ANTIPLANE DYNAMIC SOIL-BRIDGE INTERACTION FOR INCIDENT PLANE SH-WAVES

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SYNOPSIS

A study is made of the dynamic interaction between the soil and a simple bridge erected on an elastic half-space. The two-dimensional bridge model consists of a series of elastic shear beams supported by rigid end-abutments, intermediate rigid piers and rigid foundations. The nature of the dynamic interaction depends on (1) the angle of incident plane SH-waves, (2) the ratio of the rigidity of the girders to the rigidity of the soil, (3) the ratio of the girders' mass to the masses of the abutment-foundation and pier-foundation systems, and (4) the relative ratios of the different span lengths.

I. THE MODEL

The bridge model, shown in Fig. 1-a, represents an extension of the simple model (a one-span bridge) which has been considered in our previous paper [1]. The model consists of: (i) The superstructure, which consists of a series of (N-1) two-dimensional girders simply supported on N rigid supports and allowed to deform in shear only. Each beam is isotropic and homogeneous; the rigidity and velocity of the shear waves in the jth beam are given by μ_{bj} and β_{bj}, respectively. (ii) The substructure, which consists of two rigid end-abutments and (N-2) rigid intermediate piers. (iii) The N rigid foundations, of semicircular cross-section, welded to the half-space and to the bridge piers. (iv) The soil, which is represented by elastic, isotropic and homogeneous half-space and with the rigidity μ_s and the velocity of the shear waves β_s.

II. FORMULATION OF THE PROBLEM

The general dynamic soil-structure interaction problem can be divided into three parts [1]: (1) the analysis of input motions (here represented by the incident seismic waves), or equivalently the determination of the driving forces, (2) the force-displacement relationships for the foundations, i.e., the impedance or compliance functions, and (3) the dynamic analysis of the structure. Once the solutions of the first two steps have been obtained for a class of foundations, the results can be used to calculate the interaction response of different structures. Wong and Trifunac [2] have determined the driving forces induced by harmonic plane SH-waves and the impedance functions for a class of two-dimensional embedded foundations with circular cross sections at different separation distances. Their results cover steps (1) and (2) mentioned above.

It is assumed that the input excitation is in the form of plane harmonic SH-waves with amplitudes equal to one and with an angle of incidence θ

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(Fig. 1-a). For this excitation the forces which are exerted by the soil on the N-foundations and are caused by the incident waves and the motion of the neighboring foundations (Fig. 1-b) are given by [1]

$$\{F_s\} = \{F_s^*\} + [K_s]\{\Delta\} ,$$

where $\{F_s\} = \{F_{s1},\ldots, F_{sj},\ldots, F_{sN}\}$ is the vector of the forces exerted on the N-foundations, and $\{F_s^*\}$ is the vector of the driving forces; the latter corresponds to the forces that the ground exerts on the embedded foundation when the foundation is kept fixed under the action of the seismic waves. $[K_s]$ is the impedance matrix for the embedded foundations; it depends on the characteristics of the foundation and the soil and on the frequency of the motion. Finally, $\{\Delta\} = \{\Delta_1,\ldots, \Delta_j,\ldots, \Delta_N\}$ is the vector of the displacement amplitudes of the foundations; it is unknown and depends on the soil-structure interaction of each of the (N-1) girders, the vibration of the other foundations, and the characteristics of the incoming wave.

The displacements u_j and v_j, $j = 1, 2,\ldots, (N-1)$, of the bridge girders are selected to be zero, while the displacement w_j depends only on the coordinate x_j. This displacement $w_j(x_j, t)$ must satisfy the equations of motion of undamped shear beams and the displacement boundary conditions $\{\Delta\} e^{i\omega t}$ [1].

The end resisting forces, per unit length, acting on the two abutments and the piers (Fig. 1-b) can be described by

$$\{F_b(t;\omega)\} = [K_b(\omega)]\{\Delta\}e^{i\omega t} ,$$

where $[K_b(\omega)]$ is a stiffness matrix corresponding to the support forces; it is a tridiagonal matrix of order $N\times N$ and is also frequency-dependent. It is convenient to recall here that the undamped natural frequencies of the simply supported shear beams and the corresponding mode-shapes are given by

$$\omega_n^{(j)} = (n\pi/L_j)\beta_{bj} \text{ and } W_n^{(j)}(x_j) = \sin(n\pi x_j/L_j),\ n = 1,2,3,4,\ldots,\ j = 1,2,\ldots,(N-1).$$

III. DYNAMIC SOIL-BRIDGE INTERACTION

The unknown foundation displacement amplitudes $\{\Delta\}$ can be determined from the balance of forces exerted on each foundation as follows

$$-\omega^2\{\Delta\}[M_f] = -(\{F_s\} + \{F_b\}) ,$$

where the left hand side represents the inertia forces on the rigid abutment or pier-foundation systems and $[M_f]$ is a diagonal $N\times N$ mass matrix. It can be shown that this result yields

$$(\omega^2[M_f] - [K_b] - [K_s])\{\Delta\} = \{F_s^*\} .$$

The foundation displacement amplitudes Δ_j, $j = 1,2,\ldots,N$ are uniquely determined by solving the N simultaneous, complex, and nonhomogeneous equations.

Numerical examples presented in Figs. 2, 3 and 4, for a two-span bridge, depend mainly on the angle of incident waves θ and on the following parameters: (1) $\eta = (\omega/\beta_s)R_{max}$, which is the dimensionless frequency which compares the wavelength λ_s of the incident wave to the maximum radius of the foundations, and is plotted along the x-axis in these figures. (2) M_{fj}/M_{sj} (or MF_j/MS_j), $j = 1,2,3$, which is the ratio of the mass of the jth abutment or pier-foundation system to the mass of the soil replaced by the jth foundation. (3) M_{bj}/M_{sj} (or MB_j/MS_j), $j = 1,2,3$, which is the ratio of the mass of the jth beam to the mass of the soil replaced by the jth foundation. (4) $\epsilon_1 = (\beta_s/\beta_{b1})(L_1/R_1)$ (or EPS1); this ratio reflects the relative stiffness of the bridge and the soil and also describes the ratio of the span to the radius of the left foundation. Large values of ϵ_1 indicate a more flexible bridge with respect to the soil and/or a longer span, while $\epsilon_1 = 0$ implies a rigid structure.

IV. INTERPRETATION OF THE INTERACTION

The three displacement amplitudes Δ_1, Δ_2 and Δ_3, computed for the excitation by the incident plane SH-waves, have been plotted against the dimensionless frequency η in Figs. 2, 3 and 4. Some of the most important phenomena of the interaction of the bridge and the soil through the three rigid pier-foundation systems and the dynamic characteristics of the bridge girders' response are as follows: (1) In all cases shown in Figs. 2, 3 and 4 the displacement amplitudes approach the low frequency limit of $|\Delta_1| = |\Delta_2| = |\Delta_3| = 2$, which corresponds to the displacement amplitude of the surface of the half-space for incident SH-waves with unit amplitude. (2) For some input frequencies, the amplitudes of the foundation responses are significantly larger than the free field surface displacement amplitude which could be obtained for the same excitation in the absence of the bridge. (3) The simplest type of interaction occurs for the vertical incidence of SH-waves and for a symmetric bridge and its abutments. In this case, the outside foundations are in phase and have the same amplitudes, while the amplitude of the middle foundation is slightly larger (Fig. 2) due to the interference effect caused by the scattering from the outside foundations. This effect increases with an increase in the size of the outside foundations (Fig. 4). When the two outside foundations are smaller than the middle one, the latter may amplify the motion for the outside foundations (Fig. 3). (4) Three observations may be made regarding non-vertically incident waves acting on a symmetric bridge with identical foundations (Fig. 2): (4a) - The peak amplitude of the displacement Δ_1 (left foundation) increases when the flexibility of the superstructure at shallow angles of incidence increases. Furthermore, the more flexible the girders, the higher the frequency at which the peak occurs. (4b) - The displacement Δ_2 of the middle foundation has a low amplitude at the lower frequencies, but the amplitude increases as the frequency increases. The standing wave pattern that occurs during this displacement provides a possible explanation for the initial dip in the amplitude. Increasing the flexibility of the superstructure, increases the amplitude at the low frequencies, and in some cases, causes an initial peak. (4c) - As the span increases the peak value increases and the values of the peaks occur at lower frequencies, even when the flexibility is constant. In general, as the span L_i increases, there is greater degree of fluctuation in all Δ_i ($i = 1, 2, 3$) amplitudes. When L_i is constant, the fluctuation of Δ_i decreases as the angle of incidence θ approaches $90°$; this occurs because, as θ approaches $90°$, the projected wavelength on the horizontal surface $\lambda_s^* = \lambda_s/\cos\theta$ becomes infinite. (5) When the sizes of the foundations of a symmetric bridge differ, more complicated interaction phenomena occur: (5a) - When the middle foundation is larger, Fig. 3, and the wave's angle of incidence is shallow, the response of the foundation first hit by the wave tends to be amplified, while the one hit last is usually shielded. (5b) - By comparing Figs. 2 and 3, it is seen that the response of the large middle foundation is not greatly affected by the smaller outside foundations. But the smaller foundations behave as if they were interacting with the large foundation; i.e., the middle foundation drives the outside foundations by its weight and size. (5c) - On the other hand, for a small middle foundation (Fig. 4), the larger outside foundations reflect significant wave energy back to the middle one which may magnify its motion. (6) The span length and the flexibility of the bridge girders, as well as the size of foundations, greatly affect the shielding and amplification of the foundations' response. When the sizes of the foundations or the lengths of the spans differ, the nonsymmetry of the structure accentuates and complicates phenomena such as shielding, amplification due to scattering of the waves from the other foundations and the standing wave pattern of the excitation of the three bridge abutments. (7) The excitation of different modes of vibration of the bridge girders is related to the nature of the foundation movement for different angles of incident SH-waves, in particular, it depends on the relative phase of motion for the two adjacent bridge supports. When two adjacent foundations move in phase, there is a tendency to excite symmetric modes of girder vibration, while when any two adjacent foundations move out of phase, the antisymmetric modes are excited more effectively. In all cases, shown in Figs. 2, 3, and 4, when $\eta = (n\pi/\epsilon_1)(R_{max}/R_1)$, $n = 1,2,3,\ldots$, i.e., when the frequency of the incident waves corresponds to the natural frequencies of one (or all) of the girders, one finds that $|\Delta_1| = |\Delta_2| = |\Delta_3|$. The different modes of vibration for two girders of two nonsymmetric bridges have been shown for vertical and non-vertical incident waves in Figs. 5-a and 5-b, respectively. In these figures, the peak response amplitudes decrease in the higher modes for the same ϵ_1. This observation gives an idea about the effect of radiation damping which is more effective for a vertical incident wave (Fig. 5-a). Furthermore, both figures indicate that the coupling of the different modes of the two adjacent spans depends on the span lengths (for the same ϵ_1). This coupling influences the phase difference between the three amplitudes Δ_1, Δ_2 and Δ_3. (8) Finally, the solution presented in this paper may provide a simple vehicle for better understanding of certain vibrational characteristics of multi-span structures. Even though the model studied is of a two-dimensional nature, and therefore cannot be employed directly even to laterally elongated systems, the interference phenomena of waves scattered around different foundations and the different phasing of input motions along the extended structure may prove to be useful in clarifying the complexity of the dynamic response of more complicated three-dimensional systems.

REFERENCES

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(2) H. L. Wong and M. D. Trifunac, "Two-dimensional, Antiplane, Building-Soil-Building Interaction for Two or More Buildings and for Incident Plane SH-waves," Bull. Seism. Soc. Amer., 1975, 1863-1885.

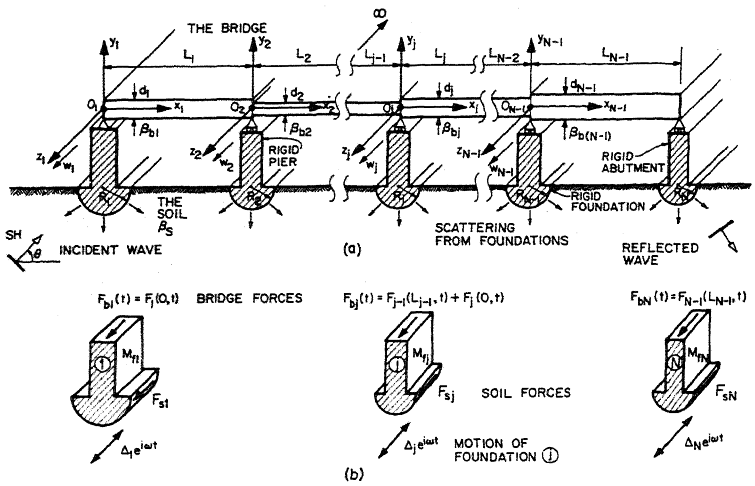
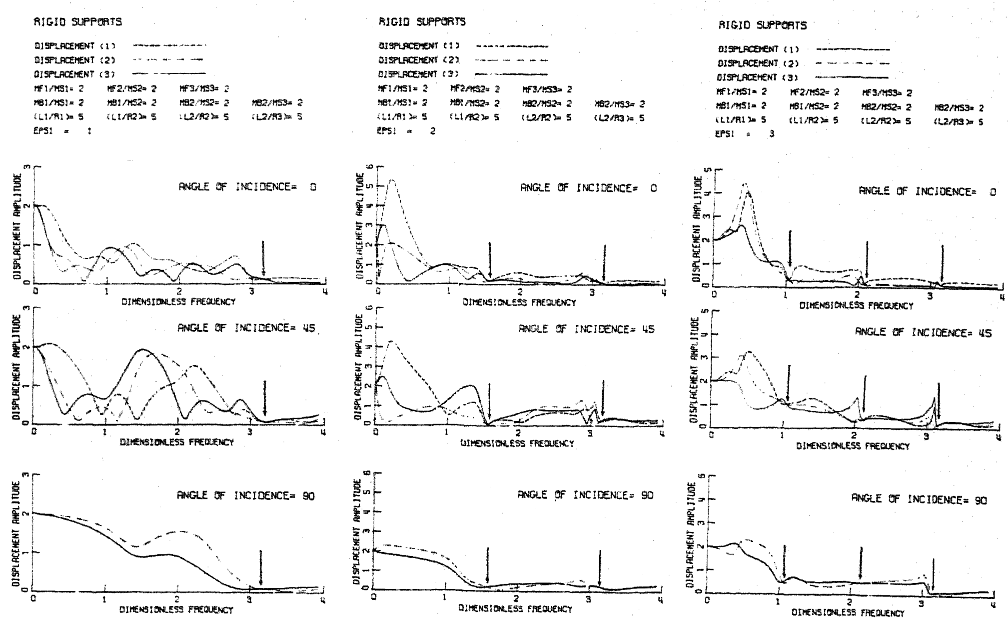


Figure 1

I-a THE MODEL OF THE BRIDGE AND FOUNDATIONS

I-b FORCES ACTING ON THE RIGID ABUTMENT / OR PIER – FOUNDATION SYSTEMS



(a) (b) (c)

Figure 2

FOUNDATION DISPLACEMENTS Δ_1 , Δ_2 , and Δ_3

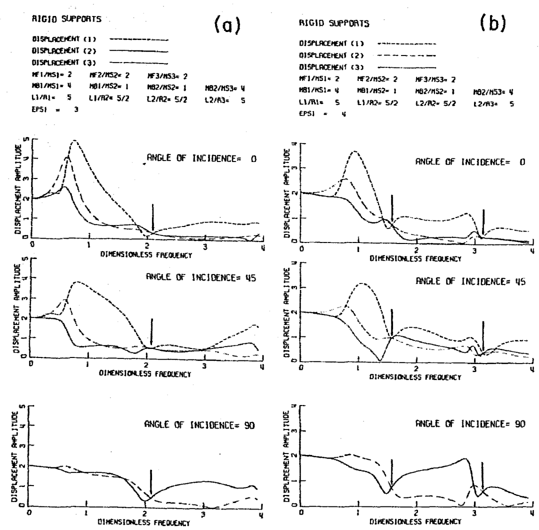


Figure 3

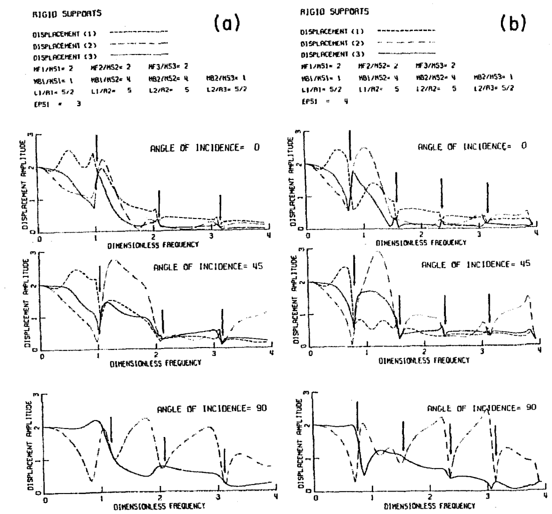
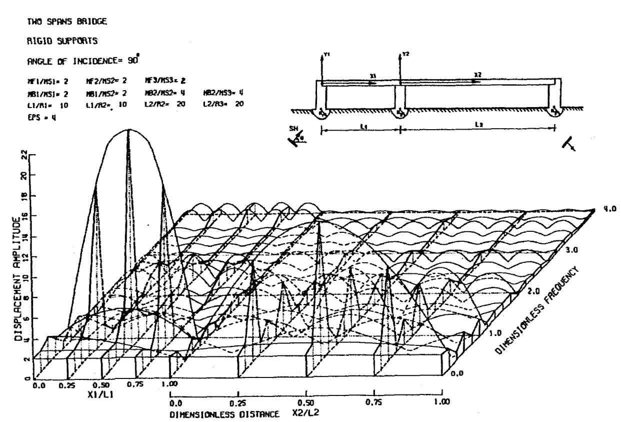
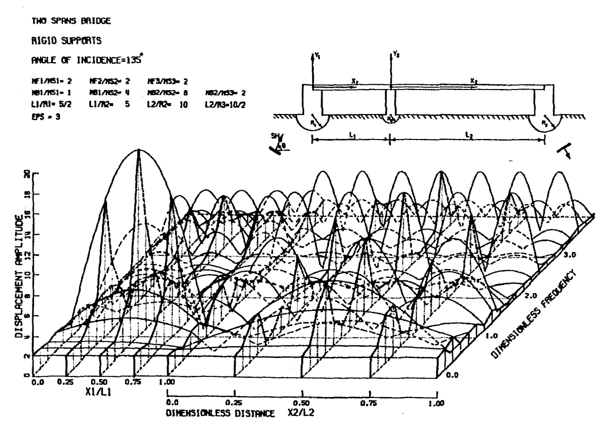


Figure 4

FOUNDATION DISPLACEMENTS Δ_1 , Δ_2 , and Δ_3



(a)



(b)

Figure 5

DISPLACEMENT AMPLITUDE AT DIFFERENT POINTS ON THE TWO-SPAN BRIDGE WITH VARIOUS MODE-SHAPES