RESPONSE OF EMBEDDED FOOTINGS AND STRUCTURES UNDER EARTHQUAKE MOTION

by

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SYNOPSIS

An approximate solution is presented for the coupled response of footings and structures embedded partially into a seminfinite medium. The response study has been made with the general equillibrium equation of motion which includes the dynamic interaction coefficients. The foundation is assumed to be flexible and supported on an elastic medium. The approach to the problem is illustrated by the solution of the coupled response involving horizontal translation, rocking and vertical translation. The interaction coefficients as obtained by Parmelee(12) are used in the analysis of embedded footings and structures. The results are presented with respect to the displacements resulting due to translational, horizontal and rocking motion ofstructure founded on flexible foundation.

INTRODUCTION

Over the past two decades the subject of structure-soil interaction of foundations and buildings subjected to seismic excitation has been receiving increasing attention. A situation is encountered with increasing in frequency in recent years with respect to the seismic design of massive structure at a considerable depth of soil deposit. This is often the case in the design of Nuclear Power Plants. The important aspect of such a structure under seismic excitation is to establish the dynamic interaction between the structure and the surrounding soil. The study is usually made by representing the effects of the soil on the structural response by a series of springs and dashpots representing a theoretical half space surrounding the structure as shown in Fig.1. Various methods have been proposed incorporating the effect of seismic excitation in the analysis and design of such structures. Arnold, Bycroft and Warburton(6), Bycroft(5) and Gladwell(7) considered all the four modes of vibration with respect to the study of vibration of circular footings subjected to horizontal and rocking vibrations. In these works coupling effect of sliding and rocking has been neglected. Karasudhi, Keer and Lee(9) presented an analytical solution for the vertical, coupled horizontal and rocking vibration of an infinitely long rigid footing resting on the surface of an elastic half space and

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proved the significance of these effects. This has also been studied by Luco and Westmann(8) with an exact solution for a particular value of poisson's ratio of 0.50. The problem has been further studied by Ang and Harper(1) and Agabein, Parmelee and Lee(2). The system being treated as a lumped parameter system consisting of mass points and stress points to simulate the semiinfinite, linearly elastic, homogeneous and isotropic media.

The effect of embedment of footing has been studied by several investigators. No rigorous analytical solution of embedded footings are available because of the mathematical computational complexities. Barnov(3) is the first investigator who determined an approximate analytical approach assuming an elastic half space under the footing base and a series of independent elastic layers between the free surface and the half space. Novak and Beredugo(14) used this model to arrive at a series solution for stiffness and damping function for the vertical vibration of a circular footing with reference to a half space or stratum, and they further extended the problem considering the coupled horizontal and rocking vibration of the same system as shown in the reference Beredugo and Novak (10). Parmelee, R.A. et al(11) studied the structure-soil interaction problem under seismic loading taking into account the foundation flexibility.

The aim of the present study is to introduce the dynamic coupling or interaction between a single storey elastic structure and the flexible elastic foundation medium, when the system is subjected to seismic loading. The foundation medium being represented by isotropic, homogeneous, elastic half space and the ground acceleration is stimulated by a transient time dependent function as shown in Fig.3, which resembles those of strong motion seismic disturbance. The embedment effect has been taken into account and a study has been made of the influence of shear wave velocity, poissions ratio and depth of embedment on the structure-soil interaction.

EQUATIONS OF MOTION

The structure foundation system as shown in(Fig.1) is a model which is circular in plan and has three degrees of freedom i.e., the horizontal translation of the top mass 'm' and the horizontal motion of the rigid 'm$_b$' and the rotation of the system about an axis on the horizontal boundry plane through the point 'b'. The deformed shape of the building is shown in Fig.2 and the corresponding equations of motion are:

$$\ddot{u} + \ddot{u}_b + \ddot{u}_\phi + 2\omega_r\lambda\dot{u} + \omega_r^2 u = -\ddot{u}_g \qquad .. (1)$$

$$\ddot{u} + \alpha\ddot{u}_b + \ddot{u}_\phi + A_1\dot{u}_b + A_2\dot{u}_\phi + A_3u_b + A_4u_\phi = -\alpha\ddot{u}_g \qquad .. (2)$$

$$\ddot{u} + \ddot{u}_b + \eta\ddot{u}_\phi + A_7\dot{u}_b + A_5\dot{u}_\phi + A_8u_b + A_6u_\phi = -\ddot{u}_g \qquad .. (3)$$

where $\alpha = (m + m_b)/m$, $\eta = 1 + \alpha(b/2h)^2$, $A_1 = d_T/m$, $A_2 = d_{RT}/m$,

$A_3 = k_T/m$, $A_4 = k_{RT}/m$, $A_5 = d_R/mh^2$, $A_6 = k_R/mh^2$, $A_7 = d_{RT}/mh^2$,

$A_8 = k_{RT}/mh^2$, $\omega_r^2 = k/m$, $\lambda = C/2\sqrt{km}$, k_T, d_T, k_R, d_R, k_{RT} and d_{RT} are the respective dynamic interaction coefficient as given by Parmelee and Kudder(12).

To study the seismic response of the interaction system, it is necessary to simulate into an actual earthquake acceleration function by an approximate acceleration function, which will be represented by a series of hormonic components. This function should have those principles which characterise a strong motion earthquake which is a random acceleration function of the following form given by Bogdanoff, Goldberg and Bernard(4)(Fig.3),

$$\ddot{u}_g(t) = 0.5\ te^{-0.333t} \sum_{j=1}^{10} \cos\ (\omega_j t + \psi_j) \quad \ldots\ldots \quad t \geq 0 \quad .. (4)$$

Expanding each term of the above equation into a fourier series the earthquake acceleration takes the form

$$\ddot{u}_g(t) = \sum_{j=1}^{10} \sum_{n=1}^{N} b_{nj} \sin \Omega_n(t) \quad \ldots\ldots \quad t \geq 0 \quad .. (5)$$

where $\Omega_n = n\pi/T$, b_{nj} is the fourier constant. N = Number of terms required to represent the function. T = Time interval over which the function is to be represented.

The equation of motion 1,2 and 3 can be solved 'N' times for each harmonic $\ddot{u}_{gn}(t)$ of the input function $\ddot{u}_g(t)$. Hence the total response will be the superposition of 'N' components given by the following equations:

$$u_b(t) = \sum_{n=1}^{N} u_{bn}(t) \quad .. (6)$$

$$u_\phi(t) = \sum_{n=1}^{N} u_{\phi n}(t) \quad .. (7)$$

$$u(t) = \sum_{n=1}^{N} u_n(t) \quad .. (8)$$

RESULTS AND DISCUSSION

Numerical results are obtained using computer for various values of shear wave velocity(V_s), embedment depth ($\delta = D/b$), where 'D' the depth of embedment of the structure and 'b' the half width of the building, poisson's ratio(μ). The elastic foundation medium(ρ), fundamental period of the structure(T_s) and damping ~~factor~~ (λ) were assumed to be equal to 120pcf,0.5 sec. and 0.01 respectively. Values of shear wave velocity, embedment depth and poisson's ratio are varied from 300ft.sec. to 600ft.sec., 0.0 to 0.4 and 0.00 to 0.50 respectively. The interaction coefficients for the embedded structure are obtained by modifying the surface interaction coefficients as (Parmelee, R.A. and Kudder, R.J.) $k_{Te} = \Delta k_T$, $d_{Te} = \Delta d_T$, $k_{RTe} = \Delta k_{RT}$, $d_{RTe} = \Delta d_{RT}$, $k_{Re} = \Delta k_R$ and $d_{Re} = \Delta d_R$ where $\Delta = e^{1.10\delta}$.

In order to compare the effect of the coupled response of the interaction system of the present study with that of the uncoupled system Parmelee et al (11) the response curves are presented in Figs. 5 and 4 respectively for m = 3900lb.-sec.^{2} per ft., $\alpha = 0.42$, $\lambda = 0.01$, $V_s = 300$ft. sec. $\rho = 120$pcf, $\mu = 0.0$ and $T_r = 0.50$sec. From these figures it can be inferred that the interaction study for the response of the system is more effective if the coupled effect of the system is taken into account. The coupled response parameters u_ϕ, u_b and u are plotted against poisson's ratio for a given values of shear wave velocity and embedment depth in Fig.6 and Fig.7. It is seen from these figures the values of u_ϕ and u_b decreases considerably with increase in poisson's ratio whereas the flexural response increases. Hence the coupled response of the system reduces the amplitude of motion when compared with the motion observed at the surface. It is also seen from these figures that an increase in the value of shear velocity reduces the values of u_ϕ, u_b and u considerably. Thus it may be concluded from the results that the intermodal coupling parameters k_{RT} and d_{RT} significantly influence the response of the interaction system consisting of building type structure resting on an elastic foundation.

CONCLUSIONS

From the numerical results presented, it may be concluded that the dynamic interaction coefficients are considerably effected by ρ, V_s, b and μ. The embedment depth also affects the vibration of the interaction system by reducing the resonant amplitude. The intermodal coupling terms k_{RT} and d_{RT} effect the response of the interaction system consisting of building type structures resting on an elastic foundation medium.

ACKNOWLEDGEMENT

The assistance of Dr.G.Mogalaiah in the preparation of the paper is greatfully acknowledged.

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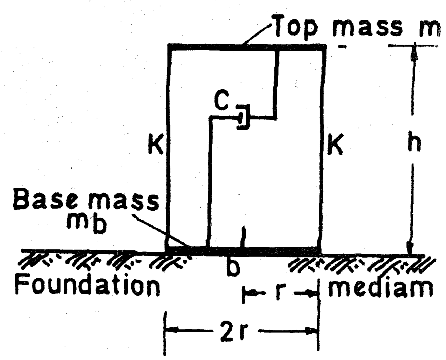


FIG. 1 - DYNAMIC MODEL

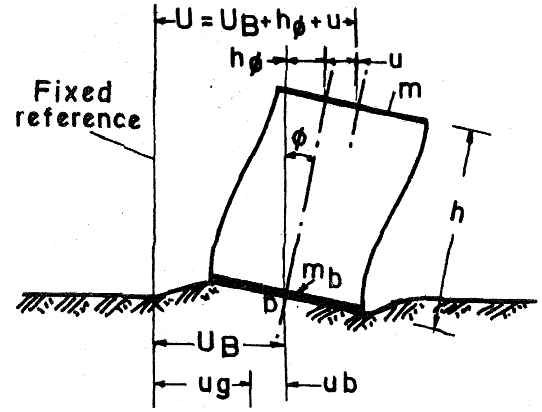


FIG. 2 - DEFLECTED POSITION

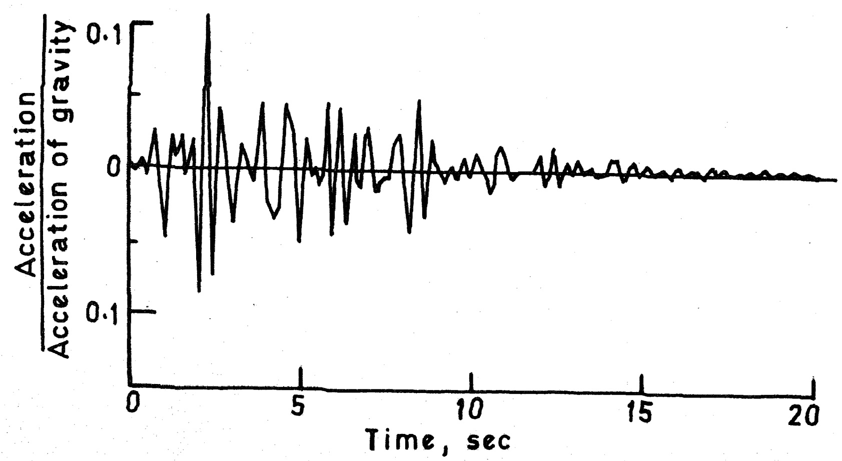


FIG. 3 - EARTHQUAKE ACCELERATION FUNCTION AS DEFINED BY EQN (4) AND (5)

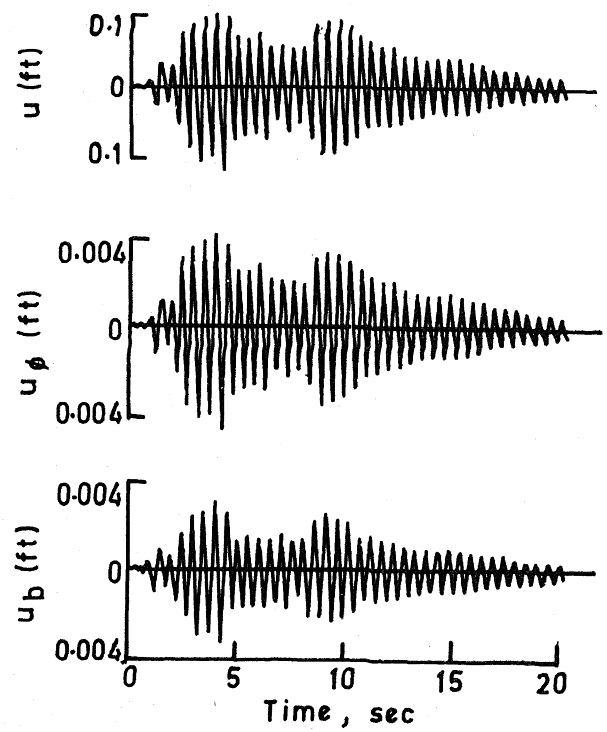


FIG. 4 - RESPONSE CURVES FOR EXAMPLE PROBLEM (Parmelee et al,11)

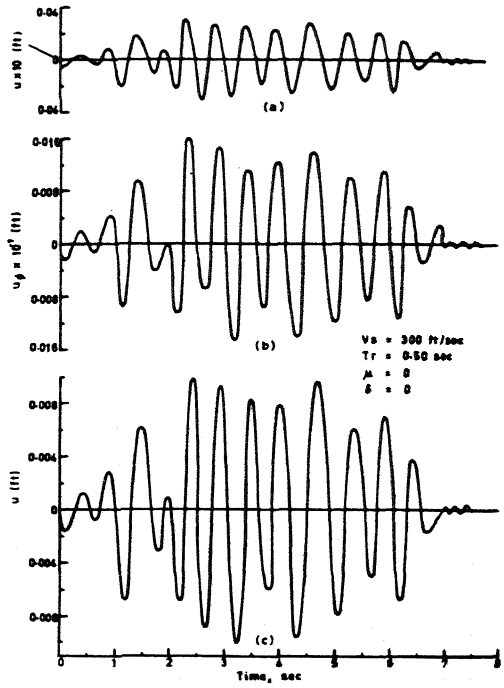


FIG. 5- RESPONSE CURVES FOR EXAMPLE PROBLEMS STUDIED

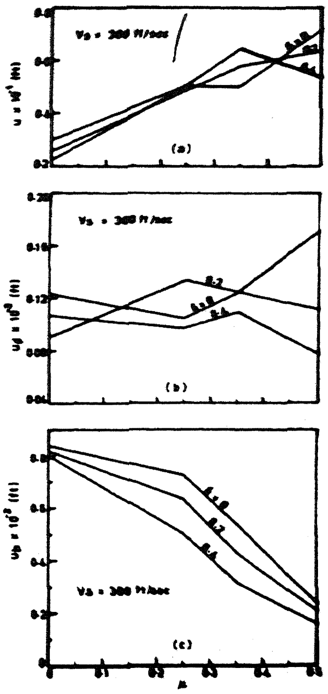
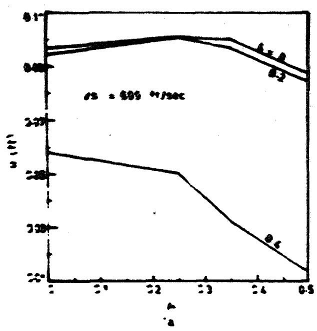
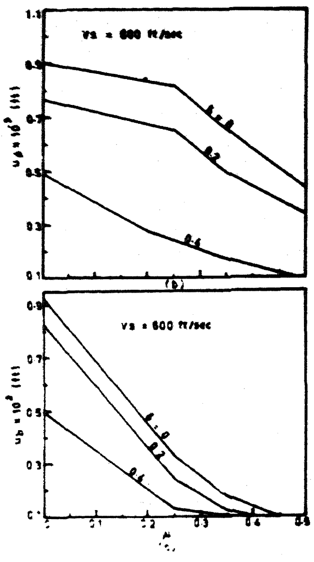


FIG 6





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