

# CALCULATION OF BEARING CAPACITY FOR THE FOUNDATIONS SUBJECTED TO SEISMIC LOADS

by

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## SYNOPSIS

A method for calculation of bearing capacity of a foundation is developed on the basis of safe stress state theory of loose media suggested by V.V. Sokolovsky and S.S. Golushkevich<sup>1</sup>. An inclined failure load is calculated as a function of bearing capacity dimensionless coefficients. For the calculation of a slope stability the expressions containing dimensionless coefficients are obtained. These coefficients help to evaluate the inclined load on the second side plane which corresponds to the safe stress state of an earth mass, provided the inclined uniform load on the first side plane and the direction of the inclined load on the second side plane are known.

## INTRODUCTION

The methods for calculation of a foundation bearing capacity and stability of slopes not subjected to seismic loads are elaborated satisfactorily<sup>2</sup>. If it is necessary to take into account seismic loads, there are many suggestions to do it on the basis of a circle slip surface. The problems connected with the application of a safe stress state theory to the calculation of foundations under seismic conditions are not sufficiently developed. The numerical methods of this theory are very tedious and not convenient for engineering practice. That is why the development of simple closed-form solutions based on this theory and taking into account seismic loads is an important task.

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## BEARING CAPACITY OF A FOUNDATION

If a structure is subjected to vertical and horizontal forces, the stress epure of the structure toe is assumed practically to be trapezoidal. In order to simplify the process of calculation<sup>3</sup>, the stress epure is replaced by a calculated uniform epure having a width equal to  $B$  (Fig. 1) and inclined to the vertical line by an angle  $\delta'$ . The angle of the reduced pressure deviation from the vertical line,  $\rho$ , within the section  $ED$  (reduced pressure is equal to a geometrical sum of a vertical pressure,  $n$ , and inclined pressure,  $q/\cos\omega$ ) is determined by the formula:

$$\rho = \arctg\left(\frac{q \operatorname{tg}\omega}{q+n}\right); \quad n = c \cdot \operatorname{ctg}\varphi; \quad q = \gamma t; \quad \omega = \arctg\frac{j}{g};$$

$$G_{3c} = \overline{ED} \cdot q / \cos\omega; \quad P = \overline{ED} \cdot n; \quad G_{ic} = G_i / \cos\omega,$$

where,  $c$  - earth cohesion;  $n$  - cohesion pressure;  $\varphi$  - angle of internal friction;  $\gamma$  - bulk weight;  $t$  - deepening of the structure toe;  $j$  - seismic acceleration;  $\omega$  - angle of deviation of the resultant gravity force and the horizontal seismic force from the vertical line;  $G_i$  - weight of a given earth zone. In order to define the locations of the slip surfaces, it is necessary to determine the angles:

$$\nu = \frac{1}{2} \left( \arccos \frac{\sin \delta'}{\sin \varphi} + \varphi - \delta' \right); \quad \varepsilon = \frac{1}{2} \left( \frac{\pi}{2} - \varphi - \rho + \arcsin \frac{\sin \rho}{\sin \varphi} \right);$$

$$\alpha = \varphi + \varepsilon; \quad \theta = \pi/2 + \nu - \alpha; \quad \overline{ED} = BK; \quad K = a / \cos \alpha; \quad a = \sin \nu \exp(\theta \operatorname{tg} \varphi).$$

The value of the failure load,  $R_{np,p}$ , which corresponds to a safe equilibrium state of a foundation can be determined by successive construction of the polygons of forces for  $ED$ ;  $ECB$  and  $EAB$  zones. However, the value of the force,  $R_{np,p}$ , and the stresses  $\sigma_{np,p}$  and  $\tau_{np,p}$  can be defined more accurately and conveniently analytically using the formulas:

$$R_{np,p} = \gamma B^2 N_\gamma + BC N_c + Bq N_q; \quad \sigma_{np,p} = \frac{R_{np,p} \cos \delta'}{B} - n; \quad \tau_{np,p} = \frac{R_{np,p} \sin \delta'}{B},$$

where,  $N_y$ ,  $N_c$  and  $N_q$  - the coefficients of bearing capacity of a foundation. The coefficients are calculated as follows:

$$b = \frac{\sin \vartheta \cos(\varphi - \vartheta)}{2 \cos \varphi \cos \omega}; \quad d = \frac{a^2 - \sin^2 \vartheta}{2 \sin 2\varphi \cos \omega}; \quad i = \frac{ka \sin \varepsilon}{2 \cos \varphi \cos \omega};$$

$$\mu_{1r} = \varepsilon + \omega - \arctg \left[ \frac{(b+d) \sin(d+\omega)}{(b+d+i) \cos(\varepsilon+\omega) \cos \varphi} - \operatorname{tg} \varphi \right]; \quad (180^\circ > \mu_{1r} > 0^\circ);$$

$$\beta = \frac{\pi}{2} - \vartheta + \arctg \left[ \frac{\exp(\theta \operatorname{tg} \varphi) - \cos \theta}{\sin \theta} \right]; \quad t = \sin(\delta' + \vartheta - \varphi);$$

$$m = \frac{a \cos \varepsilon}{\operatorname{tg} \varphi \cos d \cos(\varepsilon + \omega)}; \quad S_r = \frac{(b+d) \sin(d+\omega)}{\sin(d+\omega - \mu_{1r})}; \quad N_q = \frac{N_c \cdot k}{m \cos \omega};$$

$$\mu_r = \arctg \left[ \frac{b \cos(\omega - \beta) - S_r \cos(\omega - \beta - \mu_{1r})}{S_r \cos(\omega - \beta - \mu_{1r}) \operatorname{tg}(\vartheta + \omega) - b \sin(\omega - \beta)} \right]; \quad (180^\circ > \mu_r > 0^\circ);$$

$$N_y = \frac{b \cos(\vartheta + \omega) \sin(\mu_r - \vartheta - \omega + \varphi)}{t \cos(\mu_r - \vartheta - \omega)}; \quad N_c = - \frac{m \cos(\varepsilon + \omega) \cos(d + \beta)}{t \sin(\beta + \vartheta)}.$$

The coefficients  $N_y$ ,  $N_c$  and  $N_q$  as well as  $K$  are dimensionless and depend only on the angles  $\varphi$ ,  $\delta'$ ,  $\rho$  and  $\omega$ . By means of a computer they are calculated for all practically encountered cases. The results obtained allow to draw a diagram of a bearing capacity of a foundation and determine a safety factor <sup>3,4</sup>.

#### LIMIT STRESS STATE OF AN EARTH WEDGE AND STABILITY OF A SLOPE

Reduced seismic pressure,  $q_{1c}$ , and an angle of seismic pressure deviation from the normal,  $\rho_{1c}$ , towards  $OA$  is equal to (Fig. 2):

$$q_{1c} = \sqrt{(q'_{1c})^2 + c^2 \operatorname{ctg}^2 \varphi + 2q'_{1c} c \cdot \operatorname{ctg} \varphi \cos \rho'_{1c}};$$

$$\rho = \arctg \left( \frac{q'_{1c} \sin \rho'_{1c}}{q'_{1c} \cos \rho'_{1c} + c \cdot \operatorname{ctg} \varphi} \right),$$

where,  $q'_{1c}$  and  $\rho'_{1c}$  - known values of a real seismic pressure and an angle of seismic pressure deviation from the normal to  $OA$ . In order to locate the slip surfaces, the angles should be calculated:

$$\Delta_1 = \arcsin(\sin \rho_{1c} / \sin \varphi); \theta_1 = 0.5(\pi/2 + 2\beta_1 - \varphi - \rho_{1c} - \Delta_1);$$

$$\theta_2 = \pi/2 - \varphi - \theta_1; \rho = \pi/2 + \beta_1 - \theta_1; \eta = \pi/2 + \varphi - \rho;$$

$$\Delta_2 = \arcsin(\sin \rho_{2c} / \sin \varphi); \varepsilon = 0.5(\pi/2 - \varphi - \rho_{2c} + \Delta_2);$$

$$\omega_0 = \pi/2 - \varphi - \varepsilon; \theta = \pi + \beta_1 + \beta_2 - \rho - \varepsilon,$$

where,  $\rho_{2c}$  - a known angle of deviation of a reduced pressure from the normal to  $OD$ . The unknown force,  $P_{2c}$ , corresponding to a safe equilibrium state of an earth wedge (slope) can be determined by successive construction of the polygons for  $OAB$ ,  $OBC$  and  $OCD$  zones. However, this force can be defined more accurately and conveniently analytically:

$$P_{2c} = K_1 l_1 (\gamma l_1 N_\gamma + q_{1c} N_q); \overline{OD} = K_1 l_1,$$

where, the coefficients of slope bearing capacity,  $N_\gamma$  and  $N_q$ , as well as parameter  $K$  are dimensionless values depending on the angles  $\omega$ ,  $\varphi$ ,  $\beta_1$ ,  $\beta_2$ ,  $\rho_{1c}$  and  $\rho_{2c}$ . These coefficients are calculated as follows:

$$b = \frac{\sin \eta \sin \rho}{2 \cos \varphi \cos \omega}; d = \frac{\sin^2 \eta [\exp(2\theta \operatorname{tg} \varphi) - 1]}{2 \sin 2\varphi \cos \omega}; K_1 = \frac{\sin \eta \exp(\theta \operatorname{tg} \varphi)}{\sin \omega_0};$$

$$n = \frac{\sin(\theta_1 + \rho_{1c} - \beta_2)}{\sin(\theta_1 + \omega)}; i = \frac{\sin^2 \eta \sin \varepsilon \exp(2\theta \operatorname{tg} \varphi)}{2 \cos \varphi \sin \omega_0 \cos \omega}; f_q = \frac{n \sin(\theta_1 + \omega)}{\cos \varphi};$$

$$\mu_{1\gamma} = \operatorname{arctg} \left\{ \frac{b}{b \operatorname{ctg}(\theta_2 - \omega) + (d+i)[\operatorname{ctg}(\theta_2 - \omega) + \operatorname{ctg}(\theta_1 + \omega)]} \right\}; (180^\circ > \mu_{1\gamma} > 0^\circ);$$

$$f_\gamma = \frac{(d+i) \sin(\theta_2 - \omega)}{\sin(\theta_2 - \mu_{1\gamma} - \omega)}; \Psi = \varepsilon + \varphi + \operatorname{arctg} \left[ \frac{\exp(-\theta \operatorname{tg} \varphi) - \cos \theta}{\sin \theta} \right];$$

$$m_\gamma = \cos(\mu_{1\gamma} + \omega) - \operatorname{tg}(\Psi - \beta_2) \sin(\mu_{1\gamma} + \omega);$$

$$\mu_\gamma = \operatorname{arccctg} \left\{ \frac{i [\operatorname{tg}(\Psi - \beta_2) \cos \omega + \sin \omega] + f_\gamma m_\gamma \operatorname{tg}(\beta_2 + \omega_0 - \omega)}{i [\cos \omega - \operatorname{tg}(\Psi - \beta_2) \sin \omega] - f_\gamma m_\gamma} \right\}; (180^\circ > \mu_\gamma > 0^\circ);$$

$$N_\gamma = -\frac{i \cos(\beta_2 + \omega_0 - \omega) \cos(\mu_\gamma + \varepsilon - \beta_2 + \omega)}{K_1 \cos(\beta_2 - \mu_\gamma + \omega_0 - \omega) \cos(\rho_{2c} + \varepsilon)}; N_q = -\frac{f_q \cos(\theta_2 + \Psi - \beta_2) \cos^2 \varphi}{K_1 \sin(\Psi + \omega_0) \cos(\rho_{2c} + \varepsilon)}.$$

At first, the distribution of pressure along the side plane OD may be assumed uniform. However, the trapezoidal contour of the epure lies closer to precise solutions. Taking into account the fact that the influence of the volumetric forces at the point O is not yet felt, we may obtain

$$q_{2c}^0 = q_{1c} N_q; \quad q_{2c}^D = 2\gamma l_1 N_\gamma + q_{1c} N_q.$$

The shown pressure epure of the side plane OD helps to construct the epure of real pressure<sup>3</sup>. The obtained formulas allow to solve also a reverse problem. If the values  $P_{2c}$ ,  $\rho_{2c}$ ,  $\rho_{1c}$  and  $l_1$  are set, then

$$P_{1c} = P_{2c} \frac{1}{K_1 N_q} - \gamma l_1^2 \frac{N_\gamma}{N_q}.$$

The coefficients  $N_\gamma$ ,  $N_q$ ,  $K_1$  were computed for many practically encountered cases, the tables and diagrams were also compiled. The particular cases of this solution are: the problems of a foundation bearing capacity ( $\beta_1 = \beta_2 = 0$ ), the problems of active and passive earth pressure on the wall at  $C=0$  (in this case,  $\rho_{2c}$  is equal to the angle of earth friction along the wall). During calculation the signs of the angles  $\beta_1$ ,  $\beta_2$ ,  $\rho_{1c}$  and  $\rho_{2c}$  must be observed<sup>3</sup>. The angles  $\beta_1$  and  $\rho_{1c}$  are positive in Fig.2 while the angle  $\beta_2$  is negative.

#### REFERENCES

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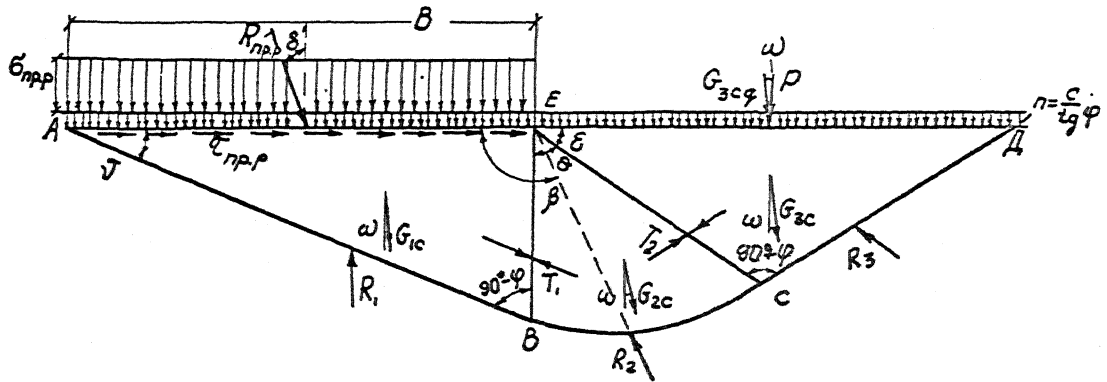


Fig.1. Slip surfaces while calculating a foundation bearing capacity

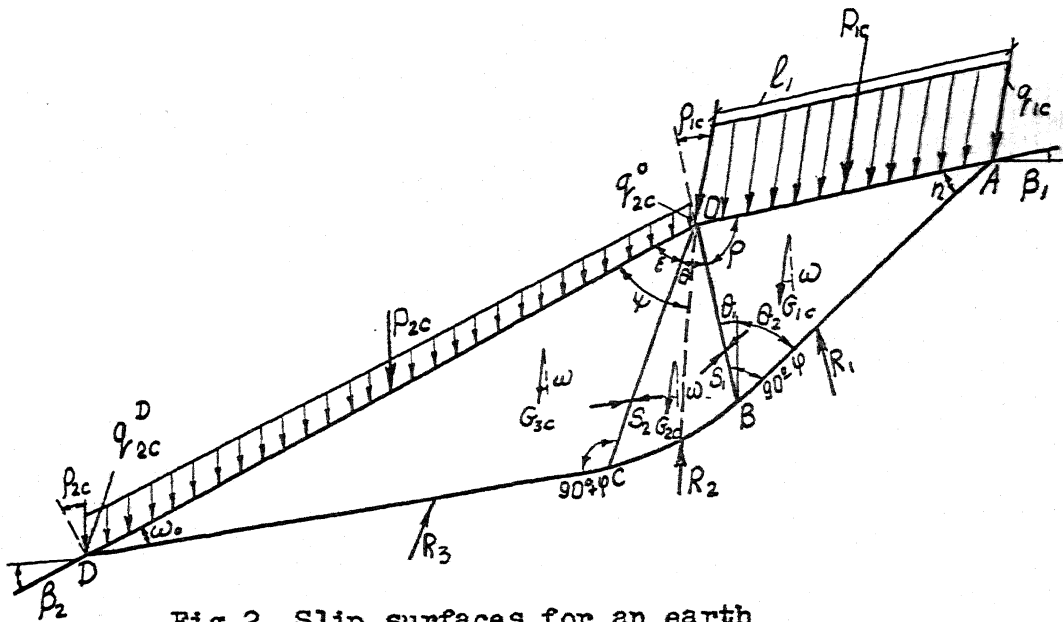


Fig.2. Slip surfaces for an earth wedge (slope)