EXTENDED APPLICATIONS OF RESPONSE SPECTRA CURVES IN SEISMIC DESIGN OF STRUCTURES

by

M. P. Singh[I], S. Singh[II] and A. H.-S. Ang[III]

SYNOPSIS

Methods are presented for obtaining accurate response of structures with closely spaced frequencies, generation of floor spectra and analysis of secondary systems with multiple supports for seismic design. The methods are based on stochastic concepts, but no stochastic seismic input is required; only structural quantities such as system frequencies, mode shapes and participation factors, and, of course, site spectra curves are required.

INTRODUCTION

For the design of important structures such as nuclear power plants, site spectra curves are most commonly used to define seismic inputs. Based on a study of recorded accelerograms, Newmark, Blume and Kapur (1) have defined the generalized shape of these curves. Fig. 1 shows such a typical spectrum curve.

So far, direct use of the response spectra has, however, been limited to the analysis of primary structures by the square-root-of-the-sum-of-squares (SRSS) method. Improvements in the SRSS method are needed before it can be applied to complex structural systems where the frequencies are close to each other. For the design of light equipment and other secondary systems with multiple supports, the design input in the form of floor spectra is obtained by time history analysis in which spectrum consistent time histories are used as seismic inputs. In the analysis of earth structures where consideration of strain dependent soil properties is important, the current methods of analyses also require that time history be used as input. To obtain an input time history for such analyses, a recorded or synthetic accelerogram is modified such that its spectra envelope the given spectra. Thus given spectra are used only indirectly. It has been shown that the results obtained from a time history analysis depend upon the input accelerogram used i. e. the results can be different for different accelerograms even though they may be all consistent with the given spectra. For design purposes such analyses should, therefore, be used with caution. The methods which do not require an accelerogram and which can be used with the given spectra directly would, therefore, be preferable.

Herein methods which extend the direct applicability of ground spectra curves beyond the conventional SRSS approach are proposed for analysis of primary structures with closely spaced frequencies, generation of floor spectra and analysis of secondary systems with multiple supports. Similar methods for the analysis of earth structures are described elsewhere (2).

[I] Senior Engr. Analyst, Sargent & Lundy, Chicago, Illinois, USA.

[II] Asst. Div. Head, Structural Analytical Division, Sargent & Lundy.

[III] Professor of Civil Engineering, Univ. of Illinois Urbana-Champaign, Urbana, Illinois, USA.

METHODS OF ANALYSIS

It is assumed that the earthquake motions represented by site spectra can be modeled by a stationary random process. Thus, there is a power spectral density (PSD) which represents the motions. Let the PSD be denoted by $\Phi(w)$. For this PSD to be consistent with the prescribed site spectrum, the maximum response of an oscillator excited by the PSD motions should be the same as the response spectrum value. The maximum response of the oscillator for a PSD is a random variable, and its value is associated with a probability of exceedance. However, to avoid using any specific number for the probability, it may be assumed that the maximum response is an amplified value of the root-mean-square response. For convenience this amplification factor can be assumed to be a constant. This defines a relationship between the response spectrum and the associated PSD; for an acceleration spectrum this can be written as:

$$C\int_{-\infty}^{\infty} \Phi(w)\ [w_o^4 + 4\beta_o^2 w_o^2 w^2] \ /\ [(w_o^2 - w^2) + 4\beta_o^2 w_o^2 w^2]dw = R^2(w_o) \qquad (1)$$

where C is the factor, w_o = oscillator frequency, $R(w_o)$ = response spectrum value at w_o, and β_o is the damping of the oscillator. The response of a primary or secondary structure can now be expressed in terms of the PSD.

ANALYSIS OF PRIMARY STRUCTURES: The design response of a structure can be written as (3),

$$R_q^2 = C\sum_{j=1}^{N} \gamma_j^2 q_j^2 \int_{-\infty}^{\infty} \Phi(w)|H_j(w)|^2 dw + 2\ C \sum_j \sum_{k=k+1} \gamma_j \gamma_k q_j q_k \int_{-\infty}^{\infty} \Phi_g(w)\{w_j^4\ \overline{A}$$

$$+ w_j^2 w^2\ \overline{B})|H_j(w)|^2/w_j^4 + (w_k^4\ \overline{C} + w_k^2 w^2\ \overline{D})|H_k(w)|^2/w_k^4\}dw \qquad (2)$$

where j, k denote mode numbers; N=significant number of modes; γ_j= participation factors; q_j=the modal value of the response quantity (like displacement, force, stresses, etc.); w_j=the frequency; $\overline{A}$, $\overline{B}$, etc.=coefficients, as defined in Ref. 3; and $|H_j(w)|^2 = 1/\{w_j^2 - w^2)^2 + 4\ \beta_j^2\ w_j^2 w^2\}$, where β_j is structural damping. The single summation term in Eq. 2 is the same as the usual SRSS and represents the contribution of the individual modes. The double summation term represents the effect of interaction between modes. For system with close frequencies, the contribution of this term is significant and should be considered in determining the response.

To define R_q in terms of the response spectrum, the integrals $I_1 = C\int w_j^4\ \Phi(w)|H_j(w)|^2 dw$ and $I_2 = C\int w_j^2\ w^2\Phi(w)|H_j(w)|^2 dw$ should be defined in terms of response spectrum value at w_j. This was done in Ref. 3, and is probably adequate; nevertheless, the following improvements are in order.

If damping is small, Eq. 1 gives $I_1 = C\int w_j^4\Phi(w)|H_j(w)|^2 dw \simeq R^2(w_j)$, but a more accurate expression is

$$I_1 \simeq R^2(w_j)\ /\ (1 + 4\ \beta_j^2) \qquad (3)$$

I_2 is defined by the relative velocity characteristics of the oscillator response. It can be shown that

$$I_2 = C\int w_j^2 w^2\Phi(w)|H_j(w)|^2 dw \simeq C\ \Phi\ (w_j)w_j^4 \int |H_j(w)|^2 dw \qquad (4)$$

For broad band spectra such as shown in Fig. 1, the PSD of the excitation will also be a broad band one. Fig. 2 shows such a PSD. Also shown in the figure is the function $w_j^4|H_j(w)|^2$ for two values of w_j. For low frequencies it can be shown that

$$I_1 = C\, w_j^4 \int \Phi(w)\,|H_j(w)|^2 dw \simeq C\, w_j^4\, \Phi(w_j) \int |H_j(w)|^2 dw \quad (5)$$

and thus $I_1 \simeq I_2 \simeq R^2(w_j)/(1+4\beta_j^2)$. For the spectrum of Fig. 1, this will be reasonably accurate for $w_j \leq 9$ cps. For $w_j > 9$ cps, the PSD of the excitation decays rapidly. For such a PSD, and for $9 \leq w_j \leq 33$ cps, the integral I_1 can be approximated by the following:

$$I_1 = C \int_{-\infty}^{w_j} \Phi(w)dw + C\, \Phi(w_j) \int_{-\infty}^{\infty} w_j^4 |H_j(w)|^2 dw = I_b + I_2 \quad (6)$$

Here, 9 cps and 33 cps are the Control Frequencies of the site spectra. For $w_j < 9$ cps, I_b is zero, and $I_1 \simeq I_2$. Also for $w_j > 33$ cps, $I_b = A_g^2$, $I_1 = A_g^2$, and thus $I_2 = 0$. A_g is the maximum ground acceleration. For $9 \leq w_j \leq 33$ cps, I_b was observed to increase exponentially. Thus, using Eq. 6, I_2 can be obtained from the following:

$$I_2 = I_1 - A_g^2\,(\ln w_j - \ln 9)/(\ln 33 - \ln 9) \quad (7)$$

Denoting $F(w_j) = I_2(w_j)/I_1(w_j)$, the design response, Eq. 2, can be written as

$$R_q^2 = \sum_j^N \gamma_j^2 q_j^2 I_1(w_j)/w_j^4 + 2 \sum_j \sum_{k=j+1} \gamma_j \gamma_k q_j q_k [\{\overline{A} + F(w_i)\ \overline{B}\} I_1(w_j)/w_j^4$$
$$+ \{\overline{C} + F(w_k)\ \overline{D}\} I_1(w_k)/w_k^4] \quad (8)$$

INPUT SPECTRA FOR SECONDARY SYSTEMS: Using the formulation developed in Ref. 3, and the definitions of I_1 and I_2 described above, the following equation is obtained for floor spectrum value at frequency w_o:

$$R_a^2(w_o, u) = \sum_j^N \gamma_j^2\, \psi_j^2(u) [\{A + F(w_o)\ B\} I_1(w_o) + \{C + F(w_j) D\} I_1(w_j) \quad (9)$$
$$+ 2 \sum_j \sum_{k=j+1} \gamma_j \gamma_k \psi_j(u) \psi_k(u) [A' + F(w_o)\ B') I_1(w_o)$$
$$+ (C' + F(w_j) D') I_1(w_j) \qquad + (E' + F(w_k) F') I_1(w_k)]$$

where $R_a(w_o,u)$ is the floor spectrum value for floor u, $\psi(u)$ is the relative modal displacement of floor u, and A, B, etc. are defined in Ref. 3.

Factors A, B, etc. are, however, not defined for a resonating case; i. e., when w_o is the same as one of the structural frequencies. In such a case, an integral $I_3 = C \int \Phi(w)\ (w_o^4 + 4\ \beta_o^2\ w_o^2\ w^2)^2 |H_o(w)|^4 dw$ is involved; and this needs to be defined in terms of response spectrum value $R(w_o)$. For various frequency ranges, I_3 can be defined as,

$$\begin{aligned} I_3 &= A_m\, R^2(w_o) \quad ; \quad w_o \leq 9 \text{ cps} \\ &= (1-A_m)(\ln w_o - \ln 9)/(\ln 33 - \ln 9) A_g^2 + A_m R^2(w_o); \quad 9 \leq w_o \leq 33 \text{ cps} \\ &= A_g^2 \quad ; \quad w_o \geq 33 \text{ cps} \end{aligned} \quad (10)$$

where

$$A_m = [1 + 4\ (\beta^2 + \beta_o^2 + \beta\ \beta_o) + 16\ \beta^2\ \beta_o^2]/[4\ \beta_o^2\ (\beta + \beta_o)(1 + 4\ \beta_o^2)] \quad (11)$$

The underlying basis of Eq. 10 is the same as that of Eq. 6. To establish the validity of Eq. 10, the integral I_3 was obtained by contour integration for a spectrum consistent PSD and by Eq. 10. The two values of I_3 were in good agreement with each other.

SECONDARY SYSTEMS WITH MULTIPLE SUPPORTS: The equation of motion of a secondary system with more than one support can be written as:

$$[M_s]\{\ddot{u}\} + [K_s]\{u\} = -[M_s]\{1\}(\ddot{Y}_o + \ddot{x}_g) - [K_c]\{Y\} \tag{12}$$

where $[M_s]$ and $[K_s]$ are the mass and stiffness matrices, $[K_c]$ = n by m cross stiffness matrix for n masses and m supports of the secondary system, Y_o = relative displacement of a reference support o, $\ddot{x}_g$ = ground excitation, and {u} and {Y} are the displacements of the masses and supports, respectively, relative to the reference support, o. The r.h.s. of Eq. 12 can be expressed in terms of the modal quantities of the primary system. Using the normal mode approach, Eq. 12 is solved to obtain the design response as:

$$\begin{aligned}R_a^2 = &\sum_{\ell=1}^{n} \psi_{s\ell}^2(u) \sum_{k=1}^{N} \gamma_{pk}^2\psi_{pk}^2[(A + B)I_1(w_{pk}) + (C + D)I_1(w_{s\ell})] \\ &+ 2\sum_{k}^{N}\sum_{j=k+1}^{N} \gamma_{pk}\gamma_{pj}\psi_{pk}\psi_{pj}[(A' + B')I_1(w_{pk}) + (C' + D')I_1(w_{pj}) \\ &+ (E' + F')I_1(w_{s\ell})] + 2\sum_{\ell}^{n}\sum_{i=\ell+1}^{n} \psi_{s\ell}\psi_{si} \sum_{k}^{N} \gamma_{pk}^2\psi_{pk}^2[(\bar{A} + \bar{B})I_1(w_{pk}) \\ &+ (\bar{C} + \bar{D})I(w_{s\ell}) + (\bar{E} + \bar{F})I(w_{si})] + 2\sum_{k}^{N}\sum_{j=k+1}^{N} \gamma_{pk}\gamma_{pj}\psi_{pk}\psi_{pj}[(\bar{A}' + \bar{B}') \\ &I_1(w_{pk}) + (\bar{C}' + \bar{D}')I_1(w_{pj}) + (\bar{E}' + \bar{F}')I_1(w_{s\ell}) + (\bar{G}' + \bar{H}')I_1(w_{si})]\end{aligned} \tag{13}$$

where the quantities with subscript p are for the primary system and those with subscript s are for the secondary system, γ=participation factors, ψ=relative displacement mode shapes. A, B, etc. are the coefficients of partial fractions which, besides the frequency ratios and damping values, also depend upon the participation factors and $[K_c]$ of the secondary system.

NUMERICAL RESULTS

Numerical results demonstrating the application of the proposed methods to certain practical situations are presented. Eq. 8 has been used to obtain the member response results for a structure with closely spaced frequencies. The results obtained by the random vibration approach using a spectrum consistent PSD are used as reference for comparison. The effectiveness of the random vibration approach itself for determining more accurate response of primary and secondary systems was demonstrated in Ref. 4 and 5. A system with torsional mode as shown in Fig. 3 is used. The system parameters can be adjusted such that the natural frequencies are close to each other, as in Table 1. Table 2 shows the results obtained by the random vibration approach, the conventional SRSS method, and the proposed method (Eq. 8). It is seen that the proposed method consistently gives better results than the SRSS method.

Application of Eqs. 9 to 11 for generation of floor spectra is straightforward. Fig. 4 shows a floor spectrum generated using Eq. 9 for a nuclear power plant floor. The comparison of this curve with the curve obtained by a more accurate stochastic approach was found excellent.

Application of Eq. 13 for obtaining member forces in a secondary system having multiple supports is likewise straightforward.

SUMMARY AND CONCLUSIONS

Viable methods which directly use site spectra curves for seismic analysis of primary and secondary structures are described. The proposed methods are rational in concept, straightforward for application and inexpensive computationally.

REFERENCES

1. Newmark, N. M., Blume, J. A. and Kapur, K. K., "Seismic Design Spectra for Nuclear Power Plants," J. Power Div., ASCE, 99, 287-303, Nov., 1973.
2. Singh, M. P., and Agrawal, P. K., "A Stochastic Method for Seismic Stability Evaluation of Earth Structures with Strain Dependent Properties," 6 WCEE, New Delhi, 1977.
3. Singh, M. P. and Chu, S. L., "Stochastic Consideration in Seismic Analysis of Structures," Int. J. of Eq. Engrg. and Str. Dyn., Vol. 4, No. 3, 1976.
4. Gungor, I., "A Study of Stochastic Models for Predicting Maximum Earthquake Structural Response," Ph. D. Thesis, Univ. of Illinois, Urbana-Champaign, Aug., 1971.
5. Singh, A. K., and Ang, A. H.-S., "Stochastic Prediction of Maximum Response of Light Secondary Systems," 2nd Intl. Conf. on Str. Mech. in Reactor Technology, Berlin, Sept., 1973.

TABLE 1

FREQUENCIES OF TORSIONAL SYSTEM

Mode	FRQ.	Mode	FRQ.
1	1.0	5	4.860
2	1.056	6	4.911
3	2.919	7	6.244
4	3.083	8	6.742
5	4.602	9	7.122

TABLE 2

SPRING FORCE IN THE TORSIONAL SYSTEM, FIG. 3

Spring No.	Random Vibration Method	SRSS Force	SRSS Col. 3 / Col. 2	Proposed Method Eq. 9 Force	Proposed Method Eq. 9 Col. 5 / Col. 3
1	8.65	7.47	.863	8.67	1.002
2	7.85	6.76	.861	7.86	1.000
3	6.63	5.67	.856	6.59	0.993
4	5.03	4.28	.851	4.97	0.987
5	4.57	4.03	.882	4.52	0.988

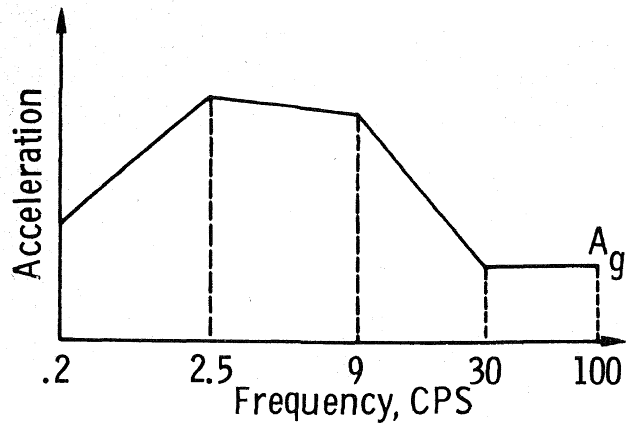

FIG. 1: A TYPICAL DESIGN RESPONSE SPECTRUM

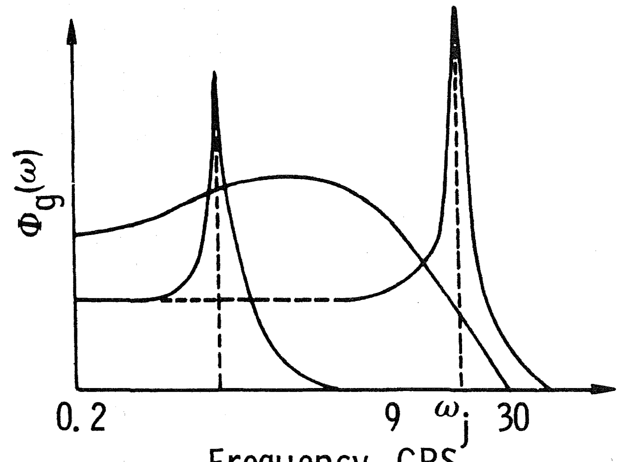

FIG. 2: A BROAD BAND SPECTRAL DENSITY FUNCTION

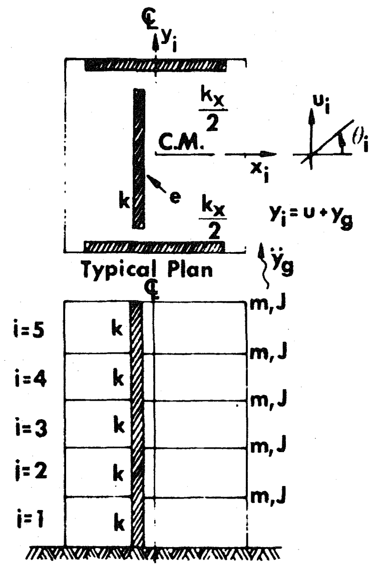

FIG. 3: A SIMPLE TORSIONAL SYSTEM

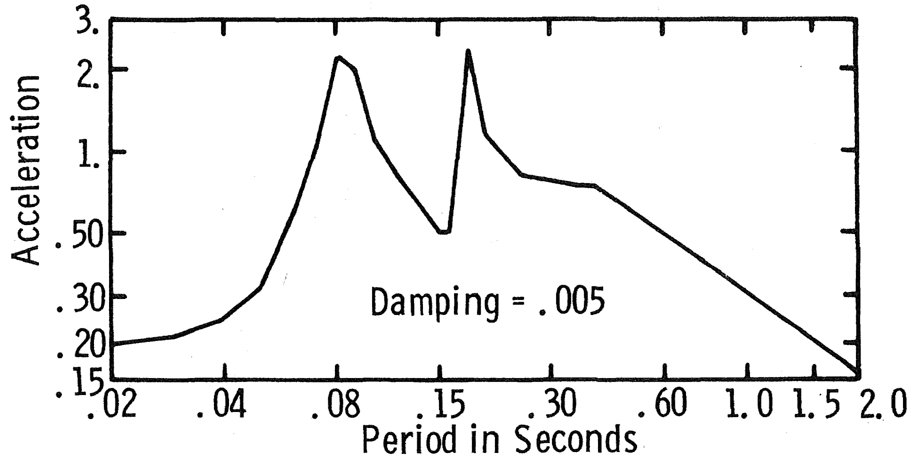

FIG. 4: FLOOR SPECTRA OF A NUCLEAR POWER PLANT BUILDING