu_m / \nu_s$) is close to unity, while for large values of ν_s, it tends to assume a value of $(1 - q/\alpha)$.

PHILOSOPHY OF THE PROPOSED ENERGY METHOD OF DESIGN

Energy released at the focus of an earthquake has high frequency content in the epicentral region and as the distance from the focus increases, the acceleration pulses and the frequency content change due to the effect of intervening medium. According to Seed et al (3), the value of the predominant period shifts from a value of 0.25 sec at epicentre of El Centro earthquake to 0.35 sec at the bed rock level under the recording station and as the waves travel upwards through the overburden alluvial soil, it elongates to 0.55 sec. For Koyna earthquake, there was almost no effect of distance and local geology, and that the predominant period was 0.3 sec. Referring back to Fig 3, the period at which λ becomes unity are found to be 1.1 sec for El Centro shock and 0.6 sec. for Koyna accelerogram. Also λ attains a value of 0.25 beyond the period range of about 3 times the predominant period at site. This shows that for structures having periods less than about 3 times T_{ps}, energy absorption takes place by cyclic hysteretic action and for structures having periods greater than this, energy absorption occurs with shift of mean position. In the latter case, the maximum energy level reached by the structure will be given by

$$\bar{E}_p = \frac{Q_y Z_y}{2} (1 + 2 \nu_m) \qquad \ldots (7)$$

By comparison with Eq.5, it is seen that this equation can be used for such systems with $\lambda = 0.25$ and $\nu_s = 2 \nu_m$.

METHOD OF DESIGN

A design procedure based on the above philosophy has been proposed, considering 5% damping:

i) Determine the yield level 'q' and the corresponding yield displacement Z_y, considering it as elasto-plastic system.

ii) Choose the peak ground acceleration 'α', velocity 'v' and the predominant period (T_{ps}) of the expected accelerogram at site, using available procedures.

iii) For this accelerogram, obtain the smoothened energy spectra considering increased energy demand in the short period range ($T < T_{ps}$).

iv) Obtain the energy coefficient λ from Eq. (6).

v) Determine the energy demand for the period T of the structure, as equal to mass times the ordinate of the energy spectra and obtain value of ν_s.

vi) Find the value of ν_m from the corresponding value of ν_s.

vii) The ductility demand is then given as $\mu = (1+\nu_m)$.

The above procedure is illustrated for four different cases for which data is given in Table 1. The results are presented in Table 2 where energy values E_p have been taken from Fig 2.

CONCLUSIONS

The characteristics of ground motion at a site such as the peak ground acceleration α and the predominant period T_{ps}, are shown to govern the mechanics of energy absorption in nonlinear structures having yield level q, through the two parameters 'Energy Coefficient' and 'Sway Ratio'.

REFERENCES

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2. Soni, R.Y., 'Energy Approach to Earthquake Resistant Design', Ph.D. Thesis being submitted to the University of Roorkee.

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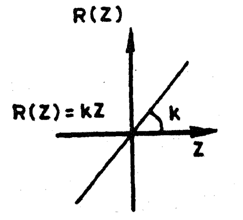
TABLE 1 - DESIGN DATA

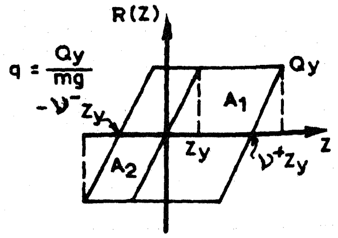
Ex.	Yield level (q)	Q_y (kg)	Z_y (cm)	Period T (sec)	Ground Motion	Peak Acceleration (g)
1	0.15	750	0.2335	0.25	Koyna	0.63
2	0.10	500	1.4000	0.75	(Longitudnal)	
3	0.15	750	0.2335	0.25	El Centro	0.32
4	0.10	500	1.4000	0.75	(N-S)	

TABLE 2 - RESULTS

Ex.	E_p (cm^2/sec^2)	λ_o	λ	ν_s	η	Ductility μ	Theoretical μ
1	1180	4.21	1.810	8.50	0.95	9.07	10.05
2	950	6.00	0.746	3.63	1.00	4.63	4.86
3	1900	2.13	1.443	18.20	0.53	10.65	9.25
4	2900	3.20	1.136	8.30	0.68	6.65	6.76



ELASTIC SYSTEM



ELASTO.-PLASTIC SYSTEM

FIG.1 _RESISTANCE FUNCTION R(Z)

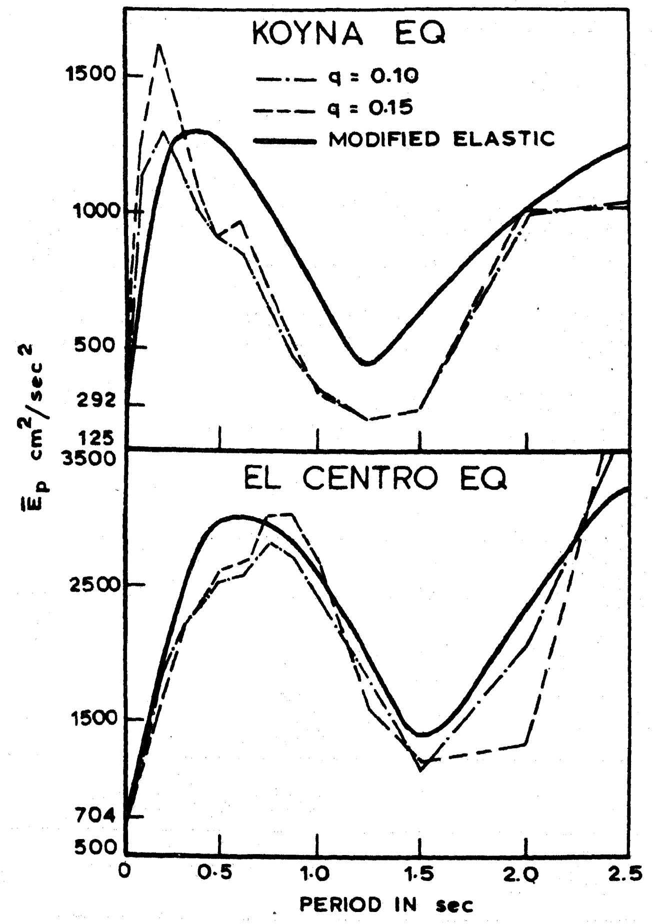


FIG. 2 _ENERGY DEMAND $\bar{E}p$ VS PERIOD T FOR $\xi = 5\%$

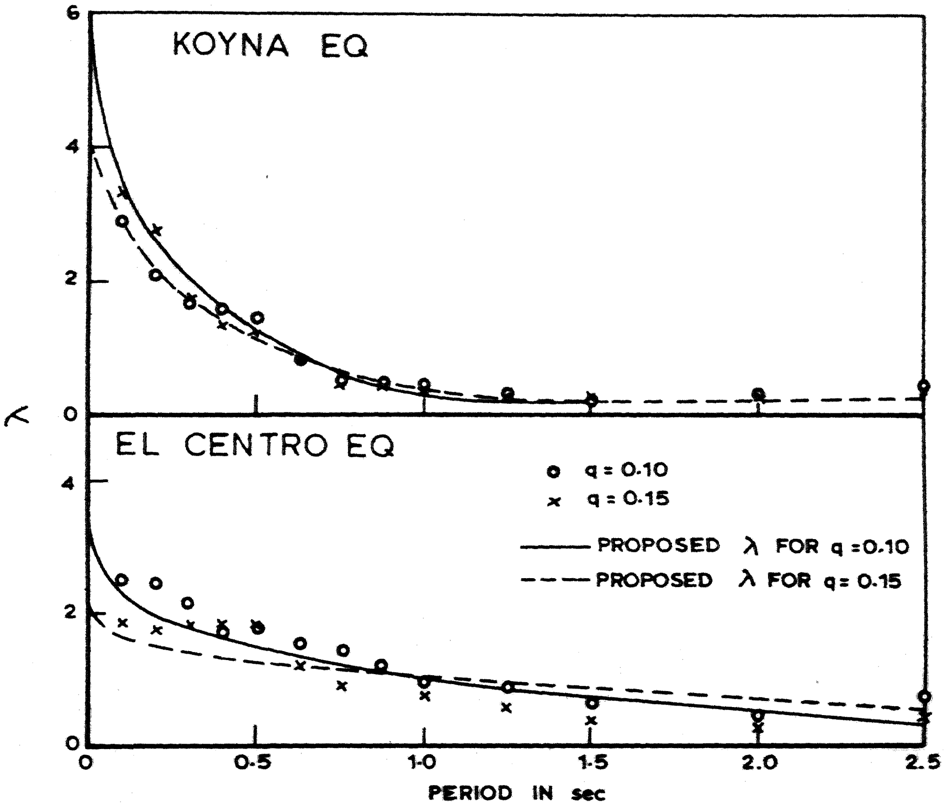


FIG. 3 _ENERGY COEFFICIENT λ VARIATION WITH PERIOD T

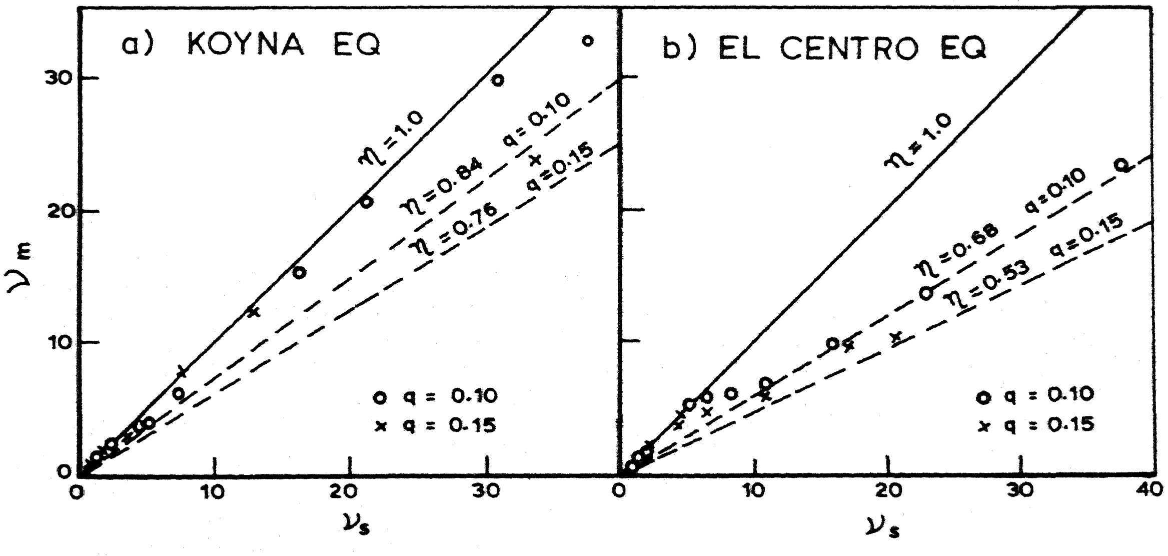


FIG. 4 _SHOWING VARIATION OF ν_m WITH ν_s