# RISK-CONSISTENT EARTHQUAKE RESPONSE SPECTRA

by

A. Der-Kiureghian[I] and A. H-S. Ang[II]

## SYNOPSIS

Earthquake response spectra corresponding to given levels of risk and their relevance in structural design are discussed. Development of such spectra based on seismic risk analysis with line-source model is introduced. It is shown that spectra developed with this technique include the significance of site conditions and risk level. For two actual sites, with different conditions, response spectra corresponding to various risk levels are developed and compared. Finally, uncertainties associated with the method are analyzed and their effects are estimated.

## INTRODUCTION

The time of occurrence, as well as location, size, and other characteristics of future earthquakes in a seismic region are not amenable to precise prediction. Therefore, in designing a structure in such a region, or in developing guidelines for this purpose, probabilistic assessments of the severity or destructive potentials of future earthquakes, and of the resulting structural responses would be necessary. Design for zero risk is neither practically possible nor economically desirable; realistically, earthquake-resistant design may be developed with a small, but finite, probability of failure within its useful life. Toward this end, and for this purpose, response spectra corresponding to specified levels of risk(III) (or return-periods) would be required. The development of such risk-consistent spectra is the subject of this paper.

## CHARACTERISTICS OF EARTHQUAKE RESPONSE SPECTRA

An earthquake response spectrum, representing the maximum response of a simple oscillator to a specified base motion, is correlated with the maximum ground motions [5]. Based on this correlation, it is a common practice in earthquake-resistant design to derive and specify constant response acceleration, velocity, and displacement for certain ranges of the system frequency by amplifying the respective maximum ground motion components. Such a response spectrum has a trapezoidal shape (on a tripartite logarithmic plot) as illustrated in Fig. 1. If a, v, and d are the maximum ground acceleration, velocity, and displacement, respectively, it can be shown that the transition frequencies $f_1$ and $f_2$ --a measure of the frequency content of the spectrum (Fig. 1)--are related to the ratios $\frac{v}{a}$ and $\frac{ad}{v^2}$ as follows:

$$f_2 \sim \frac{1}{v/a} \quad (1), \text{ and } \qquad \frac{f_2}{f_1} \sim \frac{ad}{v^2} \quad (2)$$

---

I Assistant Professor of Civil Engineering, Univ. of S. Calif, L.A., Calif.

II Professor of Civil Engineering, Univ. of Illinois at Urbana - Champaign, Urbana, Illinois.

III The word "risk" is used here to indicate the annual probability of exceeding a given threshold. The inverse of risk is the average return-period in years.

Therefore, the ratios $\frac{v}{a}$ and $\frac{ad}{v^2}$ characterize the spectrum. A large $\frac{ad}{v^2}$ (i.e. large $\frac{f_2}{f_1}$) will mean a wide range of frequencies having constant spectral velocity--i.e. a wide spectrum--whereas a small $\frac{ad}{v^2}$ (or small $\frac{f_2}{f_1}$ will imply a narrow spectrum. Also, for a given $\frac{ad}{v^2}$ (i.e. constant $\frac{f_2}{f_1}$) a larger $\frac{v}{a}$ would shift the spectrum toward lower frequencies, whereas the spectrum will tend toward higher frequencies for smaller $\frac{v}{a}$.

For several earthquakes recorded on alluvium, the ratio $\frac{v}{a}$ ranges between 35.7 and 242.2 cm/sec/g, whereas the ratio $\frac{ad}{v^2}$ ranges between 1.84 and 30.58 [4]. High frequency components of the ground motion attenuate more rapidly than the low-frequency components; therefore, the maximum ground acceleration, which is due largely to high-frequency motions, will attenuate more rapidly than the corresponding maximum ground velocity or displacement, which are more the result of low-frequency motions. As a result, the ratios $\frac{v}{a}$ and $\frac{ad}{v^2}$ will depend on the distance of the site from the earthquake source. Regression analyses of recorded ground motions [3] indicate that a, v, and d depend also on the earthquake magnitude; consequently, the ratios $\frac{v}{a}$ and $\frac{ad}{v^2}$ would also depend on the earthquake magnitude. Sepcific relations of a, v, and d with distance and earthquake magnitude [3] suggest that both $\frac{v}{a}$ and $\frac{ad}{v^2}$ increase with increasing distance, and that $\frac{v}{a}$ increases whereas $\frac{ad}{v^2}$ decreases with increasing magnitude. Besides the earthquake location and size, other factors such as the transmission characteristics of the soil and rock strata around the site also affect the ratios $\frac{v}{a}$ and $\frac{ad}{v^2}$.

Current design spectra, however, are developed with constant average values of the above ratios. For example, for horizontal ground motions on alluvium, Newmark [4] has proposed $\frac{v}{a} = 122$ cm/sec/g and $\frac{ad}{v^2} = 6$; these ratios were obtained from the average values of several earthquake records. On this basis, therefore, the transition frequencies, $f_1$ and $f_2$, of conventional design spectra would be independent of the magnitude and the distance to potential sources.

## DEVELOPMENT OF RISK-CONSISTENT SPECTRA

Let Y represent the annual maximum ground acceleration, velocity, or displacement. The corresponding annual maximum spectral response, denoted by $S_y$, can be obtained as

$$S_y = A_y Y \tag{3}$$

where $A_y$ is the amplification factor for the appropriate response component.

If the amplification factor is assumed to be deterministic, the response value associated with a given risk level can be obtained by amplifying the ground motion value corresponding to the same level of risk; i.e. if a, v, and d are the maximum ground acceleration, velocity, and displacement corresponding to T years return-period, then $s_a = A_a a$, $s_v = A_v v$, and $s_d = A_d d$ are the coordinates for the risk-consistent spectrum with T years return period.

The evaluation of probability distributions for the maximum ground acceleration, velocity, and displacement requires a seismic risk analysis. Such an analysis includes stochastic modeling of earthquakes in various sources and evaluation of resulting ground motions at a site. A line-source model for seismic risk analysis is developed by the authors [2], wherein earthquakes are assumed to originate as slips along active geologic faults. Risk estimates with this model include the significance of the distance of each source from the site, and the distribution and frequency of earthquake magnitudes at each source. As a result, the estimated values of a, v, and d, and therefore $\frac{v}{a}$ and $\frac{ad}{v^2}$, include the significance of the distance and the magnitude of potential future earthquakes.

A sensitivity analysis with the line-source model [2] revealed that low intensity ground motions, which are associated with high levels of risk, are mainly due to small magnitude earthquakes at close distances; and that high intensity ground motions, which correspond to low risk levels, are mainly due to large magnitude earthquakes including those at distant sources. This means that the ratios $\frac{v}{a}$ and $\frac{ad}{v^2}$ will also be influenced by the level of risk under consideration.

Since the ratios $\frac{v}{a}$ and $\frac{ad}{v^2}$ characterize a response spectrum, it follows that risk-consistent spectra, developed by the method outlined above, would include the significance of (i) positions of sources relative to the site, (ii) the distribution and frequency of magnitudes at each source, and (iii) the level of risk under consideration. More specifically (recalling relations of $\frac{v}{a}$ and $\frac{ad}{v^2}$ with distance and magnitude) we may conclude the following:

1-For any given risk level the ratios $\frac{v}{a}$ and $\frac{ad}{v^2}$ are larger when seismic risk is mainly due to distant sources. For such a case, therefore, the risk-consistent spectrum will be wider and will shift toward lower frequencies.

2-For any given risk level the ratios $\frac{v}{a}$ and $\frac{ad}{v^2}$ are respectively larger and smaller when seismic risk is mainly due to large magnitude earthquakes. For such a case, therefore, the risk-consistent spectrum will be narrower and will shift toward lower frequencies.

3-For low intensity ground motions associated with high levels of risk a smaller $\frac{v}{a}$ would result, indicated a risk-consistent spectrum which is shifted toward higher frequencies.

4-For high intensity ground motions associated with low levels of risk a larger $\frac{v}{a}$ and a smaller $\frac{ad}{v^2}$ would result, indicating a risk-consistent spectrum which is narrower and is shifted toward lower frequenices.

## SPECIFIC CASE STUDIES

Two actual sites, one in San Francisco, California, and the other in San Juan, Puerto Rico, were analyzed with the line-source model [2]. These sites are in greatly different geological settings. Whereas major shallow sources are located near San Francisco (about 25 km distance), the earthquake hazards for San Juan are primarily from distant sources (about 80 km). The results of risk analysis for the ground and response acceleration, velocity, and displacement for the two sites are presented in Figs. 2 and 3. The response curves were developed using mean amplification factors of Ref. 4. Specific ground motion intensities associated with various return-periods are also presented in Table 1.

The significance of the positions of sources relative to the two sites can be observed by comparing the $\frac{v}{a}$ and $\frac{ad}{v^2}$ ratios for the two sites.

Both ratios are consistently larger for San Juan than those for San Francisco, indicating that the risk-consistent spectra for San Juan will be wider and shifted toward lower frequencies than those of San Francisco. For both sites, the $\frac{v}{a}$ increases and $\frac{ad}{v^2}$ decreases with increasing return-period (i.e. decreasing risk), implying that the corresponding risk-consistent spectra will be narrower and will tend toward lower frequencies for longer return-periods. Such spectra are presented in Figs. 4 and 5 for 5% damping. For each case, the conventional response spectra are also shown, which were constructed using the constant values of $\frac{v}{a}$ and $\frac{ad}{v^2}$ shown in the bottom of Table 1.

A comparison of the respective sets of spectra indicates that the design spectra developed with the method of Ref. 4 are generally not risk-consistent.

## ANALYSIS OF UNCERTAINTY IN AMPLIFICATION FACTORS

In Ref. 2 it is shown that the maximum ground motion component, Y, can be written as:

$$Y = X_1 X_2 Y_1 \tag{4}$$

where $Y_1$ is the maximum ground motion component based on assumed attenuation and slip-length relationships, and $X_1$ and $X_2$ are random factors introducing the uncertainties associated with the above relationships, respectively. For a random amplification factor, $A_y$, Eq. 3 can be written as:

$$S_y = A_y X_1 X_2 Y_1 \tag{5}$$

if we let $Z = \ln(A_y X_1 X_2)$, the risk associated with the response level $s_y$ can be obtained as:

$$P(S_y > s_y) = P(Y_1 > s_y e^{-Z}) = \int_{-\infty}^{\infty} P(Y_1 > s_y e^{-z}) f_Z(z) dz \tag{6}$$

where the first term of the integral is the risk associated with the ground motion level $s_y e^{-z}$, and the second term is the density function of Z. It is reasonable to assume a normal distribution for Z [2]; therefore,

$$P(S_y > s_y) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} P(Y_1 > s_y e^{-z}) \exp\left[-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2\right] dz \tag{7}$$

where $\mu$ and $\sigma$ are the mean and standard deviation of Z and are obtained using first order approximations [1] as:

$$\mu \simeq \ln(\bar{A}_y \bar{X}_1 \bar{X}_2) \quad (8), \text{ and} \quad \sigma \simeq \sqrt{(\delta_{A_y})^2 + (\delta_1)^2 + (\delta_2)^2} \tag{9}$$

where $A_y$, $X_1$ and $X_2$ are the means, and $\delta_{A_y}$, $\delta_1$, and $\delta_2$ are the coefficients of variation of $A_y$, $X_1$ and $X_2$, respectively.

In Ref. 2, the values of $\delta_1$ for several attenuation equations are given ranging from 0.51 to 0.84; in the same reference, $\delta_2$ is estimated to be at most 0.30. Values of $\delta_{A_y}$ for 5% damping and for horizontal acceleration, velocity, and displacement on alluvium range between 0.26 and 0.45[4]. The above values suggest that the largest uncertainty involved is that associated with the attentuation relationship.

For the sites in San Francisco and San Juan, risks associated with various levels of response, including the uncertainty in the amplification factors, were evaluated by numerical integration of Eq. 7, and are plotted in Figs. 2 and 3.

## SUMMARY AND CONCLUSIONS

The development of risk-consistent spectra is described, based on a line-source model of seismic risk analysis [2]. In contrast to conventional design spectra, a risk-consistent spectrum includes the significance of site conditions and the level of risk associated with the ground motion intensity. The procedure is illustrated for two actual sites; namely, San Francisco, California, and San Juan, Puerto Rico.

## REFERENCES

1. Ang, A. H-S and Tang, W. H., Probability Concepts in Engineering Planning and Design, Vol. 1-Basic Principles, J. Wiley & Sons, N.Y., 1975.
2. Der-Kiureghian, A. and Ang A. H-S., "A Line-Source Model for Seismic Risk Analysis," University of Illinois, Civil Engineering Studies, SRS 419, Urbana, Illinois, 1975.
3. McGuire, R. K., "Seismic Structural Response Risk Analysis, Incorporating Peak Response Regressions on Earthquake Magnitude and Distance," M.I.T., Department of Civil Engineering, Research Report, R74-51, 1974.
4. Newmark, N.M., Consulting Engineering Services, "A Study of Vertical and Horizontal Earthquake Spectra", Prepared for the Directorate of Licensing, United States Atomic Energy Commission, Contract No. AT(49-5) -2667, WASH. 1255, April 1973.
5. Newmark, N.M. and Rosenblueth, E., Fundamentals of Earthquake Engineering, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1971.

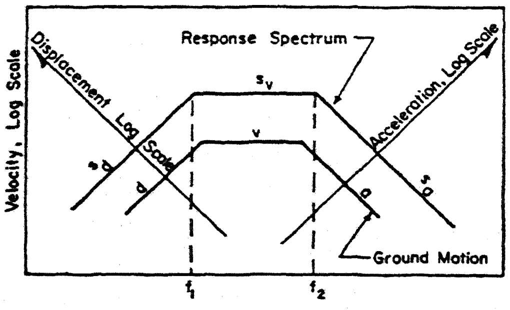

FIG.1 A TRIPARTITE RESPONSE SPECTRUM

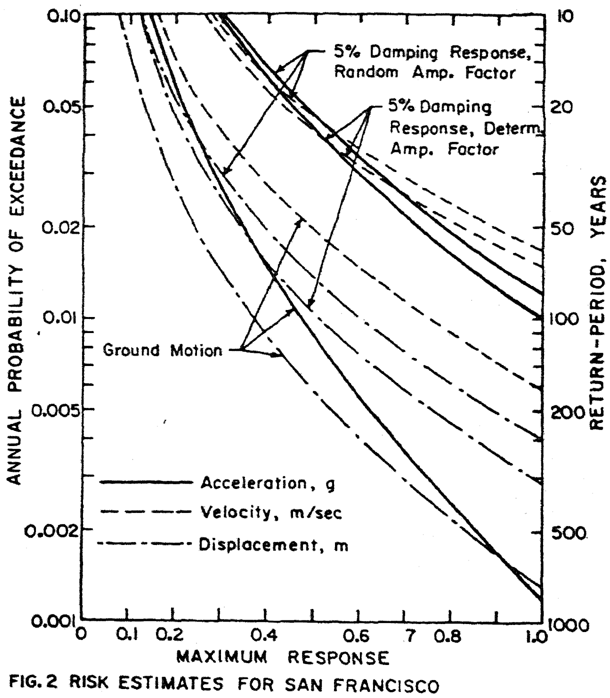

FIG.2 RISK ESTIMATES FOR SAN FRANCISCO

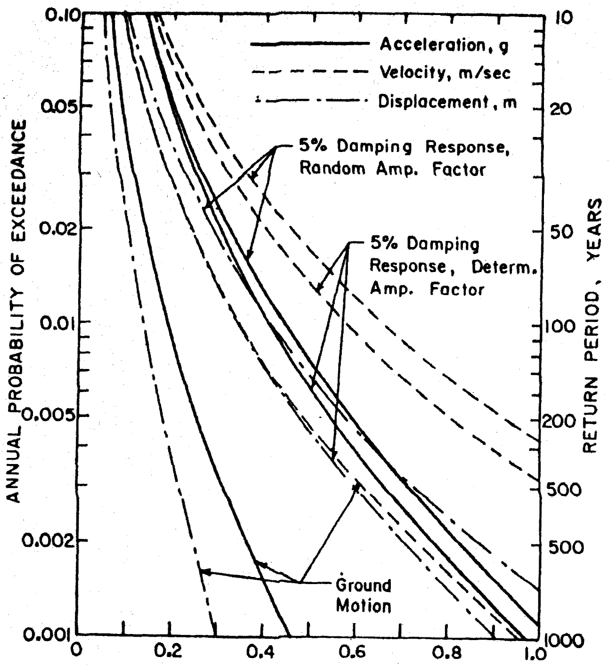

FIG. 3 RISK ESTIMATES FOR SAN JUAN

**TABLE I. Spectral Analysis of Maximum Ground Motions for San Francisco and San Juan**

| Site | Return Period (Yrs.) | a g | v cm/sec | d cm | v/a cm/sec/g | d/a cm/g | $ad/v^2$ — |
|---|---|---|---|---|---|---|---|
| San Francisco, California | 10 | 0.14 | 15.5 | 8.1 | 111 | 58 | 4.64 |
| | 20 | 0.22 | 26.5 | 13.2 | 121 | 60 | 4.06 |
| | 50 | 0.35 | 49.3 | 24.0 | 141 | 68 | 3.39 |
| | 100 | 0.47 | 72.0 | 37.0 | 153 | 79 | 3.29 |
| | 200 | 0.62 | 102.0 | 54.1 | 165 | 87 | 3.16 |
| | 500 | 0.85 | 151.0 | 82.2 | 178 | 97 | 3.01 |
| San Juan, Puerto Rico | 10 | 0.064 | 9.0 | 6.1 | 141 | 95 | 4.73 |
| | 20 | 0.093 | 14.2 | 10.2 | 153 | 110 | 4.56 |
| | 50 | 0.144 | 24.0 | 17.4 | 167 | 121 | 4.27 |
| | 100 | 0.193 | 34.8 | 24.7 | 180 | 128 | 3.86 |
| | 200 | 0.257 | 48.6 | 34.1 | 189 | 133 | 3.63 |
| | 500 | 0.362 | 72.3 | 50.0 | 200 | 138 | 3.40 |
| Values used in current spectra [4] | | | | | 122 | 91 | 6.00 |

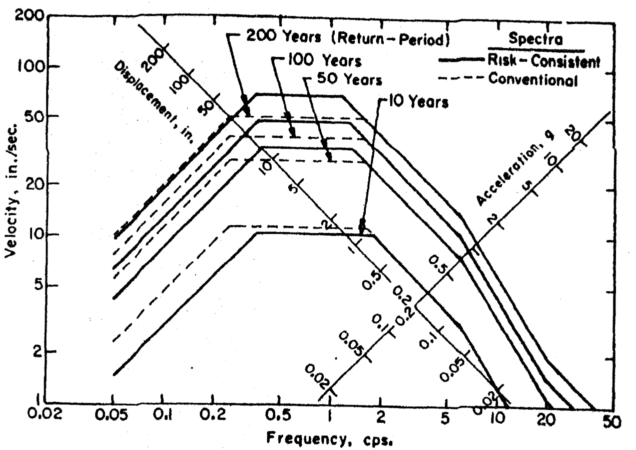

FIG. 4 RISK-CONSISTENT AND CONVENTIONAL SPECTRA FOR SAN FRANCISCO (5% DAMPING)

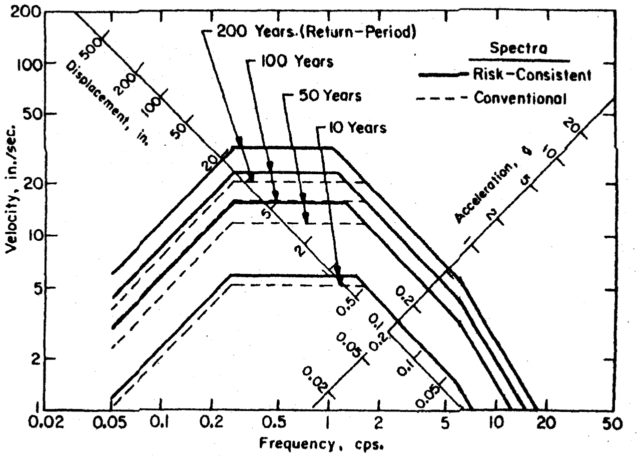

FIG. 5 RISK-CONSISTENT AND CONVENTIONAL SPECTRA FOR SAN JUAN (5% DAMPING)