ON THE IMPORTANCE OF LAYERING ON THE IMPEDANCE FUNCTIONS

by

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SYNOPSIS

The effects of layering on the vertical, horizontal and rocking impedance functions for a rigid circular foundation are examined. Two basic soil models corresponding to a single layer and a double layer on rock are studied in detail. By varying the geometrical and physical properties of the layers a number of case studies are generated. The results obtained are presented graphically and their significance discussed.

INTRODUCTION

One of the current methods of analysis of soil-structure interaction problems relies on the use of the impedance functions for a rigid massless foundation resting on the soil surface. For a very long time the only impedance functions available were those for a uniform elastic half-space. Recently, the need for more realistic soil models for the analysis of nuclear power plant structures has led to the development of general models that include multi-layer representations of the soil consisting of several parallel viscoelastic layers with hysteretic or Voigt types of material damping (1,2). The details of the method of solution are not repeated here. The present paper focuses on the analysis of the layering effects on the impedance functions over a range of frequencies. Special consideration is given to the determination of the conditions under which a layered medium may be approximately represented as a uniform half-space. These conditions are of interest since in many simplified soil-structure interaction analyses it is assumed that the soil may be represented by a uniform half-space.

CHOICE OF SOIL MODELS

Although layering conditions vary from site to site, it is possible to obtain valuable information from the analysis of a few simple models. In particular, layering conditions corresponding to a single layer and a double layer resting on rock are considered in detail here. The basic single layer on rock model geometrical and physical characteristics are shown in Fig. 1 for nine different cases. The hysteretic damping constants ξ have been chosen to represent lower bounds for moderate earthquake motions so that conservative conclusions regarding layering effects may be drawn.

The basic double layer on rock model is shown in Fig. 2. In this model, two layers of depth H_1 and H_2, respectively, are combined to form an overburden 300 ft. thick. Five different cases corresponding to the combination of H_1 and H_2 shown in Fig. 2 were studied. The characteristics of the foundation and the underlying rock were taken to coincide with those for the single layer model. The same Poisson's ratio ν of 1/3 was considered for all media.

RESULTS: SINGLE LAYER ON ROCK

The vertical, horizontal and rocking impedance functions for the nine cases shown in Fig. 1 were evaluated for different frequencies in the range

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from 0 to 5 Hz. Before proceeding with the analysis of the results for the dynamic case it is instructive to discuss the layering effects under static conditions.

One way of describing the effects of layering on the static impedance coefficients is to compute the error involved in replacing the layered medium by a uniform half-space having the same elastic properties as those for the top layer of the layered medium. One of the advantages of this procedure is that the error thus computed depends on the ratio of the shear moduli of the layer and underlying rock, G_1/G_2, on the ratio of the layer thickness to the foundation radius, H/R, and on the mode of displacement (vertical, horizontal, rocking). Since the density variations are small it is possible to use the ratio of the shear wave velocities V_{s1}/V_{s2} instead of G_1/G_2 as parameter. Fig. 3 defines the 10, 20 and 30 percent error boundaries for the vertical, horizontal and rocking static impedances as a function of V_{s1}/V_{s2} and H/R. Inspection of Fig. 3 indicates that the vertical impedances are the most sensitive to layering effects while the rocking impedances are the least affected. Fig. 3 also shows that for a given layer thickness the effects of layering increase with increasing contrast between V_{s1} and V_{s2}, and that for a given V_{s1}/V_{s2} ratio the layering effects decrease with increasing layer thickness. In particular, it is interesting to note that if an error of 20 percent in the static impedances is acceptable then a layer thicker than 4.5 times the radius of the foundation may be replaced by an elastic half-space irrespective of the velocity contrast.

In the dynamic case the layering effects are more pronounced and exhibit a strong frequency dependency as may be seen in Fig. 4. Due to space limitations the normalized stiffness coefficients (real part of the impedance function) and the normalized damping coefficients (imaginary part of the impedance functions divided by frequency) for only vertical vibrations for Cases 1, 4 and 7 of Fig. 1 are shown versus frequency. Also shown in Fig. 4 are results for a uniform half-space having the properties of the top layer (see legend), and the results for an "equivalent" half-space selected in such a way that the static stiffness coefficients for the "equivalent" half-space and the layered medium are equal (curves labeled V_s = 930, 640 and 560 ft/sec for Cases 1, 4 and 7, respectively). This static "equivalence" is easily determined here since the layered impedances at $a_o = 0$ were computed. The results presented in Fig. 4 show that as the thickness of the layer increases, the impedance functions for the layered medium approach the corresponding functions for a half-space with the properties of the top layer. However, even when the thickness of the layer is four times the foundation radius (Case 7) some strong deviations from the half-space results can be observed. For thin layers (Case 1), the stiffness coefficient for the layered medium is much higher than the corresponding coefficient for the half-space having the properties of the top layer, while a reasonable approximation is possible for the "equivalent" half-space. The damping coefficients for both uniform half-spaces grossly overestimate the damping in the layered medium.

RESULTS: DOUBLE LAYER ON ROCK

The normalized rocking, horizontal and vertical impedance functions for the five cases described in Fig. 2 are shown in Fig. 5 versus the dimensionless frequency $a_o = \omega R/V_{s1}$, where ω is the frequency in radians per second. The stiffness coefficients have been normalized by the corresponding static coefficients for a half-space having the properties of the top layer. The damping coefficients are divided by the dimensionless frequency a_o and normalized by the same factor used for the stiffness coefficients.

Inspection of the curves in Figure 5 indicates that, similar to the statics case, the vertical impedances are the most sensitive to layering effects while the rocking impedances are the least affected. Additionally, compared to the single layer, the behavior of the horizontal and vertical impedances is most complex. However, for all three types of motion, there is a distinct difference between the behavior of the stiffness and damping coefficients: whereas the reduction of stiffness is monotonic (in a gross sense) with the increase in the top layer thickness, the damping coefficient takes a drastic drop with the introduction of a thin layer with a subsequent increase as the layer thickness increases. This phenomenon is associated with the fact that the introduction of a new layer boundary prevents the full radiation of energy to the lower levels. Another interesting observation is that even a thin surface layer produces a marked reduction in stiffness. The subsequent increase in the top layer thickness has only a relatively minor effect in the further reduction of the stiffness coefficient.

CONCLUSIONS

The partial results described above lead to the conclusion that the rocking impedances are the least affected by layering while the vertical impedances are the most affected. The stiffness coefficients for a thin layer supported on harder rock are in general larger than the corresponding coefficients for a half-space having the properties of the top layer, while the damping coefficients are much lower in the layered medium. If the thickness of the layer is between one and six times the radius of foundation, the impedance functions exhibit a marked frequency dependence associated with the existence of Rayleigh and Love waves. Only if the layer thickness is larger than six times the radius of the foundation it is possible to grossly approximate the layered system by a half-space. The results from the other cases (not shown) indicate that with decreasing contrast between V_{s1} and V_{s2} this frequency dependency becomes less significant. Space does not allow the presentation of results from the other cases and the development of curves similar to Fig. 3 for the dynamic case.

Although no attempt was made to simplify the problem of the multi-layered medium for calculational purposes, an important conclusion from the study of the double layer on rock is the significance of the impact of the first layer on the impedance functions (both stiffness and damping coefficients), suggesting perhaps that more attention should be given to the determination of the geometric and material properties of the top shallow layers than the rest of the overburden.

Some of the results presented here could be useful in the finite element modeling of soils for soil-structure interaction analysis. For instance, one obvious result, which is rarely considered, is the fact that what is considered proper modeling for rocking motions may be completely inadequate for vertical or even horizontal motions.

REFERENCES

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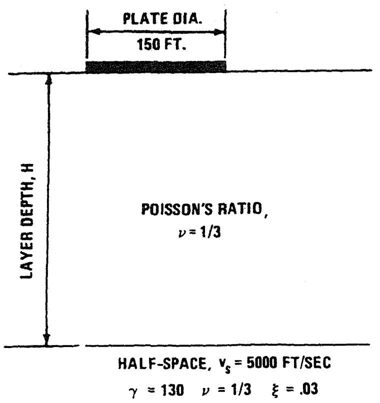
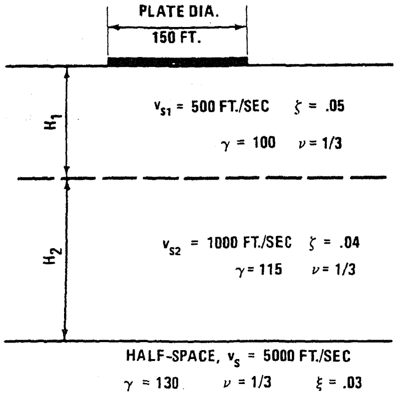


CASES STUDIED

H (FT) \ v_s	500	1000	2000
γ	100	115	130
ζ	0.05	0.04	0.03
50	CASE 1	CASE 2	CASE 3
150	CASE 4	CASE 5	CASE 6
300	CASE 7	CASE 8	CASE 9

v_s – SHEAR WAVE VELOCITY IN FT/SEC
γ – UNIT WEIGHT OF SOIL IN LBS/FT^3

FIGURE 1. PARAMETERS OF THE SINGLE LAYER STUDY



CASES STUDIED

	H_1(FT.)	H_2(FT.)
CASE 8	0	300
CASE 10	50	250
CASE 11	150	150
CASE 12	250	50
CASE 7	300	0

FIGURE 2. PARAMETERS OF THE DOUBLE LAYER STUDY

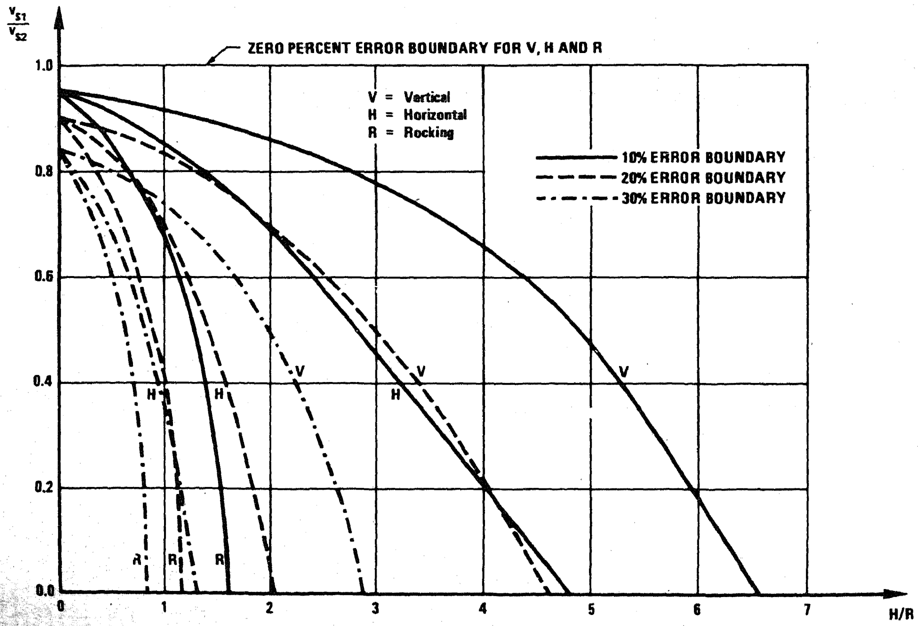


FIGURE 3. ZONES OF LAYERING EFFECTS ON STATIC IMPEDANCE COEFFICIENTS

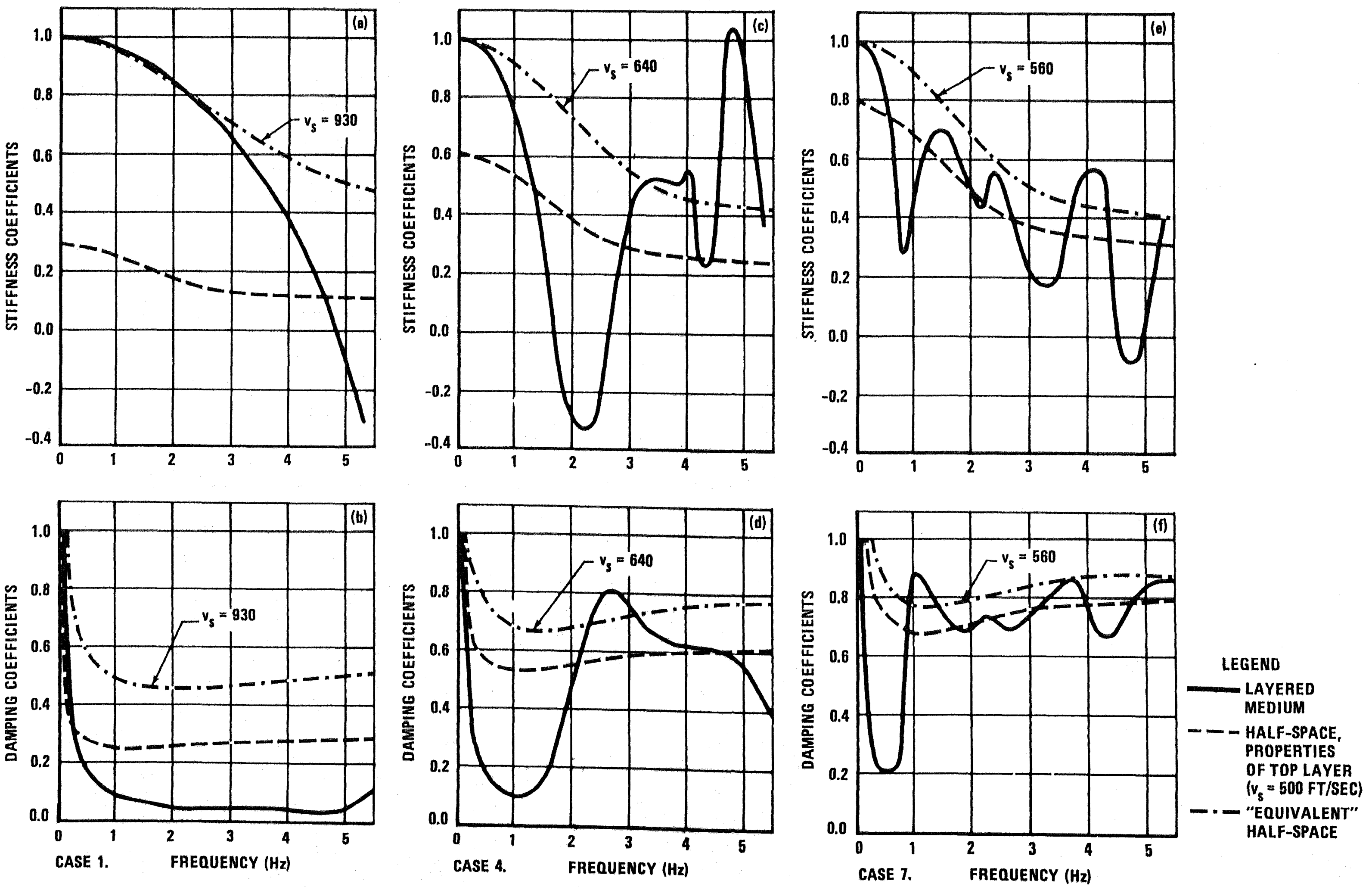


FIGURE 4. NORMALIZED VERTICAL IMPEDANCE FUNCTIONS (CASES 1, 4, 7)

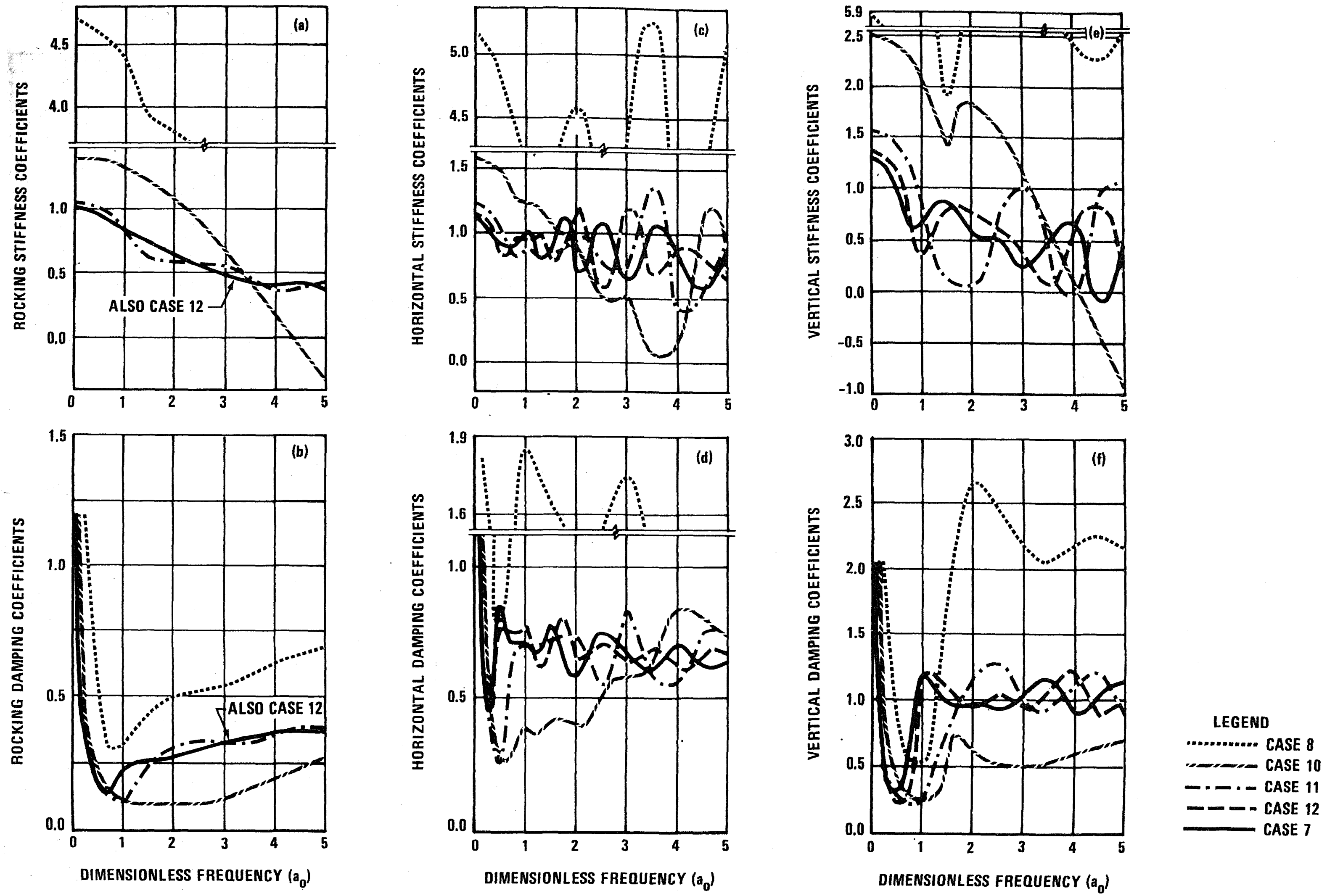


FIGURE 5. NORMALIZED IMPEDANCE FUNCTIONS (CASES 7, 8, 10, 11, 12)

DISCUSSION

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The authors should be commended for the study.

Have the authors studied the case when the rock layer be on the top of softer material like sand ? The discussor had to work on a case where the first layer was about 100 feet of lime stone rock followed by 450 feet of sandy silt, underlain by rock.

What will be the effect on impedance function in such a case ?

Author's Closure

Not received.