SEISMIC FORCE DISTRIBUTIONS FOR COMPUTATION OF SHEARS AND OVERTURNING IN BUILDINGS

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SYNOPSIS

Calculations were made for a series of buildings with varying degrees of frame and shear wall interaction considered in their design. Using a simplified response spectrum, the authors have determined shear and flexural moment ("overturning" moment) distributions over the height of these buildings and have given equations for the seismic forces to which these correspond. The results differ somewhat from those in a previous study (1), and are directly useful in preliminary design, or possibly even in final design of multi-story buildings.

GLOSSARY OF TERMS

B/L = Ratio of width of wall at base to building height

C = Base shear factor, effective weight coefficient

F_i = Equivalent static story force at i^{th} level

F_{fs} = Shear displacement at roof of frames due to a unit load at roof

F_{wf} = Flexural displacement at roof of walls due to a unit load at the roof

F_w = Total displacement at roof of walls due to a unit load at roof

i = Subscript indicating order of floors from base, where $i = 0$

J_i = Overturning moment reduction factor

K = Top story concentration coefficient

N = Number of stories in building

P = Proportion of shear displacement in total deformation of building

T = Fundamental period of vibration of building

V = Base shear, (spectral acceleration)x(total weight)

w_i = Story weight

X_i = Story level measured from base divided by height of building

α = Σ column moments of inertia/Σ wall moments of inertia.

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INTRODUCTION

Current seismic design codes use simple distributions of seismic force over the height of a building to determine shears and overturning moments. The SEAOC Recommended Lateral Force Requirements (2) represent the seismic forces that give the dynamic structural response by a linear distribution of accelerations over the height of the building with a maximum at the top. The total weight of the structure is considered effective in calculating the pseudo-static forces and a portion of the base shear may be concentrated at the top story to account for higher mode effects.

These shear distributions do not take into account the type of structure, whether its lateral load resisting systems are frames, shear walls, or combinations of these elements, and the distributions do not account for the higher mode effects in a consistent manner. The equivalent slenderness ratio required to represent the action of the building as a whole in determining the top story concentration is open to varied interpretation.

The primary parameter determining the mode shapes and their relative participation is the type of deformation the structure undergoes when loaded laterally. Uniform shear beams deform linearly when subjected to a concentrated load at the top whereas flexural beams deflect in a higher order fashion. Whether the building is represented by one of these models or a combination of the two will determine its response. The primary parameters determining the relative effects of the contributing modes of vibration are the spacing of the natural frequencies and the shape of the response spectrum. If a bilinear response spectrum is used, the response is a constant velocity response up to some frequency for which the spectral accelerations are proportional to the frequency, followed by a constant acceleration response. Then the fundamental frequency relative to the intersection of the two branches will determine the effects of the higher mode responses.

METHOD OF ANALYSIS

The structural idealizations, ranging from flexural beams to shear beams, were generated and analyzed by the modal summation method. Only lateral vibrations were considered by statically condensing the rotational effects from the elastic stiffness coefficients. For each idealization the natural frequencies and mode shapes for all significant modes were calculated. The modes were normalized, multiplied by their corresponding spectral accelerations and combined to obtain the most probable distribution of story shears. The square root of the sum of the shears of the individual modal shear distributions were normalized with respect to the fundamental frequency spectral acceleration and total weight. Thereafter new spectral accelerations were generated by effectively shifting the fundamental frequency with respect to the knee in the response spectra.

Similarly, the probable overturning moments were calculated and normalized with respect to the overturning moments obtained from the seismic forces corresponding to the most probable shear distributions. The resulting ratio represents the reduction factor J. For each structural idealization and shear distribution the strain energy due to shear deformation was compared to the strain energy due to the total deformation. A simplified model was developed to relate this value to the top story deformation of a structure due to a unit load at the top story. This model was used to identify the dynamic behavior of the structure.

RESULTS

Equations expressing equivalent static loads are presented in terms of the fundamental period, the proportion of shear deformation, and the relative height along the structure. The equations appear at the end of the text.

The proportion of the total weight effective in the equivalent static base shear is expressed by Eq. (1). After the seismic forces are normalized to unit base shear they are multiplied by this effective weight factor. In uniform flexural beams for fundamental periods less than one second as little as 0.7 of the weight is effective, whereas for fundamental periods greater than one second 1.33 times the weight may be considered effective. The introduction of shear deformation also limits the variation in effective weight.

The portion of unit base shear concentrated at the top story is determined by Eqs. (2a) and (2b). This value is a function of period and shear deformation and determines the shape of the distribution. Buildings behaving as shear beams have smaller concentrations at the top floor than do flexural beams. Similarly, increasing the fundamental period increases the concentration.

The distribution of forces along the height of the structure is expressed in Eq. (3). The forces are represented as a multiple of the story weight, distance from the base, and shape factor amplified by the top story concentration. The distribution ranges from nearly linear for low period shear beams to an "S" shape at the other extremes.

The reduction factor applied to overturning moments calculated from the equivalent static forces required to match closely the most probable overturning moments calculated from the modal response is expressed in Eq. (4).

The proportion of shear deformation in the structure can be estimated by considering the lateral resisting elements to be cantilever beams connected in parallel, each containing a component of shear and flexure deformation. Equation (5) is written for a combination of two such beams linked at the top, one representing a shear wall and the other a moment resisting frame in which a 10 percent flexural deformation was considered effective to account for column end rotations arising from flexible girders. This equation requires only the relative stiffnesses of the frame to wall elements, the relative width of the wall to the height of the building and the number of stories. It can be used at preliminary stages of design as soon as the basic framing scheme has been established. Equations (6) through (8) define quantities used in the previous equations. Equation (9) is a restatement of Eq. (5) with all quantities explicitly stated.

The most probable design shear coefficients distributed over the height of the structure, together with the distributions generated by the proposed equations are compared in Fig. 1. An envelope of the SEAOC provisions is included to indicate the effect of a 15 percent base shear concentration at the top story. The SEAOC envelope fits the probable distributions poorly for all heights of structures and offers little guidance in choosing the degree of top story concentration. It should be appreciated that the SEAOC approach does not consider the effective weight in determining the base shear values as does a modal analysis, and accordingly does not incorporate a consistent margin of safety.

The probable overturning moment coefficients distributed over the height of the structure, together with the SEAOC provisions and distributions generated from the proposed equivalent static forces and overturning moment reduction factor J are compared in Fig. 2. The overturning moments are obtained by multiplying these factors by the base shear and building height.

The equations presented herein closely represent the probable elastic shear and moment distributions over the building height. They reflect the significance of structural system and fundamental period. Effects of non-uniform stiffness distributions and yielding foundations may be accounted for when determining the fundamental period and proportion of shear deformation of the structure.

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$$C = \frac{(T-1)(5-T)(1-P)}{10} + 0.9 \qquad (1)$$

$$T > 2 \quad K = (0.4T+1.4)-(\frac{T}{3}+0.3)(\frac{0.12}{1.12-P}) \qquad (2a)$$

$$T < 2 \quad K = (0.88T+0.44)-(0.6T-0.24)(\frac{0.15}{1.15-P}) \qquad (2b)$$

$$F_i = \frac{C\{(K-C)(X_i-0.9)(X_i-0.4)/0.06+C\}X_i w_i}{\sum_{i=1}^{N}\{(K-C)(X_i-0.9)(X_i-0.4)/0.06+C\}X_i w_i} V \qquad (3)$$

$$J_i = 1 - (\frac{T}{26P+10})(1-X_i) \qquad (4)$$

$$P = 1 - \frac{(\frac{0.09}{F_{fs}} + \frac{F_{wf}}{F_w^{\,2}})}{(\frac{0.9}{F_{fs}} + \frac{1}{F_w})} \qquad (5)$$

$$F_{fs} = \frac{L^3}{12EI\alpha N^2} \qquad (6)$$

$$F_{wf} = \frac{L^3}{3EI} \qquad (7)$$

$$F_w = \frac{L^3\{1/3+0.23(B/L)^2\}}{EI} \qquad (8)$$

$$P = \frac{0.23(B/L)^2+0.9(10.8\alpha N^2)(1/3+0.23(B/L^2)^2}{1/3+0.23(B/L)^2+(10.8\alpha N^2)(1/3+0.23(B/L)^2)^2} \qquad (9)$$

REFERENCES

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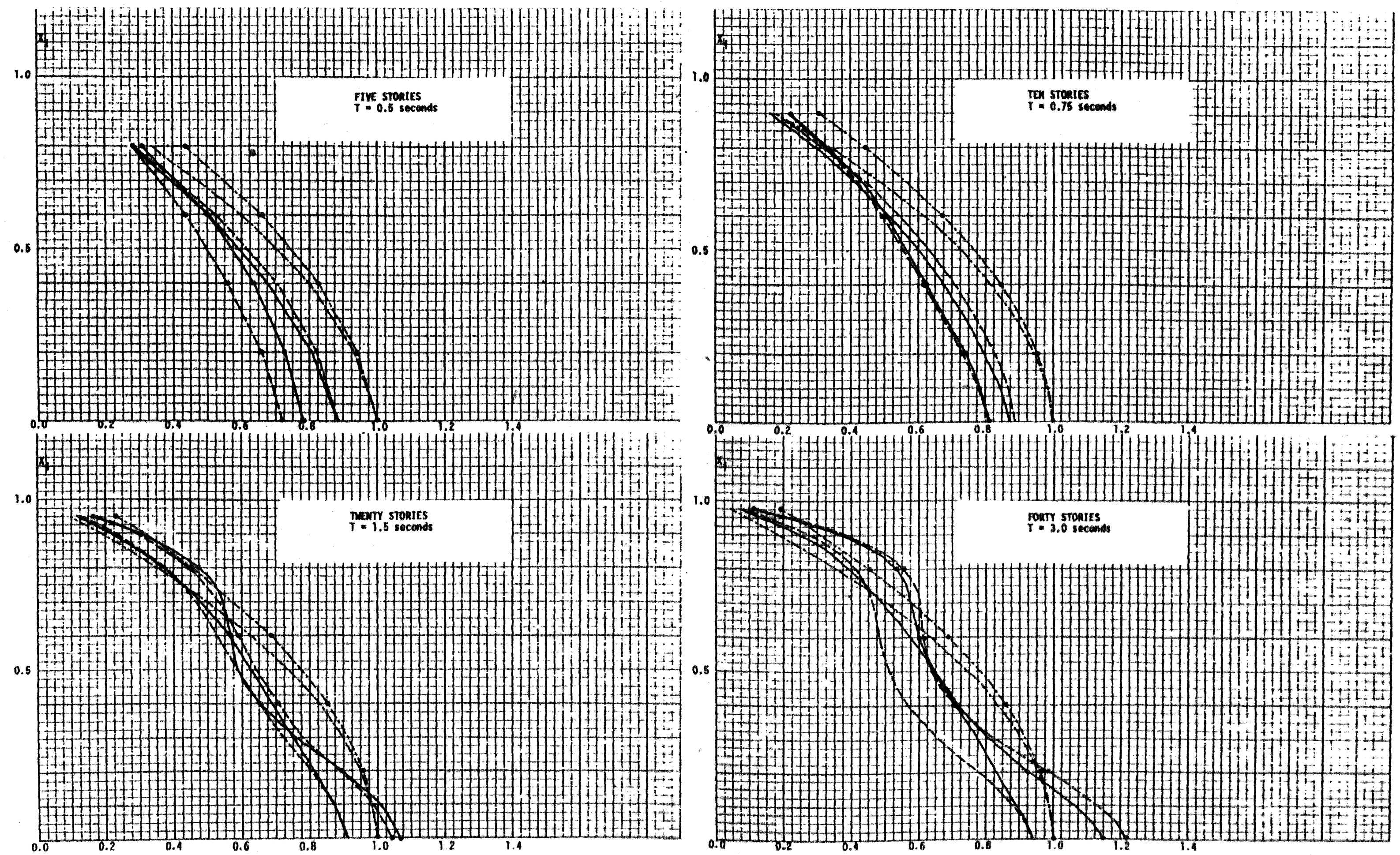

Fig. 1 SHEAR DISTRIBUTION COEFFICIENTS

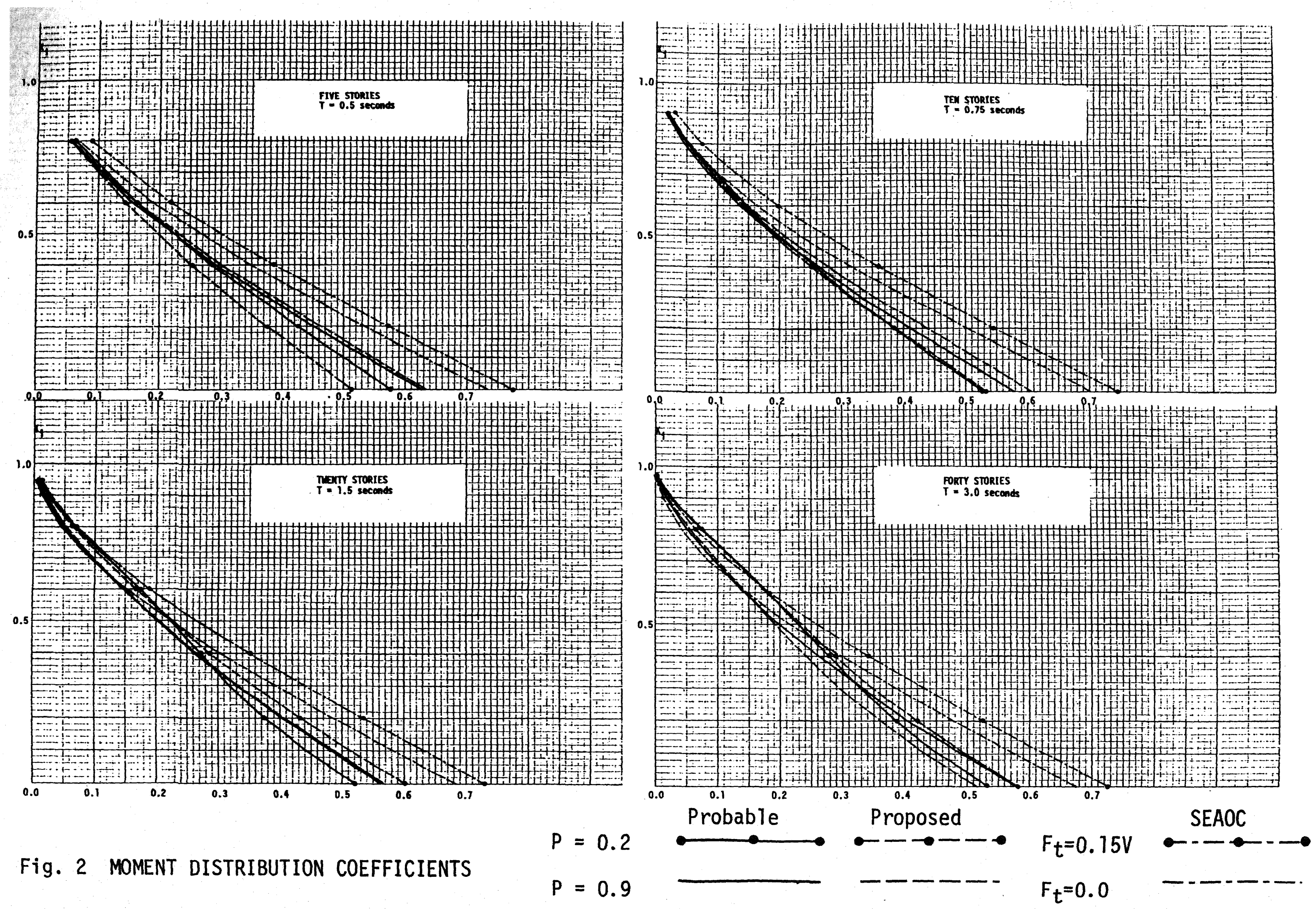

Fig. 2 MOMENT DISTRIBUTION COEFFICIENTS

DISCUSSION

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The authors have given equations for the distribution of shears and moments in multistoreyed buildings. They have stated that these could be useful, possibly even in final design of multistorey buildings. These equations will therefore be of great value to designers. Though the equations are rather cumbersome in comparison with those given in codes, for equivalent static forces, they are very simple when compared to the time and effort required for dynamic analysis. The use of the author's equations could be further simplified by developing some design aids in the form of charts. Inclusion of such provisions in the codes for earthquake resistant design of buildings in the medium height range needs to be given due consideration.

Author's Closure

Not received.